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MULTI - OBJECTIVE GEOMETRIC PROGRAMMING AND ITS APPLICATION IN GRAVEL BOX PROBLEM

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Abstract: Multi-objective geometric programming (MOGP) is a strong tool for solving a type of optimization problem. This paper develops a solution procedure to solve a multi-objective non-linear programming problem using MOGP technique based on weighted-sum method, weighted-product method and weighted min-max method .The equivalent general Multi-objective geometric programming problems are formulated to find their corresponding value of the objective functions based on duality theorem. As the numerical example Gravel- box design problem is presented to illustrate the methods.

Key Words: Multi-objective Geometric programming, Weighted-sum method, Weighted-product method, weighted min-max method, Gravel box.

INTRODUCTION

Geometric programming (GP) is a technique to solve the special class of non linear programming problems subject to linear or non-linear constraints. The original mathematical development of this method used the arithmetic–geometric mean inequality relationship between sums and products of real numbers. In 1967 Duffin, Peterson and Zener put a foundation stone to solve wide range of engineering problems by developing basic theories of geometric programming in the book **Geometric Programming [3]**. Beightler and Phillips gave a full account of entire modern theory of geometric programming and numerous examples of successful applications of geometric programming to real-world problems in their book **Applied Geometric Programming [1]**. GP method has certain advantages.

The advantage is that it is easy to solve the dual problem than primal. Multi-objective geometric programming problem is a special class of non-linear programming problem with multiple objective functions. In many real-life optimization problems, multi-objectives have to be taken into account which may be related to the economical, technical, social and environment aspects of optimization problems. In multi-objective optimization , the trade –off information between different objective functions is probably the most important piece of information in a solution procedure to reach the most preferred solution .GP Liu, JB Yang, JF Whidborne gave an account with multi-objective geometric programming in their book **Multi-objective Optimization and Control [5].** In this field a paper named Multi-objective geometric programming problem being cost coefficient as a continuous function with mean method by A.K. Ojha, A.K. Das has been published in the journal of computing 2010 [7].In 1992 M.P.Bishal [**9**] and in 1990 R.k.verma [**10**] has studied fuzzy programming technique to solve multi-objective geometric programming problems. In our paper we have discussed the basic concepts and principles of multi-objective optimization problem and then developed typical multi-objective methods.

FORMULATION OF MULTIOBJECTIVE GEOMETRIC PROGRAMMING PROBLEM

A multi-objective geometric programming problem can be defined as

Find $x=(x_1, x2,\ldots\ldots x_n)^T$,so as to

Min: $f_{k0}(x)= \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$ $k=1,2,\ldots\ldots,p$ ……….(1)

such that $f_i(x) = \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$, $i=1,2,\ldots\ldots,m$

$x_j > 0$, $j= 1,2,\ldots\ldots,n$

Where $c_{k0t} >0$ for all k and t . a_{itj} ,a_{k0tj} are all real ,for all i,k,t,j .

If $f_{10}(x)$,$f_{20}(x)$,……………. $f_{p0}(x)$ are p objective functions for any vector $X=(x_1, x_2, \ldots\ldots, x_n)^T$.

Let $w=(w: w\in R^n, w_k >0, \sum_{k=1}^{n} w_k = 1)$ be the set of non-negative weights . Using weighted sum method the above multi-objective functions in (1) can be written as

Min $\sum_{k=1}^{p} w_k \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$.

So multi-objective optimization problem reduces to a single objective geometric programming problem as,

Min $\sum_{k=1}^{p} w_k \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$. ……………….(2)

Such that $f_i(x)=\sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$, $i= 1, 2, \ldots\ldots, m$

$X_j>0$; $j= 1, 2,\ldots\ldots,n$

Solution procedure of Multi-Objective Geometric Programming Problem based on weighted sum method (MOGPP$_{ws}$):

The corresponding dual problem of (2) is Maximize d(w) =

$$\prod_{k=1}^{p}\prod_{t=1}^{T_{k0}}\left(\frac{w_k\, c_{k0t}}{w_{kt}}\right)^{w_{kt}} \prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{c_{it}}{w_{it}}\right)^{w_{it}} \prod_{i=1}^{m}\lambda_i(w)^{\lambda_i(w)}$$

Where $\lambda_i(w) =\sum_{t=1}^{T_i} w_{it}$, $i= 1,2,\ldots\ldots m$

Subject to $\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} w_{kt} =1$,

$\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} a_{k0tj}\ w_{kt} + \sum_{i=1}^{m}\sum_{t=1}^{T_i} a_{itj}\ w_{it} =0$, $j=1,2,\ldots\ldots,n$

$W_{kt} \ge 0$, $k= 1,2,\ldots\ldots,p$
$t=1,2,\ldots\ldots, T_{k0}$

$w_{it}\ge 0$, $i= 1,2,\ldots\ldots,m.$

Using weighted product method the multi-objective functions in (1) can be written as,

Min $\prod_{k=1}^{p}(\sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}})^{w_k}$

So multi-objective optimization problem reduces to a single objective geometric programming problem as,

Min $\prod_{k=1}^{p}(\sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}})^{w_k}$ …………………. (3)

Such that $f_i(x) = \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$, i= 1, 2, ……………..,m

$x_j > 0$, j=1,2,………………….,n

Solution procedure of multi-objective geometric programming problem based on Weighted product method :(MOGPT$_{wp}$):

The corresponding dual problem of (3) is

Max d (w) = $\left(\frac{1}{w_0}\right)^{w_0} \prod_{k=1}^{p}\prod_{t=1}^{T_{k0}}\left(\frac{c_{k0t}}{w_{kt}}\right)^{w_{kt}} \prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{c_{it}}{w_{it}}\right)^{w_{it}} \prod_{k=1}^{p} w_k^{w_k} \prod_{i=1}^{m}\lambda_i^{\lambda_i}$ Where λ_i(w) $=\sum_{t=1}^{T_i} w_{it}$, i= 1, 2,…………….,m

Subject to $w_0 = 1$,

$w_0 w_k - \sum_{t=1}^{T_{k0}} w_{kt} = 0$, k=1,2,……………….p

$\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} a_{k0tj} w_{kt} + \sum_{i=1}^{m}\sum_{t=1}^{T_i} a_{itj} w_{it} = 0$, j=1,2,………………..,n

$w_{kt} \ge 0$, $\binom{k=1,2,\ldots\ldots p}{t=1,2\ldots\ldots T_{k0}}$

$w_{it} \ge 0$, $\binom{i=1,2,\ldots\ldots m}{t=1,2,\ldots\ldots T_i}$

Using Min-max method the multi-objective optimization functions in (1) can be written as,

Min-max $w_k \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$ k=1,2,……………,.p

Let λ =max $w_k \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$

So multi-objective optimization problem reduces to a single objective geometric programming problem as,

Min λ ……………..(4)

Such that $\frac{1}{\lambda} w_k \sum_{t=1}^{T_{k0}} g_k c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}} \le 1$, k=1,2,…………….,p

And $f_i(x)= \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$, i= 1,2,……………,m

Solution procedure of multi-objective geometric programming problem based on Min-max method :(MOGPT$_{mm}$):

The corresponding dual problem of (4) is

Max d (w) = $\left(\frac{1}{w_0}\right)^{w_0} \prod_{k=1}^{p}\prod_{t=1}^{T_{k0}}\left(\frac{w_k c_{k0t}}{w_{kt}}\right)^{w_{kt}} \prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{c_{it}}{w_{it}}\right)^{w_{it}} \prod_{k=1}^{p}\lambda_p^{\lambda_p} \prod_{i=1}^{m}\lambda_i^{\lambda_i}$

Where λ_i(w) $=\sum_{t=1}^{T_i} w_{it}$, i= 1, 2,…………,m and λ_p(w) $=\sum_{t=1}^{T_i} w_{kt}$, k= 1,2,…………,p

Subject to $w_0 = 1$,

$w_0 - \sum_{t=1}^{T_{k0}} w_{kt} = 0$, k=1,2,……………,p

$\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} a_{k0tj} w_{kt} + \sum_{i=1}^{m}\sum_{t=1}^{T_i} a_{itj} w_{it} = 0$, (j=1,2,…………n)

$w_{kt} \ge 0$, $\binom{k=1,2,\ldots\ldots p}{t=1,2\ldots\ldots T_{k0}}$

$w_{it} \ge 0$, $\binom{i=1,2,\ldots\ldots m}{t=1,2,\ldots\ldots T_i}$

Degrees of Difficulty:

Degrees of difficulty play an important role to solve the multi-objective geometric programming problems. It is defined as follows.

DD (degrees of difficulty)=Total number of terms – (Number of variables+1)

Degrees of difficulty of problem (1.1) based on weighted sum method

i.e. $DD_{Ws} = \sum_{i=1}^{p} T_{i0} + \sum_{i=1}^{m} T_i$ - (n+1)

Degrees of difficulty of problem (1.1) based on weighted product method i.e. $DD_{Wp} = 1 + \sum_{i=1}^{p} T_{i0} + \sum_{i=1}^{m} T_i$ – (n+p+1)

Degrees of difficulty of problem (1.1) based on weighted min-max method i.e. $DD_{mm} = 1 + \sum_{i=1}^{p} T_{i0} + \sum_{i=1}^{m} T_i$ – (n+1+1).

Clearly $DD_{Wp} \le DD_{Ws} \le DD_{mm}$.

Example 1: min $g_{01} = \frac{40}{x_1x_2x_3} + 40x_2x_3$

min $g_{02} = \frac{800}{x_1x_2x_3}$

Such that $x_1x_2 + 2x_1x_3 \le 1$,

$x_1, x_2, x_3 > 0$.

The corresponding primal geometric programming problem based on weighted-sum method is

Min g $= w_1(\frac{40}{x_1x_2x_3} + 40x_2x_3) + w_2(\frac{800}{x_1x_2x_3})$

Such that $x_1x_2 + 2x_1x_3 \le 1$,

$x_1, x_2, x_3 > 0$.

The corresponding primal geometric programming problem based on weighted-product method is

Min $g(x_1, x_2, x_3, x_4) = \left(\frac{800}{x_1x_2x_3}\right)^{w_1} x_4^{w_2}$

Such that $\frac{1}{4}x_1x_2 + \frac{1}{2}x_1x_3 \le 1$,

$\frac{40}{x_1x_2x_3x_4} + \frac{40x_2x_3}{x_4} \le 1$,

$x_1, x_2, x_3 > 0$.

The correponding primal geometric programming problem based on weighted-min-max method is

Min λ

Such that

$$\frac{1}{\lambda} w_1 \left(\frac{40}{x_1x_2x_{34}} + 40x_2x_3\right) \le 1,$$

$$\frac{1}{\lambda} w_2 \frac{800}{x_1x_2x_3} \le 1,$$

$\frac{1}{4}x_1x_2 + \frac{1}{2}x_1x_3 \le 1,$

$x_1, x_2, x_3 > 0.$

Here $DD_{wp}=0$, $DD_{ws}=0$, $DD_{mm}=1$

Example 2:

$\min G_{01}=x_1{}^2x_2{}^{-1} + x_1{}^3x_3$

$\min G_{02}=x_1{}^{-1}x_2{}^{-1}x_3{}^{-1}$

such that $2x_1x_2+x_1x_3 + x_2x_3 \le 1$

$x_1, x_2, x_3 > 0.$

The corresponding primal geometric programming problem based on weighted-sum method is

Min g $=W_1(x_1{}^2x_2{}^{-1} + x_1{}^3x_3) + w_2(x_1{}^{-1}x_2{}^{-1}x_3{}^{-1})$

such that $2x_1x_2+x_1x_3 + x_2x_3 \le 1$,

$x_1, x_2, x_3 > 0$

The corresponding primal geometric programming problem based on weighted-product method is

$\min g = (x_4)^{w_1}(x_1{}^{-1}x_2{}^{-1}x_3{}^{-1})^{w_2}$

Such that $2x_1x_2+ x_1x_3 + x_2x_3 \le 1,$

$\frac{1}{x_4}(x_1{}^2x_2{}^{-1} + x_1{}^3x_3) \le 1,$

$x_1, x_2, x_3 > 0.$

The corresponding primal geometric programming problem based on weighted-min-max method is

Min λ

Such that $\frac{w_1}{\lambda}(x_1{}^2x_2{}^{-1} + x_1{}^3x_3) \le 1,$

$\frac{w_2}{\lambda}(x_1{}^{-1}x_2{}^{-1}x_3{}^{-1}) \le 1,$

$2x_1x_2+ x_1x_3 + x_2x_3 \le 1,$

$x_1, x_2, x_3 > 0.$

Here $DD_{wp}=1$, $DD_{ws}=2$, $DD_{mm}=2$

Clearly above examples show $DD_{Wp} \le DD_{Ws} \le DD_{mm}$.

MULTI-OBJECTIVE GRAVEL BOX DESIGN PROBLEM

Here we have taken gravel box design problem with minor modification from [1]. A total of 800 cubic-meters of gravel is to be ferried across a river on a barrage. A box (with an open top) is to be built for this purpose. The transport cost per round trip of barrage of box is Rs .05; the cost of materials of the ends of the box are Rs20/m^2. and other two sides and bottom are made from available scrap materials . Find the dimension of the box that is to be built for this purpose to minimize the transport cost and material cost.

Let length = x_1 m, width = x_2 m , height = x_3 m. The area of the ends of the gravel box = x_2x_3 m^2 . Area of the sides = x_1x_3 m^2. Area of the bottom = x_1x_2 m^2 .The volume of the gravel box=$x_1x_2x_3$ m^3. Transport cost: Rs $\frac{800}{x_1x_2x_3}$. Material cost: $40x_2x_3$. So the multi-objective geometric programming is

$$\min g_{01}(x_1, x_2, x_3) = \frac{40}{x_1x_2x_3} + 40x_2x_3 \qquad \ldots\ldots\ldots\ldots.(5)$$

$$\min g_{02}(x_1, x_2, x_3) = \frac{800}{x_1x_2x_3},$$

Such that $x_1x_2 + 2x_1x_3 \le 4,$

$x_1, x_2, x_3 > 0.$

Solution procedure of the above example by Weighted sum method:

According to $MOGPT_{ws}$

$$\text{Min } g(x_1, x_2, x_3) = w_1\left(\frac{40}{x_1x_2x_3} + 40x_2x_3\right) + w_2 \frac{800}{x_1x_2x_3} \ldots\ldots\ldots\ldots(6)$$

$$= \frac{40w_1+800w_2}{x_1x_2x_3} + 40w_1x_2x_3$$

Such that $\frac{1}{4}x_1x_2+\frac{1}{2}x_1x_3 \le 1,$

$x_1, x_2, x_3 > 0.$

Here DD =4-(3+1)=0

DGPP (dual geometric programming problem) of (6) is

$$\text{Max } d(w) = \left(\frac{40w_1+800w_2}{w_{01}}\right)^{w_{01}} \left(\frac{40w_1}{w_{02}}\right)^{w_{02}} \left(\frac{1}{4w_{11}}\right)^{w_{11}} \left(\frac{1}{2w_{12}}\right)^{w_{12}} \times (w_{11}+w_{12})^{(w_{11}+w_{12})} \ldots\ldots\ldots\ldots(7)$$

Such that $w_1+ w_2 =1,$

$w_{01}+ w_{02} =1,$

$- W_{01}+w_{11}+w_{12}=0,$

$- w_{01}+w_{02}+w_{11}=0,$

$- w_{01}+w_{02}+w_{12}=0,$

$w_{01}, w_{02}, w_{11}, w_{12} \ge 0.$

Solving the above normal and orthogonal conditions we get $w_{01} = \frac{2}{3}$, $w_{02}=\frac{1}{3}$, $w_{11}= \frac{1}{3}$, $w_{12}=\frac{1}{3}$.

Primal- dual variable relations are:

$\frac{40w_1+800w_2}{x_1x_2x_3} = w_{01}\, d(w),$

$40w_1x_2x_3 = w_{02}\, d(w),$

$\frac{1}{4}x_1x_2 = \frac{w_{11}}{w_{11}+w_{12}}$,

$\frac{1}{2}x_1x_3 = \frac{w_{12}}{w_{11}+w_{12}}$,

$X_3 = \left(\frac{w_1+20w_2}{8w_1}\right)^{\frac{1}{3}}$, $x_2 = 2x_3$, $x_1 = \frac{1}{x_3}$.

Table -1:Optimal solution of problem (5) by weighted sum method

Weights W_1,w_2	**Optimal dual variables**	**Optimal primal variables**			**Optimal objective functions**	
		X_1^*	**X_2^***	**X_3^***	**g_{01}^***	**g_{02}^***
$W_1=.1$ $W_2=.9$	$w^*_{01}=\frac{2}{3}$ $W^*_{02}=\frac{1}{3}$	.3535	5.6566	2.8283	647.01	141.45
$W_1=.2$ $W_2=.8$		.4622	4.3267	2.1633	383.64	184.92
$W_1=.3$ $W_2=.7$		.5516	3.6258	1.8129	273.96	220.64
$W_1=.4$ $W_2=.6$		.6366	3.1413	1.5706	210.08	254.71
$W_1=.5$ $W_2=.5$		.7249	2.7589	1.3794	166.72	289.99

W_1=.6 W_2=.4	$W^*_{11}=\frac{1}{3}$ $W^*_{12}=\frac{1}{3}$	.8233	2.4291	1.2145	134.47	329.37
W_1=.7 W_2=.3		.9419	2.1232	1.0616	109.00	376.81
W_1=.8 W_2=.2		1.1006	1.8171	0.9085	88.04	440.30
W_1=.9 W_2=.1		1.3540	1.4770	0.7385	70.71	541.67

The table-1 shows different optimal solutions for different weights of the problem (5) by weighted-sum method. First objective gives better optimal result when w_1 increases. Similarly second objective gives better optimal result when w_2 increases.

Solution procedure of the problem (5) by weighted product method:

According to $MOGPT_{wp}$

Min $g\,(x_1,x_2,x_3) = \left(\frac{800}{x_1x_2x_3}\right)^{w_1}\left(\frac{40}{x_1x_2x_3}+40x_2x_3\right)^{w_2}$

Such that $x_1x_2+2x_1x_3 \le 4$.

Let $\frac{40}{x_1x_2x_3}+40x_2x_3 \le x_4$

Then the above geometric programming problem becomes

Min $g \quad (x_1,x_2,x_3,x_4) = \left(\frac{800}{x_1x_2x_3}\right)^{w_1} \quad x_4^{\,w_2}$ …………………(8)

Such that $\frac{1}{4}x_1x_2+\frac{1}{2}x_1x_3 \le 1$,

$\frac{40}{x_1x_2x_3x_4}+\frac{40x_2x_3}{x_4} \le 1,$

$X_1, x_2, x_3, x_4 > 0.$

Here DD = 5-(4+1) =0

DGPP of (8) is Max d(w) = $\left(\frac{800^{w_1}}{w_{01}}\right)^{w_{01}}\left(\frac{1}{4w_{11}}\right)^{w_{11}}\left(\frac{1}{2w_{12}}\right)^{w_{12}}$

$(w_{11}+w_{12})^{w_{11}+w_{12}}\left(\frac{40}{w_{21}}\right)^{w_{21}}\left(\frac{40}{w_{22}}\right)^{w_{22}}+(w_{21}+w_{22})^{w_{21}+w_{22}}$ ……….(9)

Such that $w_1+w_2 = 1$,

$W_{01} = 1$,

$-W_1\, w_{01}+w_{11}+w_{12}-w_{21}=0$,

$-w_1\, w_{01}+w_{11}-w_{21}+w_{22}=0$,

$-w_1\, w_{01}+w_{12}-w_{21}+w_{22}=0$,

$w_2\, w_{01}-w_{21}-w_{22} = 0$,

$W_{01}, w_{11}, w_{12}, w_{21}, w_{22} \ge 0$.

Primal dual variable relations are:

$\left(\frac{800}{x_1x_2x_3}\right)^{w_1} x_4^{\,w_2} = w_{01}\, d(w)$,

$\frac{x_1x_2}{4} = \frac{w_{11}}{w_{11}+w_{12}}$,

$\frac{x_1x_3}{2} = \frac{w_{12}}{w_{11}+w_{1x2}}$,

$\frac{40}{x_1x_2x_3x_4} = \frac{w_{21}}{w_{21}+w_{22}}$,

$\frac{40x_2x_3}{x_4} = \frac{w_{22}}{w_{21}+w_{22}}$.

Solving the above DGPP (9) subject to the normal and orthogonal conditions we get

$W_{01}=1$, $w_{11}=\frac{1}{3}$, $w_{12}=\frac{1}{3}$, $w_{21}=\frac{2w_2-w_1}{3}$, $w_{22}=\frac{1}{3}$.

So $x_1 = [4(2-3w_1)]^{\frac{1}{3}}$, $x_2=\frac{2}{x_1}$, $x_3=\frac{x_2}{2}$.

$W_1 < \frac{2}{3}$, i.e. $w_1 < 0.6$

Table-2: Optimal solution of problem (5) by weighted product method

Weights	Optimal dual variables	Optimal primal variables			Optimal objective functions	
w_1 ,w_2		X_1^*	X_2^*	X_3^*	g_{01}^*	g_{02}^*
W_1=0.1 W_2=0.9	$w^*_{01}=1$, $w^*_{11}=\frac{1}{3}$, $w^*_{12}=\frac{1}{3}$, $w^*_{21}=\frac{17}{30}$, $w^*_{22}=\frac{1}{3}$	1.89	1.05	0.52	60.60	775.23
W_1=0.2 W_2=0.8	$w^*_{01}=1$, $w^*_{11}=\frac{1}{3}$, $w^*_{12}=\frac{1}{3}$, $w^*_{21}=\frac{7}{15}$, $w^*_{22}=\frac{1}{3}$	1.77	1.12	0.56	61.11	720.62
W_1=0.3 W_2=0.7	$w^*_{01}=1$, $w^*_{11}=\frac{1}{3}$, $w^*_{12}=\frac{1}{3}$, $w^*_{21}=\frac{11}{30}$, $w^*_{22}=\frac{1}{3}$	1.63	1.22	0.61	62.74	659.49
W_1=0.4 W_2=0.6	$w^*_{01}=1$, $w^*_{11}=\frac{1}{3}$, $w^*_{12}=\frac{1}{3}$, $w^*_{21}=\frac{4}{15}$, $w^*_{22}=\frac{1}{3}$	1.47	1.35	0.67	66.26	601.67
W_1=0.5 W_2=0.5	$w^*_{01}=1$, $w^*_{11}=\frac{1}{3}$, $w^*_{12}=\frac{1}{3}$, $w^*_{21}=\frac{1}{2}$, $w^*_{22}=\frac{1}{3}$	1.25	1.60	0.8	76.20	500.00

The table-2 shows different optimal solutions of the problem (5) by weighted-product method for different weights. If we increase the weights w_1 and w_2 both the optimal objective functions will increase. Here objective functions are inversely related to the weights.

Solution procedure of the problem (5) by weighted min-max method:

According to $MOGPT_{mm}$

min max $\left(w_1\left(\frac{40}{x_1x_2x_3}+40x_2x_3\right), w_2\frac{800}{x_1x_2x_3}\right)$

Such that

$\frac{1}{4}x_1x_2+\frac{1}{2}\,x_1x_3 \le 1,$

$x_1, x_2, x_3 > 0.$

Let max $\left(w_1\left(\frac{40}{x_1x_2x_3}+40x_2x_3\right), w_2\frac{800}{x_1x_2x_3}\right) = \lambda$

Then the above problem becomes

Min λ

Such that $w_1\left(\frac{40}{x_1x_2x_{34}}+40x_2x_3\right) \le \lambda,$

$w_2 \frac{800}{x_1 x_2 x_3} \leq \lambda,$

$\frac{1}{4} x_1 x_2 + \frac{1}{2} x_1 x_3 \leq 1,$

$x_1, x_2, x_3 > 0.$

The corresponding primal geometric programming problem is

Min λ ……………… (10)

Such that

$$\frac{1}{\lambda} w_1 \left(\frac{40}{x_1 x_2 x_{34}} + 40 x_2 x_3\right) \leq 1,$$

$$\frac{1}{\lambda} w_2 \frac{800}{x_1 x_2 x_3} \leq 1,$$

$$\frac{1}{4} x_1 x_2 + \frac{1}{2} x_1 x_3 \leq 1,$$

$X_1, x_2, x_3, \lambda > 0.$

Here DD =6 – (4+1) =1

DGPP of (10) is

Max d(w) = $\left(\frac{40w_1}{\lambda w_{11}}\right)^{w_{11}} \left(\frac{40w_1}{\lambda w_{12}}\right)^{w_{12}} \left(\frac{800w_2}{\lambda w_{21}}\right)^{w_{21}} {w_{21}}^{w_{21}} \left(\frac{1}{4w_{31}}\right)^{w_{31}} \left(\frac{1}{w_{32}}\right)^{w_{32}} (w_{11}+w_{12})^{w_{11}+w_{12}} (w_{31}+w_{32})^{w_{31}+w_{32}}$ …………(11)

Such that $w_1 + w_2 = 1,$

$w_{01} = 1,$

$w_{01} - w_{11} - w_{12} - w_{21} = 0,$

$-w_{11} - w_{21} + w_{31} + w_{32} = 0,$

$-w_{11} + w_{12} - w_{21} + w_{31} = 0,$

$-w_{11} + w_{12} - w_{21} + w_{32} = 0,$

$w_{01}, w_{12}, w_{31}, w_{32}, w_{11}, w_{21} \geq 0$

Primal dual variable relations are:

$\lambda = w_{01}\, d(w)$

$\frac{40w_1}{\lambda x_1 x_2 x_3} = \frac{w_{11}}{w_{11}+w_{12}},$

$\frac{40 w_1 x_2 x_3}{\lambda} = \frac{w_{12}}{w_{11}+w_{12}},$

$\frac{800w_2}{\lambda x_1 x_2 x_3} = \frac{w_{21}}{w_{21}} = 1,$

$\frac{1}{4} x_1 x_2 = \frac{w_{31}}{w_{31}+w_{32}},$

$\frac{1}{2} x_1 x_3 = \frac{w_{32}}{w_{31}+w_{32}},$

Solving the above normal and orthogonal conditions

$w_{01}=1,\ w_{12}=w_{31}=w_{32}=\frac{1}{3},\ w_{11}=\frac{w_1}{3(20w_2-w_1)},\ w_{21}=\frac{40w_2-3w_1}{3(20w_2-w_1)}.$

$x_2 = \left[\frac{2(20w_2-w_1)}{w_1}\right]^{\frac{1}{3}}, \quad x_3 = \frac{x_2}{2}, \quad x_1 = \frac{1}{x_3}$

Table-3:Optimal solution of problem(5)by weighted min-max method

Weights	Optimal dual variables	Optimal primal variables			Optimal objective functions	
W_1,W_2		X_1^*	X_2^*	X_3^*	g^*_{01}	g^*_{02}
$W_1=.1$ $W_2=.9$	$w^*_{01}=1, w^*_{11}=0.0018, w^*_{12}=0.33, w^*_{21}=0.664, w^*_{31}=0.33, w^*_{32}=0.33$	0.2816	7.1005	3.5502	1013.96	112.69
$W_1=.2$ $W_2=.8$	$w^*_{01}=1, w^*_{11}=0.0042, w^*_{12}=0.33, w^*_{21}=0.662, w^*_{31}=0.33, W^*_{32}=0.33$	0.3699	5.4061	2.7030	591.90	148.00
$W_1=.3$ $W_2=.7$	$W^*_{01}=1, w^*_{11}=0.0072, w^*_{12}=0.33, w^*_{21}=0.659, w^*_{31}=0.33, W^*_{32}=0.33$	0.4441	4.5034	2.2517	414.49	177.64
$W_1=.4$ $W_2=.6$	$W^*_{01}=1, w^*_{11}=0.0114, w^*_{12}=0.33, w^*_{21}=0.655, w^*_{31}=0.33, W^*_{32}=0.33$	0.5166	3.8708	1.9354	309.99	206.71
$W_1=.5$ $W_2=.5$	$W^*_{01}=1, w^*_{11}=0.0175, w^*_{12}=0.33, w^*_{21}=0.649, w^*_{31}=0.33, W^*_{32}=0.33$	0.5949	3.3619	1.6809	237.93	237.96
$W_1=.6$ $W_2=.4$	$W^*_{01}=1, w^*_{11}=0.0270, w^*_{12}=0.33, w^*_{21}=0.639, w^*_{31}=0.33, W^*_{32}=0.33$	0.6870	2.9109	1.4554	183.20	274.86
$W_1=.7$ $W_2=.3$	$W^*_{01}=1, w^*_{11}=0.0440, w^*_{12}=0.33, w^*_{21}=0.622, w^*_{31}=0.33, W^*_{32}=0.33$	0.8084	2.4740	1.2370	138.58	323.36
$W_1=.8$ $W_2=.2$	$W^*_{01}=1, w^*_{11}=0.0833, w^*_{12}=0.33, w^*_{21}=0.583, w^*_{31}=0.33, W^*_{32}=0.33$	1.0000	2.0000	1.0000	100.00	400.00
$W_1=.9$ $W_2=.1$	$W^*_{01}=1, w^*_{11}=0.2727, w^*_{12}=0.33, w^*_{21}=0.393, w^*_{31}=0.33, W^*_{32}=0.33$	1.4847	1.3470	0.6735	65.980	593.94

The table-3 shows different optimal solutions of the problem (5) by weighted min-max method for different weights. First objective gives better optimal result when w_1 increases. Similarly second objective gives better optimal result when w_2 increases. . Here the objective functions are directly related to the weights.

Table-4: Optimal solutions of problem (5) for equal weights

Methods	**Degrees of difficulty**	**Optimal Dual Variables**	**Optimal primal variables**	$\sum_{i=1}^{3} x_i^*$	**Optimal objective Functions**	
					g_{01}^*	g_{02}^*
Weighted sum method	0	$W^*_{01}=0.66$ $W^*_{02}=0.33$ $W^*_{11}=0.33$ $W^*_{12}=0.33$	$X_1=0.7249$ $X_2=2.7589$ $X_3=1.3794$	4.8632	166.72	289.99
Weighted product method	0	$W^*_{01}=1$ $W^*_{11}=0.33$ $W^*_{12}=0.33$ $W^*_{21}=0.5$ $W^*_{22}=0.33$	$X_1=1.25$ $X_2=1.60$ $X_3=0.8$	3.65	76.20	500.00
Weighted min-max method	1	$W^*_{01}=1$ $W^*_{11}=0.0175$ $W^*_{12}=0.33$ $W^*_{21}=0.6491$ $W^*_{31}=0.33$ $W^*_{32}=0.33$	$X_1=0.5949$ $X_2=3.3619$ $X_3=1.6809$	5.6377	237.93	237.96

We see that DD is Minimum for weighted product method. Among three methods (weighted sum method, Weighted product method and weighted min- max method), optimal value of first objective function$\square_{01}$(x_1,x_2,x_3) gives better result by weighted product method and optimal value of second objective function$\square_{02}$(x_1,x_2,x_3) gives better result by weighted min-max method. For minimum total optimal variables weighted product method gives better result and for maximum total optimal variables weighted min-max method gives better result.

CONCLUSION

Here we have discussed multi-objective geometric programming based on the weighted sum method, weighted product method, weighted min-max method, We have also formulated the multi-objective optimization model of the gravel-box design problem and solved this problem by multi-objective geometric programming technique based on said three methods. The different objective functions are combined into a single objective function by the above three methods. The GP technique is used to derive the optimal solutions for different preferences on objective functions. In tables 1-4 we have shown the optimal solution of our problem for different preference values of the objective functions. This multi-objective optimization model may also be solved by multi-objective geometric programming technique based on global criterion method.

REFERENCES

[1]. C.S. Beightler and D.T.Phillips : applied geometric programming , John Wiley and sons, New York 1976

[2]. C.S. Beightler and D.T.Phillips, D.J.Wilde:foundation of optimization , Prentice-hall, New Jersy ,1979

[3]. R.J.Duffin , E.L.Peterson and C.M.Zener : geometric programming theory and application , Wiely , New York,1967

[4]. A.K.Ojha and A.K.Das : geometric programming problem with coefficients and exponents associated with binary numbers , international journal of computer science, volm.7, issue1,2010

[5]. G.P.Liu ,J.B.Yang, J.F.whidborne , Uk: Multiobjective optimization and control.

[6]. Claude Mc-Millan ,Jr John wiley and sons: Mathematical programming,An introduction to the design and application of optimal design machines ,1970.

[7]. A.K.Ojha and A.K. Das : Multi-objective geometric programming problem being cost coefficients as continuous function with mean method, Journal of computing 2010.

[8]. S.B.Sinha ,A.Biswas ,and M.P.Bishal :geometric programming problem with negative degrees of difficulty ,Europian journal of operation research .28,pp.101-103 , 1987.

[9]. M.P.Bishal, fuzzy programming technique to solve multi-objective geometric programming problems, fuzzy sets and systems, 51: 67-71,1992.

[10]. R.k.Verma, fuzzy geometric programming with several objective functions, fuzzy sets and systems, 35:115-120,1990.