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Evaluating the Viability of Solutions to the Hubble Tension

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Abbreviations: ΛCDM: Lambda Cold Dark Matter; CM: Cosmic Microwave Background; CDM: Cold Dark Matter; SNeIa: Type Ia Supernovae; σ: Sigma Standard Deviation; WMAP: Wilkinson Microwave Anisotropy Probe; AC: Atacama Cosmology Telescope; SPT: South Pole Telescope; BAO: Baryon Acoustic Oscillations; SH0ES: Supernovae; H0: for the Equation of State of Dark energy; Cepheids PLR: Cepheids Period-Luminosity Relation; DEB: Detached eclipsing binaries; Gaia DR1/DR2: Gaia Data Release 1/Data Release 2; H0LiCOW: H0 Lenses in COSMOGRAIL's Wellspring; rs: Comoving Sound Horizon at the Surface of Last Scattering; θs: Angular Size of Sound Horizon from the Surface of Last Scattering; rsearly: The Sound Horizon at Radiation Drag Derived Assuming only the Early Universe Model; DE: Dark Energy; rEQ: Comoving Size of the Horizon at Matter-Radiation Equality; ΔNeff: Change in Effective Number of Relativistic Species; EDE: Early Dark Energy

ABSTRACT

The model dependent and independent measurements of the Hubble constant H0 are in a 4.4σ tension, which increases to 5.3 σ after the inclusion of independent time-delay cosmography data. This tension has been one of the most important discordance in cosmology, suggesting unknown systematic errors or potentially new physics beyond the standard model of cosmology. We analyze the current literature of independent experimental results, the reproduciblility of which leads to the conclusion that theoretical solutions would be more promising than unknown systematic errors. Several exotic theoretical scenarios not affecting the ΛCDM model are then evaluated and ruled out, leaving modifications to ΛCDM to be the more likely category. As a result of the tight constraints imposed by the CMB data, it is concluded that new pre-recombination mechanisms would be more promising than alterations of late universe expansion history as predicted by ΛCDM. Then, two relatively promising early universe modifications are evaluated, giving the conclusion that ΔNeff models are weakly preferred over EDE models and could therefore be a route for future research, though it is noted that neither theories can yet fully resolve the Hubble tension. For definitive confirmation or rejection of most models, data from the next generation experiments with a higher precision would be needed.

INTRODUCTION

Since the beginning of the twenty-first century, cosmology has entered an age of unprecedented precision. Measurements have provided stringent tests for the standard model of cosmology: the ΛCDM (Lambda Cold Dark Matter) model. Although predictions

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of the flat ΛCDM model agree with most experimental results, especially the Cosmic Microwave Background (hereinafter CMB) anisotropy power spectrum [1], to an exceptional precision, with tightening bounds on uncertainties several discordance have emerged in the recent decade, most notably the discrepancy between the early and late universe measurements of the Hubble constant, H_0.

The Hubble Constant describes the expansion rate of the universe:

$$H^0 = \frac{\dot{a}}{a}[2]$$

where a is the scale factor of the universe. The value of H0 is of central significance to cosmology, essential to the determination of both the past expansion history and the future evolution of the universe, the determination of which also offers tests of the standard ΛCDM model.

The model-dependent early universe value of the Hubble Constant, inferred from Planck Satellite's CMB spectrums supposing flat ΛCDM, is H_0 = 67.4 ± 0.5 km s^{-1} Mpc^{-1}, with a 1% precision [1]. Meanwhile, the late universe H_0 value can be calculated from direct empirical observations with little or no model dependence: the most precise measurement to date is calculated using the cosmic distance ladder method of the SH0ES project, giving H_0=74.03 ± 1.42 km s^{-1} Mpc^{-1} [2,3]. The early and late universe H0 differ by 6.6 ± 1.5 kms^{-1} Mpc^{-1}, corresponding to a 4.4 σ tension, or a Gaussian error of P=99.999%, indicating that the tension is highly unlikely to be caused by random errors. This discrepancy is termed the Hubble tension.

No definite explanation has been given to the Hubble tension, albeit a wide range of proposals have been suggested in the literature. In this paper, we review and analyse the potential solutions to the tension by eliminating unlikely categories to arrive at features of the most viable solution to the Hubble tension, given the data available to date.

POSSIBILITY OF SYSTEMATIC ERRORS

Since the H_0 discrepancy is a disagreement between the model dependent and independent derivations, it could be either a hint to new physics beyond the concordance model of cosmology or a result of unknown systematic errors. To evaluate the feasibility of the two cases, potential sources of systematics are considered first.

Early Universe Systematics

In the early universe measurement, possibilities of unknown systematic errors have been raised: The ΛCDM parameters inferred from the Planck high multipole and low multipole spectrums are inconsistent with a significance of σ=2.5, the cause of which is not understood fully [4] and could be attributed to systematic errors simultaneously causing the Hubble tension. However, the case against a CMB spectrum systematic error is considerably stronger. Firstly, in a recent study [5] have combined Baryon Acoustic Oscillations (BAO) results with Wilkinson Microwave Anisotropy Probe (WMAP), Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT) CMB data to obtain a consistently low H0 value in tension with the local measurements, independent of the Planck data, suggesting that the systematic errors within Planck measurements are unlikely to be the dominant cause of the Hubble tension. Moreover, the tension cannot be alleviated by removing any single set of data, meaning that the tension is unlikely to be caused by systematic errors in any specific experiments [5].

This finding is corroborated by derivations of H0 from a combination of BA_0 and the primordial deuterium abundance data [5] which also exhibit a 2.5~3.0 σ tension with SH0ES results, independent of any CMB anisotropy measurements used in the previous studies and hence complementary to them.

Another derivation of H_0 by different authors [6] from a combination of BA_0 and Type I_a Supernovae (SNeIa) data, employing the inverse distance ladder technique, also produces a model dependent value in tension with the late universe measurements, diminishing the chance of early universe systematics by the reproducibility of results using different datasets. Moreover, upon analysis performed by the Planck team [7] the tension is insensitive to the choice of frequency channel, meaning that systematic errors involving the choice of channels are largely irrelevant.

Combining all the results above, it can be concluded that the chance of unknown systematic errors in the early universe measurements as the cause of the discrepancy are negligible to date.

Late Universe Systematics

Then, the chance of systematics errors, if any, depend on the model-independent measurements. The most precise determination of the late universe H_0 comes from the SH0ES project, using distance ladder techniques, which can be divided into three principal steps [6]:

1. Direct distances to Cepheids can be deduced using geometric parallax in Milky Way, water masers in NGC 4258, and Detached Eclipsing Binaries (DEBs) in the Large Magellanic Cloud. These are the three anchors of the ladder

2. Absolute luminosity of Cepheids can be calculated from their apparent luminosity and distance; combined with their period of pulsation, the Cepheids' Period-Luminosity Relation (PLR) can be calibrated

3. Using the Cepheids in the host galaxies of the much rarer SNeIa, light curves of the SNeIa can be calibrated, which then act as standard candles at higher redshifts

The first and second steps compose the first rung of the ladder. In step 1, direct distances to anchor Cepheids are determined using three independent methods; therefore, checking the internal consistency between the sets of data would provide a method for ruling out systematic errors in each. In three values of H_0 have been calculated by randomly removing one of the three anchors at a time. The resultant values agree within 0.7%; comparing to the 9% discrepancy, this suggests that systematics within any of the anchors are unlikely to be a major cause.

The conclusion is supported by an analysis carried out by independent scholars [8]: utilizing the statistical method of Bayesian hyper-parameters, it is found that both the zero points and the PLR gradients of the three cepheid anchors are consistent with each other, similar to the conclusions made by Riess et al. [3].

To further diminish the probability of late-time systematic errors in SH0ES, one could increase the sample size using more independently measured cepheid samples. The Gaia Data Release 1 (DR1) parallax sample of 212 Cepheid distances is used to derive an independent value of H0, which is consistent within 0.3% with the SH0ES determination [9]; the Gaia DR2 parallax has been analyzed and is also in tension with the Planck inferred H_0 value.

To examine potential systematic errors in Cepheids calibration in Step 2, one could examine the statistical treatment of the data, the assumptions made, or the validity of modelling choices.

A variety of assumptions made about Cepheids could affect the determination of luminosities, hence distances; for example, Cepheids PLR are empirically determined to be dependent on metallicity, so metallicity parameter priors could bias the data. Nonetheless, in Cardona, Kunz, and Pettorino [8-10], using a new statistical method of Bayesian hyper-parameters to avoid arbitrary rejection of outliers in the analysis, H0 values are calculated without some of the assumptions made each time (including metallicity, cut in Cepheid period data, etc.). The H_0 tension persists through the removal of any assumptions, thus disfavoring the possibility of inaccurate underlying assumptions about Cepheid variables. Photometric modelling bias are also effectively ruled out as a solution to the Hubble tension, since H0 derived are robust to a range of modelling choices shown by Rosser WGV [11]. There remains the possibility of systematic errors in the photometry of telescopes, which is diminished by independent measurements of different Cepheids carried out by Gaia and Hubble Space Telescope giving consistent results [9], suggesting the reproducibility of the results using different photometric systems.

The second rung of the distance ladder (step 3) involves the calibration of SNeIa, which is slightly more problematic since the underlying mechanism of their progenitors and explosions are not fully understood in theory. Empirical data analysis has demonstrated a dependence of SNe luminosity on host galaxy star formation rate, the correction of which is shown to be able to alleviate the Hubble tension significantly. Nonetheless, the effect of the star formation bias is tested by removing the cause of such mechanism through sample selection. Applying a late-type only galaxy selection criteria, the star formation rate is controlled; the resultant change $\Delta H_0<0.3\%$ suggests the insignificance of the star formation dependence effect. Furthermore, the effect of host galaxy age and mass have been ruled out as the major cause.

These results are complemented by a near-infrared survey of SNeIa producing results consistent with SNeIa results in the optical range [9-15], ruling out systematic errors caused by dust extinction which behaves differently in different wavelength range.

By these results, one could infer that SNeIa systematics are not a major cause behind the Hubble tension. This conclusion is further validated by a 2.4% precision determination of H0 from the H_0LiCOW project, using time-delay cosmography with data from six strongly gravitationally lensed quasars, which gives

H kms^{-1} Mpc^{-1} independent of both the distance ladder and CMB derivations in agreement with the distance ladder results, meanwhile also in a significant 3.1σ tension with the Planck value [16]. Combining SH0ES data and the independent lensing measurements, a stronger tension of 5.3σ is then concluded the errors and uncertainties in the lensing results have been evaluated and found to be valid.

Evaluating the combination of all independent studies considered above, it is evident that this significant discrepancy cannot be easily explained by any appeal to systematic errors in early or late universe measurements. Consequently, to resolve the unexplained Hubble tension, theoretical solutions are favored over experimental refinements, based on the information discussed.

THEORETICAL SOLUTIONS

The theoretical solutions proposed in the literature can be categorized by the active period of the new mechanism: a class of theoretical solutions involves an extra mechanism added to ΛCDM, active before the epoch of recombination (pre-recombination); a competing class of solution involves mechanisms causing modifications to the expansion history of the universe at later times (post-recombination); additionally, several relatively uncommon theoretical considerations not involving changing the concordant model have also been proposed.

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Physics Within ΛCDM or Beyond

We first consider the possibilities of the theoretical proposals not affecting the standard model.

The effect of sample variance on the SNeIa observed volume [15-19]. The observable SNeIa datasets are constrained to a small fraction of the Hubble volume, therefore data of SNeIa are dependent on local density distributions. Due to cosmic inhomogeneity on the scale considered, there exists a possibility of a void of underdensity surrounding our cosmic neighborhood, which would theoretically cause a higher local H0 than the global value extrapolated using CMB data. Nonetheless, in a recent large volume n-body simulation quantifying the density fluctuations using SNe data, it is found that the scale of local underdensity required ($\delta \sim -0.8$, radius~150 Mpc) to explain the Hubble tension is not probable in a flat ΛCDM universe, the condition of which is also beyond the constraints of existing direct observations of the local density. Hence sample variance is effectively ruled out as the cause of the tension.

Next, we consider the effect of assuming a slow-roll inflationary model. The calculation of H0 from CMB data uses the primordial power spectrum law predicted by the simplest slow-roll inflation model, thus features beyond slow-roll inflation could alter the H0 values derived. This possibility is tested where 64 extra degrees of freedom are allowed in the power law but a 4.9 σ tension persists, undermining the possibility of the inflationary theory being the cause.

Then, since no theory to date has been able to resolve the tension without introducing modifications to the standard model of cosmology, the chances of the ΛCDM picture being insufficient to fully describe the universe are high.

Modification to Early or Late Universe ΛCDM Physics

To identify routes for modifying ΛCDM in order to raise the early universe H0, we examine the derivation of the value from CMB measurements, and conclude two variable approaches: refer to Appendix B for details.

These two approaches rely on changing ΛCDM at different eras: pre- or post-recombination. The assumptions of both early and late parts of the ΛCDM model are involved in the CMB inferred value of H0, making it difficult to decide between the two. Hence, a method of separating the assumptions are employed by translating the H0 tension into a tension between model dependent and independent values of the comoving sound horizon at the surface of last scattering, rs [10]. In a value of rsearly, sensitive to only early models, are used together with BAO, SNeIa and the empirically measured H0 value to model the Hubble parameter H(z) independent of any late universe model assumptions. The resultant shape of H(z) when not using SNeIa data is best fitted by curve with an increase in the acceleration rate at z<0.2, where z stands for the redshift, which is explained by a late time phantom dark energy mechanism. However, when SNeIa data are also included, H(z) is constrained tightly around the prediction of the ΛCDM model, disfavoring a late universe solution using phantom dark energy.

This conclusion disfavoring late universe resolutions is confirmed by the more general theoretical examination [12-15] concluding that any late universe solutions would be tightly constrained by existing data.

The details of the theoretical reasoning are presented in Appendix C.

A more recent independent study by Knox et al. further corroborates the conclusion drawn from the studies considered above [2]. In the paper five more late universe solutions have been examined, finding no promising late universe solutions.

POTENTIAL EARLY UNIVERSE MECHANISMS

By method of elimination of unlikely scenarios, we infer that solutions involving modifications to prerecombination ΛCDM picture remain most likely by far. Unlike the case of late universe solutions, to date, there has been no robust evidence against a general early universe solution to the Hubble tension in the literature; although as this paper is written, Krishnan et al. analyzed a combination of late universe data and found a correlation between redshift and the corresponding H_0 value similar to the trend suggested by the H_0LiCOW team which potentially favours a late universe solution [4] the trend is currently only at a level of 2.1 σ and consequently does not offer enough evidence against the existence of early universe solutions. It is also worth noting no independent evaluations of this paper have yet been published.

Therefore, possible candidates of early universe solutions are evaluated next, to identify some of the most promising solutions of the Hubble tension. While a variety of early universe theory modifications have been proposed including exotic scenarios such as allowing a time-varying fine structure constant, most are tightly constrained by existing CMB and BAO data to be very unlikely, There then remains two potential solutions which are shown to be relatively promising and remain in debate in the literature, both involving additional energy components which would increase H(z) in a redshift window just before the epoch of recombination, and hence decrease the conformal time to the end of the baryon-drag epoch, ηd, and rs. Since the dominant energy contributions in the rs integral is during the two decades just before recombination, the theories proposed are active during this window to maximize the effect on rs. One such mechanism involves allowing an extra relativistic degree of freedom, whilst the other approach adds a component of extra Early Dark Energy (EDE).

Extra Relativistic Degree of Freedom

The parameter Neff connotes the effective number of relativistic species, and it is generally agreed within the literature that a ΔNeff>0 could mitigate the Hubble tension because light, relativistic particles like neutrinos behave as dark radiations increasing the radiation energy density before recombination, thus decreasing ηd combining conclusions from previous studies [2,7], it is concluded that a change between the range 0.4<ΔNeff<1.0 could alleviate the tension partially to completely. However, Neff is tightly constrained by the Planck CMB data to be Neff~3.046, which is consistent with the prediction of the standard model. Accounting for the probabilistic restrictions posed by the CMB data, although small ΔNeff allowed by the data does ameliorate the tension, it is unable to resolve the tension completely and the discrepancy persists beyond 3σ [1]. Therefore, the standard ΔNeff mechanism is unlikely to solve the Hubble tension fully. Nonetheless it allowing for a neutrino self-interaction mechanism which delays the free streaming would result in Neff=4.02 ± 0.29 (68% C.L.) being preferred by the CMB data when combined with BAO and measured H0. This solution would be able to resolve the H0 tension as well as the S8 tension simultaneously, but several problems also exist with this approach [2,8]: firstly, inclusion of Planck polarization data appears to deviate the fit of the model, a full statistical analysis of the significance of which is yet to be completed; next, the Bayes factor of the model is lower than that of ΛCDM, albeit this is more due to intrinsic features of Bayesian statistics; lastly, the model does not yet have a certain underlying mechanism and could be restricted by neutrino physics [2,8].

Although the potential problems of this approach considered above decrease its likelihood of success, the ΔNeff approach with self-interacting neutrinos is not completely ruled out by any definitely; therefore, it remains in consideration as a potential cause of the Hubble tension B.

Early Dark Energy

An alternate new mechanism active in approximately the same redshift window involves an evolving scalar field φ which behaves like cosmological constant before recombination, and quickly decays away after a critical redshift zc. Such a scalar field EDE model provides an extra energy injection which decreases the sound horizon at the surface of last scattering, and is inspired by a stringaxiverse scenario where a simple fisher analysis concludes that EDE models could alleviate the tension, without being able to fully resolve it. Nonetheless, Poulin et al. developed effective fluid modelswhich enabled simulation of perturbation growth in the EDE fluid, the inclusion of which led their analysis to conclude that a field accounting for 5% of the total energy density at z~5000 can explain the discrepancy without being constrained by existing datasets. The model has a positive Bayesian evidence when compared with ΛCDM, and unlike the ΔNeff solution, when only considering the CMB data, it provides a fit as well as the ΛCDM model. However, recently a paper used a full analysis of EDE field's equation of motion without the fluid approximations previously employed and showed proved the approximation used in previous papers invalid, thus arriving at a different result reached the conclusions that a scalar field with potential $V \propto \varphi^4$ could reduce the Hubble tension significantly to a 2 σ level, albeit not being able to resolve the tension completely. The modified EDE model still fits the CMB data better than the ΔNeff model, producing a slightly higher H_0 value.

Several problems arise when evaluating the EDE class of models. Firstly and most importantly, all analysis of EDE models consistently show that such models would increase the S8 tension, which concerns the disagreement between the matter perturbation predicted by ΛCDM from CMB data and its local measurement from Large Scale Structure (LSS) data. Therefore, although when using the datasets used in previous studies EDE models reduce the tension to approximately 1.9 σ, including further LSS data like Dark Energy Survey Year 1 results that encapsulate S8 information results in no obvious evidence for the EDE being concluded, raising the tension above 2σ again. Moreover, just like ΛCDM, EDE models suffer from both the fine-tuning problem and the coincidence problem, which are not easily explained by theories without chance, whereas a ΔNeff solution does not face these problems. Hence, considering the weaknesses of EDE models based on existing data, we conclude that ΔNeff would be a relatively satisfactory solution despite tight constraints posed by the CMB data. Though it must be noted that no solutions proposed can fully solve the Hubble tension to date, and the early universe solutions considered here are only able to alleviate the tension.

DISCUSSION

Even though it is unlikely that late universe systematics would be the major cause of the Hubble tension, based on available evidence considered in this paper, it must be acknowledged that this possibility is not ruled out completely and hence should be explored further under closer examinations in the future. The detailed mechanisms behind Cepheids and SNeIa are not yet fully understood theoretically, which could potentially leave rooms for unknown systematic errors. A new late universe mechanism of a fifth force impacting the distance ladder measurements published when his paper is being written might be a candidate for future evaluations.

Although to date, no new theoretical solutions have been able to ameliorate the tension satisfactorily without changing the ΛCDM model, this probability is not completely eliminated and more successful theories in this class could emerge in the future.

Furthermore, the results should be examined further with more precise data; if future late universe data continue to suggest such a correlation with higher statistical significance, it could be taken as an argument for a late universe solution instead.

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The comoving angular diameter distance to the surface of last scattering, DA* , is a quantity related to both the experimentally determined CMB spectrums assuming pre-recombination ΛCDM physics and values of the Hubble parameter, H(z), from which a model-dependent early universe value of H_0 can be deduced from CMB. DA* can be calculated from rs, which is estimated using early universe ΛCDM assumptions, together with θs, which is empirically estimated from the spacing between CMB spectrum peaks, by the formula:

$$D_A = \frac{r_s}{\theta_s} \tag{B1}$$

DA* is also related to the Hubble parameter by:

$$D_A^* = \int_0^{z^*} \frac{dz}{H(z)} \tag{B2}$$

where z* is the redshift of the surface of last scattering. Combining the two and rearranging gives:

$$r_s = \theta_s \int_0^{z^*} \frac{dz}{H(z)} \tag{B3}$$

From theory, then, there are two promising routes to modifying ΛCDM producing an increase in the inferred Hubble constant. Firstly, decreasing the sound horizon rs would cause an increase in the extrapolated early universe H_0 from equation B_3. Because rs is determined by ΛCDM physics before recombination, this approach requires altering the pre-recombination ingredients of the ΛCDM model. More mathematically:

$$r_s = c_s \eta_d \tag{B4}$$

where ηd is the conformal time to the end of the baryondrag epoch and cs is the speed of sounds, so decreasing the time ηd would resolve the tension theoretically.

Alternatively, H_0 can be increased through altering the post-recombination ΛCDM prediction of the low-redshift expansion history of the universe; specifically by adding an extra driving force which only comes into effect at a relatively recent era to increase the recent value of H_0, reducing by equation B2. For example, a phantom dark energy component with equation of state parameter ω<−1 could in principle resolve the tension, by increasing the predicted acceleration rate at z<0.2, as the results concluded from a parameter space search with 12 cosmological parameters shown.

CONCLUSION AND EVALUATION

In this paper, different categories of potential solutions to the H0 tension is evaluated, considering a comprehensive set of existing studies and experimental results in the literature.

In Section II, the possibility of systematic errors explaining the tension is evaluated, dividing into early universe and late universe measurements errors. Based on existing independent studies, significant systematic errors in the Planck CMB data are highly unlikely. The case of distance ladder measurements is less certain, due to the complexity of rungs of distance ladder calibrations. Under closer examination, no significant sources of systematic errors can be identified in the distance ladder measurements; independent time-delay cosmography results also support the SH0ES data. Hence, we conclude that systematic errors in late universe measurements have a low probability of being the cause.

Theoretical solutions are then considered in Section III. Several exotic scenarios which do no modify the ΛCDM model are considered and effectively ruled out due to constraints set by existing data, leaving us with modifications of ΛCDM. By the effective period, these solutions are grouped into early and late universe mechanisms; the Planck CMB power spectrums tightly constrains any general late time modifications, leaving us with early time solutions being the most probable class.

Having narrowed down the properties and active periods of potential solutions, remaining early universe solutions are considered in Section IV. ΔNeff and EDE models are evaluated since they are commonly considered to be the most promising early universe candidates. The criterions assessed include the theory's 1) ability to solve the fine-tuning and coincidence problem of ΛCDM; 2) fit to existing data; 3) ability to mitigate Hubble tension; 4) influence on other existing tensions; 5) background mechanism's certainty. Both theories cannot resolve the tension completely and do not have confirmed supporting background theories; although EDE provides a slightly better fit to the CMB data and can alleviate the Hubble tension slightly better, it fails to meet criterion 1) which ΔNeff models solve, and worsens the S8 more than the ΔNeff model. Therefore, from existing evidence, ΔNeff would be weakly preferred over the EDE as the most viable solution for the Hubble tension, though both theories should still remain in consideration until more precise data rejects or confirms either definitely.

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