ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

COMMON FIXED POINT THEOREMS FOR SIX SELF MAPS UNDER SUB COMPATIBLE AND SUB SEQUENTIALLY CONTINUOUS MAPS IN FUZZY METRIC SPACES BY USING IMPLICIT RELATION

M. Rangamma [2], G. Mallikarjun Reddy [1], P. Srikanth Rao [3]

Professor, Department of Mathematics, O.U., Hyderabad, India[2]

Assistant professor, Department of Mathematics, CVR Engineering College, Ibrahimpatan, Hyderabad, India[1]

Professor, Department of Mathematics, B.V.Raju Engineering College, Narsapur, Hyderabad, India[3]

Abstract: The purpose of this paper is to prove common fixed point theorems for the new concepts of sub compatibility and sub sequential continuity in fuzzy metric spaces using Implicit Relation for six self mappings which are weaker than occasionally weak compatibility and reciprocal continuity. In general all known results on commuting, weakly commuting, compatible, weak compatible, semi compatible and occasionally weak compatible maps in fuzzy metric spaces are generalized in this note.

Keywords: Compatible maps, R-weakly commuting maps, Weakly compatible maps, Sub compatible maps, Sub sequentially continuous maps.

I. INTRODUCTION

In 1965, the concept of fuzzy sets was introduced by Zadeh[1] which laid the foundation of fuzzy mathematics. Kramosil and Michalek [2] introduced the notion of a fuzzy metric space by generalizing the concept of the probabilistic metric space to the fuzzy situation. George and Veeramani [3] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [2]. There are many view points of the notion of the metric space in fuzzy topology for instance one can refer to Kaleva and Seikkala [4], Kramosil and Michalek [2], George and Veeramani [3]. Popa ([5]-[6]) introduced the idea of implicit function to prove a common fixed point theorem in metric spaces. Singh and Jain [7] further extended the result of Popa ([5]-[6]) fuzzy metric spaces. Using the concept of R-weak commutative mappings, R.Vasuki [8] proved the fixed point theorems for Fuzzy metric space. Recently in 2009, using the concept of sub compatible maps, H.Bouhadjera et. al. [9] proved common fixed point theorems. In 2010 and 2011, B.Singh et. al. [7] proved fixed point theorems in Fuzzy metric space and Menger space using the concept of semi-compatibility, weak compatibility and compatibility of type (β) respectively. In this paper, we prove fixed point theorems by using concepts of sub compatibility and sub sequential continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity in Fuzzy metric space using Implicit Relations. With them, we establish a common fixed point theorem for six maps, which extends the results of Kamal Wadhwa.et al.[10] and others.

II. PRELIMINARIES

Definition 1.1 ([1]): Let X be any non empty set. A fuzzy set M in X is a function with domain X and values in [0, 1].

Definition 1.2 ([12]): A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is called a continuous t-norm if * is satisfying the following conditions

a) * is commutative and associative,

b) * is continuous,

c) $a*1=a$ for all $a\in[0,1]$,

d) $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$ for all $a,b,c,d\in[0,1]$.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

Definition 1.3 ([3])**:** A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set. * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$, $s, t > 0$

(fm1) $M(x, y, t) > 0$,

(fm2) $M(x, y, t) = 1$ if and only if $x = y$,

(fm3) $M(x, y, t) = M(y, x, t)$,

(fm4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,

(fm5) $M(x, y, \bullet) : [0, \infty) \to [0,1]$ is left continuous,

then M is called a fuzzy metric on X.

Then M is called a fuzzy metric on X. The function M(x, y, t) denote the degree of nearness between x and y with respect to 't'. We identify x=y with M(x, y, t)=1 for all t>0 and M(x, y, t)=0 with ∞ and we can find some topological properties and examples of fuzzy metric spaces in paper of George and Veeramani [3].

Example 1.4 (Induced fuzzy metric [3]): Let (X, d) be a metric space. Define a*b= ab for all a, b ∈ [0, 1] and let M_d fuzzy sets on $X^2 \times$(o, ∞) defined as follows, $M_d(x, y, t) = \dfrac{t}{t + d(x, y)}$ then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by the metric d, the standard fuzzy metric. On the other hand note that there exists no metric on X satisfying the above $M_d(x, y, t)$.

Definition 1.5 ([3])**:** Let $(X, M, *)$ be fuzzy metric space then,

a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each $\varepsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \ge n_0$.

b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence for each $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \varepsilon$ for all $m, n \ge n_0$.

c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 1.6 (remark 2.1 of [17]): Since * is continuous, it follows from (fm4) that the limit of the sequence in fuzzy metric space is uniquely determined. Let $(X, M, *)$ is a fuzzy metric space with the following condition (fm6) $\underset{t \to \infty}{Lt}$ M(x, y, t)=1 for all x, y ∈ X.

Lemma 1.7 ([15])**:** For all $x, y \in X$, $M(x, y, \bullet)$ is a non decreasing function.

Lemma 1.8 ([17])**:** Let $(X, M, *)$ be a fuzzy metric space if there exists $k \in (0,1)$ such that $M(x, y, kt) \ge M(x, y, t)$ then x=y.

Lemma 1.9([2]): Let $\{y_n\}$ be a sequence in a fuzzy metric space $(X, M, *)$ with the condition (fm6). If there exists a number $k \in (0,1)$ such that $M(y_n, y_{n+1}, kt) \ge M(y_{n-1}, y_n, t)$ for all t>0 and $n \in N$, then $\{y_n\}$ is a Cauchy sequence in X.

Proposition 1.10([2]): In a fuzzy metric space $(X, M, *)$, if $a * a \ge a$ for all $a \in [0,1]$ then $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$.

Definition 1.11([2]): Two self maps A and S of a fuzzy metric space $(X, M, *)$ are called compatible if $\underset{n \to \infty}{Lt} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\underset{n \to \infty}{Lt} Ax_n = \underset{n \to \infty}{Lt} Sx_n = x$ for some x in X.

Definition 1.12([9]): Two self maps A and S of a fuzzy metric space $(X, M, *)$ are called weakly commuting if $M(ASx, SAx, t) \ge M(Ax, Sx, t)$ for all x in X and t>0.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

Definition 1.13 ([9]): Two self maps A and S of a fuzzy metric space $(X, M, *)$ are called R-weakly commuting if there exists R> 0 such that $M(ASx, SAx, t) \geq M\left(Ax, Sx, \frac{t}{R}\right)$ for all x in X and t>0.

Remark 1.14 ([9])**:** Clearly, point wise R-weakly commuting implies weak commuting only when $R = 1$.

Remark 1.15 ([9]): Compatible mappings are point wise R-weakly commuting but not conversely.

Definition 1.16 ([25])**:** Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence points that is the pair (A, S) is weakly compatible pair if $Ax = Sx$ implies $ASx = SAx$ for all $x \in X$.

Definition 1.17 ([25])**:** Two self maps are said to be occasionally weakly compatible if there exists at least one x in X for which $Sx = Tx$ implies $STx = TSx$.

Definition 1.18 ([15]): Two self maps A and S on a fuzzy metric space are called reciprocal continuous if $\underset{n\to\infty}{Lt}\ ASx_n = At$ and $\underset{n\to\infty}{Lt}\ SAx_n = St$ for some t in X whenever $\{x_n\}$ is a sequence in X such that $\underset{n\to\infty}{Lt}\ Ax_n = \underset{n\to\infty}{Lt}\ Sx_n = t$, t in X.

Remark 1.19 ([14]): If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for point wise R-weakly commuting maps satisfying contractive conditions, continuity of one of the mappings A or S implies their reciprocally continuity but not conversely.

In this paper, we weaken the above notion by introducing a new concept called sub compatibility just as defined by H. Bouhadjera [16] in metric space, as follows.

Definition 1.20 ([16])**:** Self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be sub compatible if and only if there exists a sequence $\{x_n\}$ in X such that $\underset{n\to\infty}{Lt}\ Ax_n = \underset{n\to\infty}{Lt}\ Sx_n = t$, t in X and satisfy

$\underset{n\to\infty}{Lt}\ M(ASx_n, SAx_n, t) = 1$ for all t>0.

Obviously two occasionally weakly compatible maps are sub compatible maps, however the converse is not true in general.

Example 1.21 ([16])**:** Let $X = [0, \infty)$, $S(x) = x^2$,

$$T(x) = \begin{cases} x+2 \text{ if } x \in [0,4] \cup [9,\infty) \\ x+12 \text{ if } x \in (4,9) \end{cases}$$ and $x_n = 2 + \frac{1}{n}$ for $n \in N$ then

$\underset{n\to\infty}{Lt}\ S(x_n) = 4 = \underset{n\to\infty}{Lt}\ T(x_n)$ and $ST(x_n) = S(x_n + 2) = 16 = TS(x_n) = T(x_n^2)$ as $n \to \infty$ thus

$\underset{n\to\infty}{Lt}\ M(STx_n, TSx_n, t) = 1$ i.e. S and T are sub compatible.

On the other hand we have $Sx = Tx \Leftrightarrow x = 2$ and

$ST(2) = S(4) = 16, TS(2) = T(4) = 6$ then $S(2) = 4 = T(2)$ but $ST(2) \neq TS(2)$

hence the mappings S and T are not OWC.

Now, Kamal Wadhwa etal. [10] introduced a new notion called sub sequential continuity in fuzzy metric spaces by weakening the concept of reciprocal continuity introduced by pant [15].

Definition 1.22 ([10]): Two self maps A and S on a fuzzy metric space are said to be sub sequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that $\underset{n\to\infty}{Lt}\ Ax_n = \underset{n\to\infty}{Lt}\ Sx_n = t$ for some t in X and satisfy

$\underset{n\to\infty}{Lt}\ ASx_n = At$ and $\underset{n\to\infty}{Lt}\ SAx_n = St$.

Clearly, if A and S are continuous or reciprocally continuous then they are obviously sub sequentially continuous. The next example shows that there exist sub sequential continuous pairs of maps which are neither continuous nor reciprocally continuous.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

Example1.23 ([26])**:** Let X=R, endowed with metric d and $M_d(x, y, t)= \frac{t}{t+d(x,y)}$ for all x, y$\in$X, t>0 define the self maps A, S as

$$Ax=\begin{Bmatrix} 2, x<3 \\ x, x\geq 3 \end{Bmatrix}, Sx=\begin{Bmatrix} 2x-4, x\leq 3 \\ 3, x>3 \end{Bmatrix}.$$

Consider a sequences $x_n = 3+\frac{1}{n}$ then $Ax_n = (3+\frac{1}{n}) \to 3, Sx_n = 3, SAx_n = S(3+\frac{1}{n}) = 3 \neq S(3) = 2$ as $n \to \infty$.
Thus A and S are not reciprocally continuous but, if we Consider a sequence $x_n = 3-\frac{1}{n}$, then

$$Ax_n = 2, Sx_n = 2(3-\tfrac{1}{n})-4 = (2-\tfrac{2}{n}) = 2,$$

$$ASx_n = A(2-\tfrac{2}{n}) = 2 = A(2), SAx_n = S(2) = 0 = S(2) \text{ as } n \to \infty.$$

Therefore, A and S are sub sequentially continuous.
Remark 1.24 ([26])**:** If A and S are continuous or reciprocally continuous then they are obviously sub sequentially continuous.
Implicit Relations 1.25 ([5]**):**

(1.25.1) Let Φ be the set of all real continuous functions $\phi:(R^+)^6 \to R^+$ satisfying the condition ϕ(u, u, v, v, u, u) ≥0 imply u≥ v, for all u, v ∈ [0, 1].

(1.25.2) Let Φ be the set of all real continuous functions $\phi:(R^+)^5 \to R^+$ satisfying the condition ϕ(u, u, v, u, u) ≥0 imply u≥ v, for all u, v ∈ [0, 1].

The following theorem proved by Kamal Wadhwa et al. [10].
Theorem1.26: Let A, B, S and T be four self maps of fuzzy metric space $(X,M,*)$ with continuous tnorm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (A, S) and (B, T) are sub compatible and sub sequentially continuous then

(1.26.1) A and S have a coincidence point
(1.26.2) B and T have a coincidence point
(1.26.3) for some function ϕ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Ax,By,t), M(Sx,Ty,t), M(Sx,Ax,t), \\ M(Ty,By,t), M(Sx,By,t), M(Ty,Ax,t) \end{Bmatrix} \geq 0$$

then A, B, S and T have a unique common fixed point.

III. MAIN RESULTS

In this section we extend the **theorem 1.26** using implicit relations 1.25.1 and 1.25.2 which generalizes the results of [10], [11], [15] and [18].
Theorem 2.1: Let P, Q, S, T, A and B be six self maps of a complete fuzzy metric space $(X,M,*)$ with continuous t-norm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (P, AB) and (Q, ST) are sub compatible and sub sequentially continuous then

(2.1.1) P and AB have a coincidence point,
(2.1.2) Q and ST have a coincidence point,
(2.1.3) the pairs (P, T), (AB, T), (Q, B), (ST, B) are commutes,
(2.1.4) for some $\phi \in \Phi$ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Px,Qy,t), M(ABx,STy,t), M(ABx,Px,t), \\ M(STy,Qy,t), M(ABx,Qy,t), M(STy,Px,t) \end{Bmatrix} \geq 0,$$

then P, Q, S, T, A and B have a unique common fixed point .
Proof: Since (P, AB) and (Q, ST) are sub compatible and sub sequentially continuous then there exists two

sequences $\{x_n\}$, $\{y_n\}$ in X such that

(2.1.5) $\underset{n\to\infty}{Lt} Px_n = \underset{n\to\infty}{Lt} ABx_n = z$, $z \in X$ and satisfy

$\underset{n\to\infty}{Lt} M(P(AB)x_n,(AB)Px_n,t) = M(Pz, ABz,t) = 1$

(2.1.6) $\underset{n\to\infty}{Lt} Qy_n = \underset{n\to\infty}{Lt} STy_n = z^1$, $z^1 \in X$ and satisfies

$\underset{n\to\infty}{Lt} M(Q(ST)y_n,(ST)Qy_n,t) = M(Qz^1, STz^1,t) = 1$.

Therefore $Pz = ABz$ and $Qz^1 = STz^1$ i.e. z is a coincidence point of P , AB and z^1 is a coincidence point of Q, ST.

Now we prove $z = z^1$.

Put $x = x_n$ and $y = y_n$ in (2.1.4) we get

(2.1.7) $\phi\begin{Bmatrix} M(Px_n,Qy_n,t), M(ABx_n,STy_n,t), M(ABx_n,Px_n,t), \\ M(STy,Qy_n,t), M(ABx_n,Qy_n,t), M(STy_n,Px_n,t) \end{Bmatrix} \geq 0$.

Taking the limit as $n \to \infty$ and using (2.1.5), (2.1.6) we get

$\phi\begin{Bmatrix} M(z,z^1,t), M(z,z^1,t), M(z,z,t), \\ M(z^1,z^1,t), M(z,z^1,t), M(z^1,z,t) \end{Bmatrix} \geq 0$

(2.1.8) $\phi\{M(z,z^1,t), M(z,z^1,t),1,1, M(z,z^1,t), M(z,z^1,t)\} \geq 0$.

In view of implicit relations (1.25.1) we get

(2.1.9) $z = z^1$.

Again we claim that $Pz = z$.

Substitute $x = z$ and $y = y_n$ in (2.1.4) we get

(2.1.10) $\phi\begin{Bmatrix} M(Pz,Qy_n,t), M(ABz, STy_n,t), M(ABz, Pz,t), \\ M(STy_n,Qy_n,t), M(ABz,Qy_n,t), M(STy_n, Pz,t) \end{Bmatrix} \geq 0$.

Taking the limit as $n \to \infty$ and using (2.1.6), $Pz = ABz$ we get $\phi\begin{Bmatrix} M(Pz,z^1,t), M(Pz,z^1,t), M(Pz,Pz,t), \\ M(z^1,z^1,t), M(Pz,z^1,t), M(z^1,Pz,t) \end{Bmatrix} \geq 0$

(2.1.11) $\phi\begin{Bmatrix} M(Pz,z^1,t), M(Pz,z^1,t),1, \\ 1, M(Pz,z^1,t), M(Pz,z^1,t) \end{Bmatrix} \geq 0$.

In view of implicit relations (1.25.1) and using (2.1.9) we get

(2.1.12) $Pz = z^1 = z$ implies $ABz = Pz = z$.

Again we claim that $Qz = z$.

Substitute $x = z$ and $y = z$ in (2.1.4), (2.1.12) and $STz = Qz$ we get

(2.1.13) $\phi\begin{Bmatrix} M(Pz,Qz,t), M(ABz, STz,t), M(ABz, Pz,t), \\ M(STz,Qz,t), M(ABz,Qz,t), M(STz, Pz,t) \end{Bmatrix} \geq 0$

$\phi\begin{Bmatrix} M(z,Qz,t), M(z,Qz,t), M(z,z,t), \\ M(Qz,Qz,t), M(z,Qz,t), M(Qz,z,t) \end{Bmatrix} \geq 0$

(2.1.14) $\phi\{M(z,Qz,t), M(z,Qz,t),1,1, M(z,Qz,t), M(Qz,z,t)\} \geq 0$.

In view of implicit relations (1.25.1) and using (2.1.12) we get

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

(2.1.15) $z = Qz$ implies $STz = Qz = z$

and also $ABz = Pz = Qz = STz = z$.

which shows that z is a common fixed point of AB, P, Q and ST.

Now we claim $Tz = Sz = z$.

Put $x = Tz$ and $y = z$ in (2.1.4) and using (2.,1.3), (2.1.15) we get

(2.1.16) $\phi\begin{Bmatrix} M(PTz,Qz,t), M(ABTz,STz,t), M(ABTz,PTz,t), \\ M(STz,Qz,t), M(ABTz,Qz,t), M(STz,PTz,t) \end{Bmatrix} \geq 0$

$$\phi\begin{Bmatrix} M(TPz,Qz,t), M(\mathrm{T}ABz,STz,t), M(\mathrm{T}ABz,\mathrm{TP}z,t), \\ M(STz,Qz,t), M(\mathrm{T}ABz,Qz,t), M(STz,TPz,t) \end{Bmatrix} \geq 0$$

$$\phi\begin{Bmatrix} M(Tz,z,t), M(Tz,z,t), M(Tz,Tz,t), \\ M(z,z,t), M(Tz,z,t), M(z,Tz,t) \end{Bmatrix} \geq 0$$

(2.1.17) $\phi\{M(Tz,z,t), M(Tz,z,t),1,1, M(Tz,z,t), M(Tz,z,t)\} \geq 0$.

In view of implicit relations (1.25.1) and using (2.1.15) we get

(2.1.18) $Tz = z$ and $STz = z$ implies $Sz = z$.

Again we claim that $Az = Bz = z$.

Let $x = z$ and $y = Bz$ in (2.1.4) and using (2.1.3), (2.1.15) we get

(2.1.19) $\phi\begin{Bmatrix} M(Pz,QBz,t), M(ABz,STBz,t), M(ABz,Pz,t), \\ M(STBz,QBz,t), M(ABz,QBz,t), M(STBz,Pz,t) \end{Bmatrix} \geq 0$

$$\phi\begin{Bmatrix} M(z,BQz,t), M(z,BSTz,t), M(z,z,t), \\ M(BSTz,BQz,t), M(z,BQz,t), M(BSTz,z,t) \end{Bmatrix} \geq 0$$

$$\phi\begin{Bmatrix} M(z,Bz,t), M(z,Bz,t),1, \\ M(Bz,Bz,t), M(z,Bz,t), M(Bz,z,t) \end{Bmatrix} \geq 0$$

(2.1.20) $\phi\{M(z,Bz,t), M(z,Bz,t),1,1, M(z,Bz,t), M(z,Bz,t)\} \geq 0$.

In view of implicit relations (1.25.1) we get

(2.1.21) $Bz = z$ and $ABz = z$ implies $Az = z$.

Therefore $Pz = Qz = Az = Bz = Sz = Tz = z$,

z is a common fixed point of P, Q, A, B, S and T.

For uniqueness of z, let ω be another common fixed point $(\omega \neq z)$

put $x = z$ and $y = \omega$ in (2.1.4) we get

$$\phi\begin{Bmatrix} M(Pz,Q\omega,t), M(ABz,ST\omega,t), M(ABz,Pz,t), \\ M(ST\omega,Q\omega,t), M(ABz,Q\omega,t), M(ST\omega,Pz,t) \end{Bmatrix} \geq 0$$

$$\phi\begin{Bmatrix} M(z,\omega,t), M(z,\omega,t), M(z,z,t), \\ M(\omega,\omega,t), M(z,\omega,t), M(\omega,z,t) \end{Bmatrix} \geq 0$$

$$\phi\{M(z,\omega,t), M(z,\omega,t),1,1, M(z,\omega,t), M(\omega,z,t)\} \geq 0.$$

In view of implicit relations (1.25.1) we get $z = \omega$.

Therefore there exists a unique common fixed point exists for P, Q, S, T, A and B.

If we put $B = T = I_X$ (the identity map on X) in the **theorem 2.1** we have the following.

Corollary 2.2: Let P, Q, S, A be four self maps of a complete fuzzy metric space $(X, M, *)$ with continuous t-norm *

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (P, A) and (Q, S) are sub compatible and sub sequentially continuous then

(2.2.1) P and A have a coincidence point
(2.2.2) Q and S have a coincidence point
(2.2.3) for some $\phi \in \phi$ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Px,Qy,t), M(Ax,Sy,t), M(Ax,Px,t), \\ M(Sy,Qy,t), M(Ax,Qy,t), M(Sy,Px,t) \end{Bmatrix} \geq 0$$

then P, Q, S, A have a unique common fixed point.

If we put $P = Q$, $B = T = I_X$ in the **theorem 2.1** we have the following.

Corollary 2.3: Let P, S, A be three self maps of a complete fuzzy metric space $(X, M, *)$ with continuous t-norm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (P, A) and (P, S) are sub compatible and sub sequentially continuous then

(2.3.1) P and A have a coincidence point
(2.3.2) P and S have a coincidence point
(2.3.3) for some $\phi \in \phi$ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Px,Py,t), M(Ax,Sy,t), M(Ax,Px,t), \\ M(Sy,Py,t), M(Ax,Py,t), M(Sy,Px,t) \end{Bmatrix} \geq 0$$

then P, S, A have a unique common fixed point .

If we put $A = S$ and $B = T = I_X$ in the **theorem 2.1** we have the following.

Corollary 2.4: Let P, Q, S be three self maps of fuzzy metric space $(X, M, *)$ with continuous t-norm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (P, S) and (Q, S) are sub compatible and sub sequentially continuous then

(2.4.1) P and S have a coincidence point
(2.4.2) Q and S have a coincidence point
(2.4.3) for some $\phi \in \phi$ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Px,Qy,t), M(Sx,Sy,t), M(Sx,Px,t), \\ M(Sy,Qy,t), M(Sx,Qy,t), M(Sy,Px,t) \end{Bmatrix} \geq 0$$

then P, Q, S have a unique common fixed point .

If we put $P = Q, A = S$ and $B = T = I_X$ in the **theorem 2.1** we have the following.

Corollary 2.5: Let P, S be two self maps of fuzzy metric space $(X, M, *)$ with continuous t-norm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pair (P, S) is sub compatible and sub sequentially continuous then

(2.5.1) P and S have a coincidence point
(2.5.2) for some $\phi \in \phi$ and for all $x, y \in X$ and every t>0

$$\phi\begin{Bmatrix} M(Px,Py,t), M(Sx,Sy,t), M(Sx,Px,t), \\ M(Sy,Py,t), M(Sx,Py,t), M(Sy,Px,t) \end{Bmatrix} \geq 0$$

then P, S have a unique common fixed point .

Now we can prove the following theorem using implicit relations (1.25.2).

Theorem 2.6: Let P, Q, S, T, A and B be six self maps of fuzzy Metric space $(X, M, *)$ with continuous t-norm * defined by $t*t \geq t$ for all $t \in [0,1]$. If the pairs (P, AB) and (Q, ST) are sub compatible and sub sequentially continuous then

(2.6.1) P and AB have a coincidence point
(2.6.2) Q and ST have a coincidence point,
(2.6.3) the pairs (P, T), (AB, T), (Q, B), (ST, B) are commutes,
(2.6.4) for some function ϕ in Φ and for all $x, y \in X$ and every t>o,

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 7, July 2013

$$\phi\left\{\begin{array}{l} M(Px,Qy,t), M(ABx,STy,t), \\ \dfrac{M(ABx,Px,t)+M(STy,Qy,t)}{2}, \\ M(ABx,Qy,t), M(STy,Px,t) \end{array}\right\} \geq 0,$$

then P, Q, S, T, A and B have a unique common fixed point.

Now replacing self maps by the identity maps in **theorem 2.6** we get the particular cases for four, three and two self mappings which generalizes the results of Kamal Wadhwa et al.[10] and others.

IV.CONCLUSION

This article investigates common fixed point theorems for six self mappings. The concept of sub compatible maps and sub sequentially continuous maps in Fuzzy metric spaces using implicit relations has also been used. Several Fixed point theorems in Fuzzy metric spaces such as fixed point theorems for four, three and two self mappings have been derived in the present study as particular cases.

REFERENCES

[1] Zadeh, L. A., “Information and Control”, Fuzzy Sets, No.8, pp.338-353, 1965.
[2] Kramosil, I., Michalek, J., “Fuzzy metric and statistical metric spaces”, Kybernetica, No.11, pp.336-344, 1975.
[3] George, A., Veeramani, P., “On some results in fuzzy metric spaces”, Fuzzy Sets and Systems, No.64, pp.395-399, 1994.
[4] Kaleva, O., Seikkala, S., ”On fuzzy metric spaces”, Fuzzy Sets Systems, No.12, pp.215-229, 1984.
[5] Popa, V., “A general coincidence theorem for compatible multi valued mappings satisfying an implicit relation”, Demonsratio Math., No.33, pp.159-164, 2000.
[6] Popa, V., “Some fixed point theorems for compatible mappings satisfying on implicit relation”, Demonsratio Math., No.32, pp.157 – 163, 1999.
[7] Singh, B., Jain, A., and Lodha, B., “On common fixed point theorems for semi compatible mappings in Menger space”, Commentationes Mathematicae, 50 (2), pp.127-139, 2010.
[8] Singh, B., Jain, A., and Masoodi, A. A., “Semi-compatibility, weak compatibility and fixed point theorem in fuzzy metric space”, International Mathematical Forum, 5(61), pp.3041-3051, 2010.
[9] Vasuki, R., “Common fixed points for R-weakly commuting maps in fuzzy metric spaces”, Indian J.Pure.Appl.Math. No. 30 pp.419-423, 1999.
[10] Kamal Wadhwa, Farhan Beg and Hariom Dubay, “Common Fixed Point Theorem for sub compatible and sub sequentially continuous maps in fuzzy metric space using implicit relation” , IJRRAS, 9(1), pp.87-92, 2011.
[11] Kutukcu, S., Sharma, S., and Tokgoz, H., “A fixed point theorem in fuzzy metric spaces”, Int.J.Math.Analysis, 1(18), pp.861-872, 2007.
[12] Klement, E. P., Mesiar R., and Pap, E., “Triangular Norm”, Kluwer Academic Publishers, Dordrecht., In Trends in Logic, vol.8, 2000.
[13] Grabiec, M., “Fixed points in fuzzy metric spaces”, Fuzzy Sets and Systems, No.27, pp.385-399, 1988.
[14] Pant, R. P., Common fixed points of four mappings, Bull.Cal.Math.Soc., No.90, pp.281- 286, 1998.
[15] Pant, R. P., Jha, K., “A remark on Common fixed points of four mappings in a fuzzy metric space”, J.FuzzyMath.12(2), pp.433-437, 2004.
[16] Bouhadjera, H., and Godet-Thobie, C., “Common fixed point theorems for pairs of sub compatible maps”, arXiv:0906.3159v1 [math.FA] 17 June 2009.
[17] Cho, Y. J., Pathak, H. K., Kang, S.M., and Jung, J.S., “Common fixed points of compatible maps of type (β) on fuzzy metric spaces”, Fuzzy Sets and Systems, No.93, pp.99-111, 1998.
[18] Balasubramaniam, P., MuraliSankar, S., and Pant, R. P., “Common fixed points of four mappings in a fuzzy metric space”, J.Fuzzy Math. 10(2), pp.379-384, 2002.
[19] Sharma, S., Deshpande, B., “Compatible multi valued mappings satisfying an implicit relation”, Southeast Asian .Bull.Math. No.30, pp.535-540, 2006.
[20] Jungck, G., “Compatible mappings and common fixed points (2)”, Internat.J.Math. Math. Sci, pp.285- 288, 1988.
[21] Sharma, S., “Common fixed point theorems in fuzzy metric spaces”, Fuzzy Sets and Systems No.127, pp.345-352, 2002.
[22] Sharma, S., Deshpande, B., “Discontinuity and weak compatibility in fixed point consideration on non-complete fuzzy metric Spaces”, J.Fuzzy Math.11(2), pp.671-686, 2003.
[23] Jungck, G., and Rhodes, B.E., “Fixed point for occasionally weakly compatible mappings”, Fixed point theory 7(2), pp.287- 296, 2006.
[24] Al-Thagafi, M. A., Naseer Shahzad, “Generalized I-Non expansive self maps and invariant approximation”, Acta Mathematica sinica, English Series 24(5), pp.867-876, 2008.
[25] Al-Thagafi, M. A., and Naseer Shahzad, “A note on occasionally weakly compatible maps”, Int.J. Math. Anal. 3(2), pp.55-58, 2009.
[26] Alamgir Khan, M., Sumitra,”Sub Compatible and Sub Sequentially Continuous Maps in Fuzzy Metric Spaces”, Applied Mathematical Sciences., 29(5), pp.1421-1430, 2011.