



Voltage Collapse Sensitivity in Power Systems

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ABSTRACT: Voltage instability and voltage collapse have been considered as a series threat to power system operation. Fast response and accurate voltage stability indications in power systems are still a challenging task to achieve, particularly when power systems operated close to its transmission capacity limits. A successful avoidance of system collapse is based on method accuracy and its low computation time. This paper presents simple, fast and efficient indices for analysing power system voltage stability and successfully predicting system voltage collapse. Four indices are proposed; which are based on the gradient of generated powers with respect to load components producing sensitive voltage stability indications. The S_{Pq} and S_{Qp} response sensitively when real load power changes with clear and readable indications while S_{Pq} and S_{Qq} measured system sensitivity to any demanded reactive power changes. This describes clearly the dynamics of power transfer through the transmission systems and how the system responds to load changes indicating how the system regain its load-generation equilibrium when load rate changes. A demonstration on the IEEE 14-bus, 57-bus and 118-bus systems are presented to validate the propose indices' efficiency and accuracy.

Keywords: Voltage stability analysis, voltage collapse, voltage stability index, sensitivity to voltage collapse

NOMENCLATURE

P_{gv}, P_{dt}	Total sending and receiving active powers at system buses.
Q_{gt}, Q_{dt}	Total sending and receiving reactive powers at system buses.
S_{T-Loss}	Total network losses.
P_g, P_d	Sending and receiving real powers at buses i and j .
Q_g, Q_d	Sending and receiving reactive powers at buses i and j .
S_g, S_d	Total sending and receiving apparent powers at buses i and j .
Y_{ij}	$(G+jB)$ system admittance between bus i and j .
V_i, V_j	Sending and receiving voltages at system buses i and j .
δ_i, δ_j	Sending and receiving voltage angles at system buses i and j .
$\Delta P, \Delta Q$	The changes in the real and reactive powers.
$\Delta V, \Delta \delta$	The deviations in bus voltage magnitude and angle.
ζ, η	The left and the right eigenvectors.
Λ	The diagonal eigenvector matrix of J_r matrix.
P_D^{total}, Q_D^{total}	The total demand active and reactive powers.
Q_G^{total}	The total generated reactive powers.
Q_D^i	The reactive power demand vector.
n_{mp}	The continuation direction of demand increase.
Q_D^i, P_D^i	The reactive and active perturbations at load bus i .
nG, nD	The numbers of generation and load buses.
S_i	Apparent powers at system bus i .
V_k, V_m	Sending and receiving voltages at system buses k and m .
δ_k, δ_m	Sending and receiving voltage angles at system buses k and m .
Y_{km}	system admittance between bus k and m .

I. INTRODUCTION

Electric power utilities are being under pressures by governments' agency, politics, economy and residential and industrial customers to provide reliable and uninterrupted service from power plants to loads. Unfortunately, a few utilities are able to construct new power plants and advance their systems targeting high reliability standard while others are forced to operate near their functioning design limits due economic and environmental constraints or because of the shortage in power delivery investment. Due to such operation, new types of instability have been formed



characterized new behaviour of system dynamics such voltage instability in load areas which could be the major cause to partial or total system collapse.

Several blackout events associated with voltage instability have been recorded worldwide costing millions of dollars, and still a threat to power system security. Blackouts events have occurred recently in Germany in 2006 , Russia in 2005 [1-3] and Greece in 2004 [4, 5]. In 2003, several blackouts occurred in Europe: in Italy [1], Sweden-East Denmark Sweden-East Denmark[1], London- UK [6] and Croatia and Bosnia Herzegovina [7] while a major blackout were recorded in north America, USA and Canada [8]. These recent blackout incidents are mostly caused by faults, equipment and device failures, unwanted relay operations, rapid and unexpected load increase, shortage in reactive powers, human errors, or by the lack of smart power system tools and intelligent protection devices.

As a voltage collapse problem has become a point of concern for utilities and academic researchers, several models have been employed in voltage collapse studies. Dynamic and static approaches are the most dominant use in voltage stability studies. The dynamic analysis is very useful in providing an insight into the nature of voltage collapse event coordinating between protection and control devices and testing in remedial measures while the static analysis provides an insight into the nature of voltage instability and determining the key contributed factors [9, 10].

Although voltage instability in power system is a dynamic phenomenon and its analysis is favored by some utilities, voltage instability has been viewed as a steady-state problem suitable for static analysis methods. Static voltage stability analysis is commonly used in research and on-line applications providing a fast and clear insight of voltage stability problem. Some methods proposed in the literature use the singularity of power flow model, Jacobian matrix, [11-13] or to calculate the reduce Jacobian determinates [14], compute the eigenvalue [15, 16], determine the smallest singular value of the dynamic state Jacobian matrix [17, 18] or identify the critical buses using tangent vector [19]. Another approach then were taken to determine maximum loadability point [20], estimate system collapse based on quadratic approximation of *PV*-curves [21] or minimize load voltage deviation [22].

In recent research, a second order approximation of the saddle nod bifurcation is introduced in [23] while instability detection using anti-colony optimization established in [24]. Reference [25] develops voltage collapse prediction index (*VCPI*) to evaluate voltage stability problem and reference [26] presents linear and nonlinear analysis tools to evaluate voltage stability whenever a small disturbance is occurred, while an improvement to voltage stability index designated as L_{ij} with the influence of load modelling is presented in [27].

A new protection scheme with a deviation approach was presented in [28] introducing two sensitivities terms as $TRGG_p$ and $TRGG_Q$ to screen maximum available margin. A non-iterative approach was proposed in [29] as a tool of voltage stability evaluation while an equivalent local network model with node index proposed in [30] to detect the point of voltage collapse and identify the critic node bus while Reference [31] introduced a performance index for distributed monitoring of system-wide quasi-static voltage instability based on the smallest singular value sensitivity of power flow Jacobian matrix.

These methods are different in their approaches, applications or applied conditions making their analyses somehow vulnerable. Some of them might be robust or precise, but may be time consuming for large power systems others may fail if any power system element is involved like control devices. Clear indications, lower computation time and accuracy are needed to prevent such blackout event and avoid voltage instability.

This paper proposes new indices to conduct power system sensitivity to voltage collapse based on generated real and reactive powers predicting how the system response to load dynamic behaviour. Four indices are proposed; two for indicating the sensitivity of generated real powers to load components designated as S_{Pp} and S_{Pq} while S_{Qp} and S_{Qq} indicate the sensitivity of generated reactive powers to load active and reactive powers. These developed indices provide how the power system reacts due to any load changes and at which point the system is collapsed based on generation power availability. S_{Pp}, S_{Pq}, S_{Qp} and S_{Qq} indices produce sensitive indications of voltage stability for the system as a whole and a separate bus analysis can be achievable. Simplicity, speediness and accurate voltage collapse prediction or detection is confirmed in the results section, reducing computation time and allowing operators and controls to act with sufficient time. The performance of these indices have been demonstrated on the *IEEE* 14-bus, *IEEE* 57-bus and 118-bus Test System to show their effectiveness and efficiency.

The paper is organized as follows: Section *II* presents the proposed voltage collapse indices while section *III* demonstrates a quick review of voltage stability methods. Then, paper results are discussed in section *IV* and the conclusions follow in section *V*.

II. THE PROPOSED INDICES:

The paper proposed four indices of conducting generated powers sensitivity to voltage collapse. Two sensitive indices are for the generated real powers to loads donated as SPp and SPq while the second sensitive set are for the generated reactive powers to load components designated as SQp and SQq . These sensitive tools are based on the behaviour of load power components and used to measure how far the system is from its collapse point. Here, the



generated real power sensitivity set is derived first from load flow equations and, then, the derivative of generated reactive power sensitivity set is followed.

A. Generated real powers Sensitivity Set:

The system shown in, Fig.1 is representative of general power system connecting a generator with load through transmission-line. As the equivalent generator modelled in normal state, it is assumed that the generator voltage, E , is in normal condition and equal to the voltage at generation bus V_i , preserving constant value using generator excitation systems.

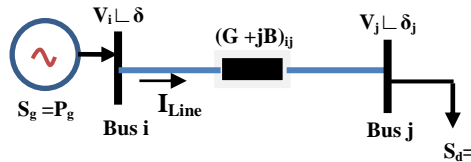


Fig.1 Simple power system at bus

This model can be extended to an n-bus power system. Using a generated real power as a base, the S_{Pp} and S_{Pq} sensitive indices can be derived from load flow equations with its constraints. Power system can be represented under subject constrained as following

$$\sum S_{di} + \sum S_{Loss-i} = \sum S_{gi} \quad (1.1)$$

$$s.t. \quad P_{g-min} \leq P_g \leq P_{g-max} \quad (1.2)$$

$$Q_{g-min} \leq Q_g \leq Q_{g-max} \quad (1.3)$$

The equation (1) also can be expressed in terms of total real and reactive powers as

$$(P_{gt} + jQ_{gt}) = (P_{dt} + jQ_{dt}) + S_{T-Loss} \quad (2)$$

By taking equation (2) in terms of real generation and demanded powers, the new equation can be expressed as

$$(1 + j\alpha)P_{gt} = (1 + j\beta)P_{dt} + S_{T-Loss} \quad (3)$$

where; $\alpha = \frac{Q_{gt}}{P_{gt}}$ and $\beta = \frac{Q_{dt}}{P_{dt}}$

Then, the total real generation powers, P_{gt} , can be determined by

$$P_{gt} = \frac{(1+j\beta)P_{dt} + S_{Loss-t}}{(1+j\alpha)} \quad (4)$$

By taking the derivative of total real generation powers with respect of total real demand powers, the new equation is expressed as

$$\frac{\partial P_{gt}}{\partial P_{dt}} = \frac{(1+j\beta)}{(1+j\alpha)} \quad (5)$$

By substituting the value of α and β into equation (5), the derivative, then, is expressed as

$$\frac{\partial P_{gt}}{\partial P_{dt}} = \frac{(P_{dt} + jQ_{dt})}{(P_{gt} + jQ_{gt})} \times \frac{P_{gt}}{P_{dt}} \quad (6)$$

By arranging the above equation, the new derivative form is expressed as

$$\frac{\partial P_{gt}}{\partial P_{dt}} = \frac{S_{dt}}{S_{gt}} \times \frac{P_{gt}}{P_{dt}} \quad (7)$$

S_{dt}/S_{gt} represents the power system efficiency, η , and the new equation is expressed as

$$\frac{\partial P_{gt}}{\partial P_{dt}} = \eta \frac{P_{gt}}{P_{dt}} \quad (8)$$

η might be considered in this equation as constant factor to equation (8) and its effect to the derivative is limited. If it is assumed to be neglected, then, the derivative of total real generation powers with respect of total real demand powers is expressed as

$$S_{Pp} = \frac{\partial P_{gt}}{\partial P_{dt}} = \frac{P_{gt}}{P_{dt}} \quad (9)$$

The equation (2) also could represents the real generation power and reactive demanded powers, and can be expressed as

$$(1 + j\alpha)P_{gt} = (\gamma + j1)Q_{dt} + S_{T-Loss} \quad (10)$$

where; $\alpha = \frac{Q_{gt}}{P_{gt}}$ and $\gamma = \frac{P_{dt}}{Q_{dt}}$



By following the same procedure, the derivative of total real generation powers with respect to total reactive demand powers is expressed as

$$\frac{\partial P_{gt}}{\partial Q_{dt}} = \eta \frac{P_{gt}}{Q_{dt}} \quad (11)$$

$\eta = S_{dt}/S_{gt}$ represents the power system efficiency and might be considered as constant factor to equation (11) and its effect to the derivative is limited. If it is assumed to be neglected, then, the derivative of total real generation powers with respect to total reactive demand powers is expressed as

$$S_{Pq} = \frac{\partial P_{gt}}{\partial Q_{dt}} = \frac{P_{gt}}{Q_{dt}} \quad (12)$$

S_{Pp} and S_{Pq} vary from the initial ratio point to infinity and can be used as a sensitive indicator to predict voltage instability and voltage collapse point. The system approaches its collapse point when S_{Pp} and S_{Pq} increment gradually, causing a sharp rise to infinite values.

B. Generated reactive powers Sensitivity Set:

As a load varies up and down demanding reactive powers, several dynamic power system components are involved in action to generate and supply the reactive power shortages within system capability limits. With such capability limits, system voltage profile associated with network reactive power is needed. By using the same simple system as illustrated in Fig.1, and the same assumptions, the S_{Qp} and S_{Qq} sensitive indices are introduced here based on system generated reactive powers and derived from equation (1) under its constraints. The total real and reactive powers of power flow expressed as

$$(P_{gt} + jQ_{gt}) = (P_{dt} + jQ_{dt}) + S_{T-Loss} \quad (2)$$

By taking equation (2) in terms of reactive generation and demanded powers, the new equation can be expressed as

$$(\gamma + j1)Q_{gt} = (\sigma + j1)Q_{dt} + S_{T-Loss} \quad (13)$$

Where; $\gamma = \frac{P_{gt}}{Q_{gt}}$ and $\sigma = \frac{P_{dt}}{Q_{dt}}$

Then, the total reactive generation powers, Q_{gt} , can be determined by

$$Q_{gt} = \frac{(\sigma + j1)Q_{dt}}{(\gamma + j1)} + \frac{S_{T-Loss}}{(\gamma + j1)} \quad (14)$$

By taking the derivative of total reactive generation powers with respect of total reactive demand powers, the new equation is expressed as

$$\frac{\partial Q_{gt}}{\partial Q_{dt}} = \frac{(\sigma + j1)}{(\gamma + j1)} \quad (15)$$

By substituting the value of γ and σ into equation (15), the derivative, then, is expressed as

$$\frac{\partial Q_{gt}}{\partial Q_{dt}} = \frac{P_{dt} + jQ_{dt}}{P_{gt} + jQ_{gt}} \times \frac{Q_{gt}}{Q_{dt}} \quad (16)$$

By arranging the above equation, the new derivative form is expressed as

$$\frac{\partial Q_{gt}}{\partial Q_{dt}} = \frac{S_{dt}}{S_{gt}} \times \frac{Q_{gt}}{Q_{dt}} \quad (17)$$

S_{dt}/S_{gt} represents the power system efficiency, η , and the new equation is expressed as

$$\frac{\partial Q_{gt}}{\partial Q_{dt}} = \eta \cdot \frac{Q_{gt}}{Q_{dt}} \quad (18)$$

η might be considered in this equation as constant factor to equation (18) and its effect to the derivative is limited. If it is assumed to be neglected, then, the derivative of total imaginary generation powers with respect of total reactive demand powers is expressed as

$$S_{Qq} = \frac{\partial Q_{gt}}{\partial Q_{dt}} = \frac{Q_{gt}}{Q_{dt}} \quad (19)$$

The equation (13) also could represents the reactive generation power and real demanded powers, and can be expressed as

$$(\gamma + j1)Q_{gt} = (1 + j\psi)P_{dt} + S_{T-Loss} \quad (20)$$

where; $\gamma = \frac{P_{gt}}{Q_{gt}}$ and $\psi = \frac{Q_{dt}}{P_{dt}}$



By following the same procedure, the derivative of total reactive generation powers with respect to total real demand powers is expressed as

$$\frac{\partial Q_{gt}}{\partial P_{dt}} = \eta \frac{Q_{gt}}{P_{dt}} \quad (21)$$

$\eta = S_{dt}/S_{gt}$ represents the power system efficiency and might be considered as constant factor to equation (21) and its effect to the derivative is limited. If it is assumed to be neglected, then, the derivative of total real generation powers with respect to total real demand powers is expressed as

$$S_{Qp} = \frac{\partial Q_{gt}}{\partial P_{dt}} = \frac{Q_{gt}}{P_{dt}} \quad (22)$$

S_{Qp} and S_{Qq} vary from the initial ratio point to infinity and can be used as a sensitive indicator to predict voltage instability and voltage collapse point. The system approaches its collapse point when S_{Qp} and S_{Qq} increment gradually, causing a sharp rise to infinite values or goes to or approaching zero.

III. REVIEW OF VOLTAGE STABILITY METHODS:

This section briefly discusses three methods of conducting voltage stability analysis which are: modal analysis introduced in [32] and the voltage instability indices *TRGG*[28] and *VCPI*[25].

A Modal Analysis:

Modal analysis is used to compute eigenvalues and eigenvectors of a reduced Jacobian matrix of power flow to predict voltage instability in power systems. The eigenvalues of the reduced matrix determine the system mode while the eigenvectors give an approximate measure to system instability. The equation of power flow is given by

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (23)$$

where ΔP and ΔQ are the changes in the real and reactive powers while ΔV and $\Delta\delta$ are the deviations in bus voltage magnitude and angle. If ΔP is assumed to be zero, the *V-Q* sensitivity is expressed as

$$\frac{\Delta V}{\Delta Q} = J_r^{-1} \quad (24)$$

where, $J_r = J_{QV} - (J_{Q\delta} \cdot J_{P\delta}^{-1} \cdot J_{PV})$. By taking the right and left eigenvector matrix into account, the J_r matrix can be expressed as

$$J_r^{-1} = \xi \cdot \Lambda^{-1} \eta \quad (25)$$

where, ξ and η are the left and the right eigenvectors while Λ is the diagonal eigenvector matrix of J_r matrix. Then, the *V-Q* sensitivity is expressed as

$$\Delta V = \xi \cdot \Lambda^{-1} \eta \cdot \Delta Q \quad (26)$$

Once the eigenvectors are normalized as it did in practice, $\xi_i \cdot \eta_i = 1$ where $\forall_i = 1, 2, \dots, n$ then the equation expressed as

$$\eta \cdot \Delta V = \Lambda^{-1} \eta \cdot \Delta Q \quad (27)$$

Since $\eta = \eta^{-1}$, the final ΔV and ΔQ relationship is expressed as

$$\Delta V = \Lambda^{-1} \cdot \Delta Q \quad (28)$$

where Λ^{-1} can be expressed in matrix form as

$$\Lambda^{-1} = \begin{bmatrix} \lambda_1 & 0 & \dots & \dots & 0 \\ 0 & \lambda_2 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \lambda_n \end{bmatrix}$$



$$\text{Hence, } \Delta V|_{bus} = \lambda_i^{-1} \cdot \Delta Q, \quad \forall_i = 1, 2, \dots, n \quad (29)$$

For any i , if $\lambda > 0$, then the variation of V_i and Q_i are in the same direction and the system is stable while the system is considered unstable when $\lambda < 0$ for any i .

Once assumed $\Delta Q=0$ in Eq. (29), the V - P sensitivity is established and expressed as

$$\frac{\Delta V}{\Delta P} = J_r^{-1} \quad (30)$$

where, $J_r = J_{PV} - (J_{P\delta} \cdot J_{Q\delta}^{-1} \cdot J_{QV})$. By taking the right and left eigenvector matrix into account, the J_r matrix can be expressed as

$$J_r^{-1} = \xi \cdot \Lambda^{-1} \eta \quad (31)$$

By doing the same procedure, the V - P sensitivity is then expressed as

$$\Delta V|_{bus} = \lambda_i^{-1} \cdot \Delta P, \quad \forall_i = 1, 2, \dots, n \quad (32)$$

For any i , if $\lambda > 0$, then the variation of V_i and Q_i are in the same direction and the system is stable while the system is considered unstable when $\lambda < 0$ for any i .

B TRGGQ:

$TRGGQ$ is a voltage stability indicator used to screen the maximum available margin utilized in remedial scheme protection against cascaded voltage collapses. $TRGGQ$ represents the gradient of the total reactive power generation with respect to the total active or reactive demand perturbations termed as $TRGGQ_{mp}^Q$ and $TRGGQ_{mp}^P$ where both indices can be defined in any direction of load increase. The $TRGGQ_{mp}^Q$ is defined as the change rate of Q_G^{total} with respect to a point of Q_D in the direction n_{mp} and can be given by:

$$TRGGQ_{mp}^Q = \frac{dQ_G^{total}}{dQ_D^{total}} = \sum_{i=1}^{nD} \left[\frac{\partial Q_G^{total}}{\partial Q_D^i} \cdot \frac{Q_D^i}{\sum Q_D^i} \right] \quad (34)$$

The $TRGGQ_{mp}^P$ is defined as the change rate of Q_G^{total} with respect to a point of P_D in the direction n_{mp} and can be given by:

$$TRGGQ_{mp}^P = \frac{dQ_G^{total}}{dP_D^{total}} = \sum_{i=1}^{nD} \left[\frac{\partial Q_G^{total}}{\partial P_D^i} \cdot \frac{P_D^i}{\sum P_D^i} \right] \quad (35)$$

$TRGGQ_{mp}^Q$ and $TRGGQ_{mp}^P$ indices increase incrementally when the load increase gradually until reach to a point where both indices go to infinity or drop sharply to zero indicating voltage collapse.

C VCPI:

$VCPI$ is a voltage collapse indicator based on bus system and derived from power flow equation where the apparent power, S_s , at any bus i is expressed as:

$$S_k^* = \left[\begin{array}{l} |V_k|^2 - (|V_k| \cdot \cos \delta_k - j|V_k| \cdot \sin \delta_k) \\ \cdot \left(\sum_{\substack{m=1 \\ m \neq k}}^N (|V'_m| \cdot \cos \delta'_m - j|V'_m| \cdot \sin \delta'_m) \right) \end{array} \right] \cdot Y_{kk} \quad (36)$$

where V'_m is given by:

$$V'_m = \frac{Y_{km}}{Y} \cdot V_m, \quad \text{and } Y = \sum_{k=1, j \neq k} Y_{km}$$

Based on equation (38), the voltage collapse prediction index is expressed as

$$VCPI|_{bus} = 1 - \frac{\left| \sum_{m=1, m \neq k}^n V'_m \right|}{|V_k|} \quad (37)$$

$VCPI$ varies from zero to one, indicating the voltage stability margin. Once the value of $VCPI$ closes to unity or exceeds it, the system voltage collapses.



IV. RESULTS AND DISCUSSION

This section demonstrates an implementation of the proposed indices S_{Pp} , S_{Pq} , S_{Qp} , and S_{Qq} on IEEE 14-bus, 57-bus and 118-bus systems to approximate how far the system is from its point of collapse based on load behaviour. Those indices were compared with modal analysis indices (dV/dP and dV/dQ), TRGG ($TRGG_{Qp}$ and $TRGG_{Qq}$), and VCP to validate their accuracy. The characteristics of these methods are different, yet they have something in common. They all share system maximum power transfer and voltage stability margin starting by system normal condition and ending by system voltage collapse. Loading scenarios were considered here to validate the accuracy of the proposed indices at which each scenario represented a gradual load increase until the system reached to voltage collapse point.

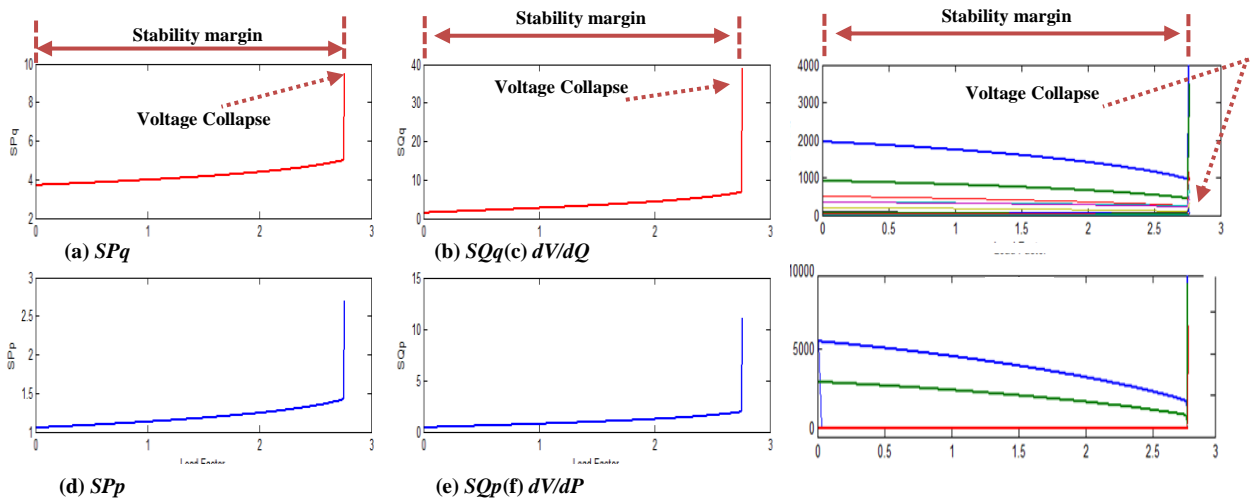


Figure 21st scenario SPq , SPp , SQq and SQp and Modal Method vs. load factor kon IEEE 14-bus system

A. IEEE 14-bus system:

Figures (2) and (3) show the performance of S_{Pp} , S_{Pq} , S_{Qp} , and S_{Qq} on IEEE 14-bus system and their sensitive indications were compared to the results of modal analysis indices, dV/dP and dV/dQ , to verify their accuracy of estimating the point of voltage collapse for two loading scenarios. In each of these figures, S_{Pq} , S_{Pp} , S_{Qq} and S_{Qp} are shown in sub-figures (a), (b), (d), (e) respectively, while dV/dP and dV/dQ are shown in subfigures (c) and (f) respectively.

Figure 2 illustrates the first loading scenario where the loads at all bus were incrementally increased with identical loading rate k until the IEEE 14-bus system collapsed. The results showed that S_{Pp} , S_{Pq} , S_{Qp} , and S_{Qq} predicted the point of voltage collapse at loading rate $k = 2.75$; as dV/dP and dV/dQ indices predicted. Voltage stability margin was also estimated by all indices starting with steady-state condition and ending by system voltage collapse. S_{Pp} , S_{Pq} ,

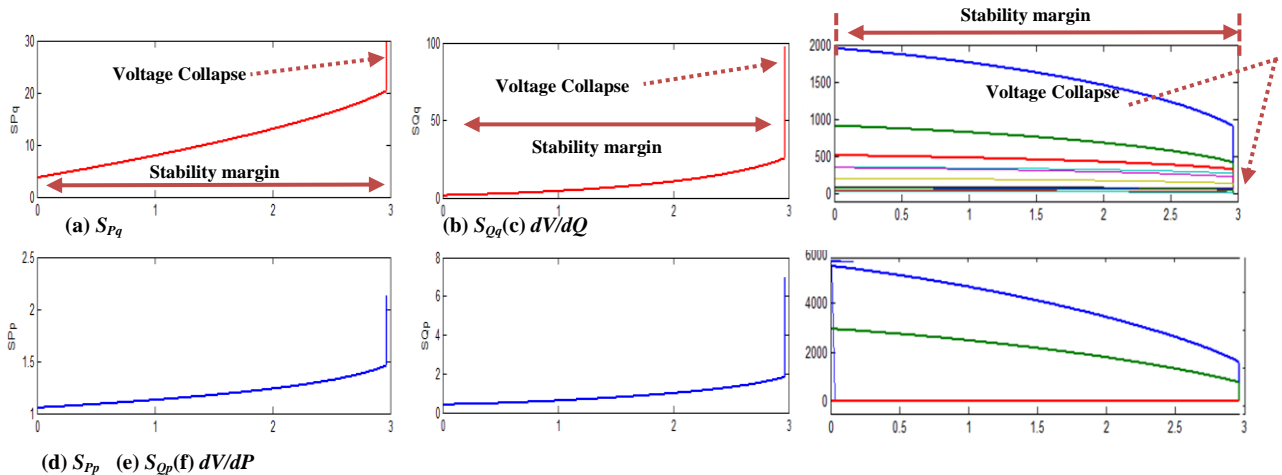


Figure 3 2nd scenario SPq , SPp , SQq and SQp and Modal Method vs. load factor kon IEEE 14-bus system



S_{Op} , and S_{Oq} started at the initial system state with clear and readable voltage stability indications and increased gradually along with load increase until reached a point where they went sharply to infinity.

Figure 3 is an illustration of second scenario when only the real powers at all buses were increased gradually till system collapse. All performed indices predicted accurately the point of voltage collapse at loading rate $k = 2.96$ sharing similar voltage stability margin. The eigenvalues of dV/dP and dV/dQ dropped from high to low values approaching to zero, where S_{Pp} , S_{Pq} , S_{Op} , and S_{Oq} went sharply to infinity indicating system voltage collapse point.

dV/dP and dV/dQ in subfigures (2.c) and (2.f) approximated the sensitive measures to system voltage instability for number of system buses. The sensitivity of the generated real powers to changeable demanded (active and reactive) powers were demonstrated in subfigures (2.a) and (2.d) while S_{Op} and S_{Oq} illustrated in subfigures (2.b) and (2.e) showed the response of system generated reactive powers to load increase.

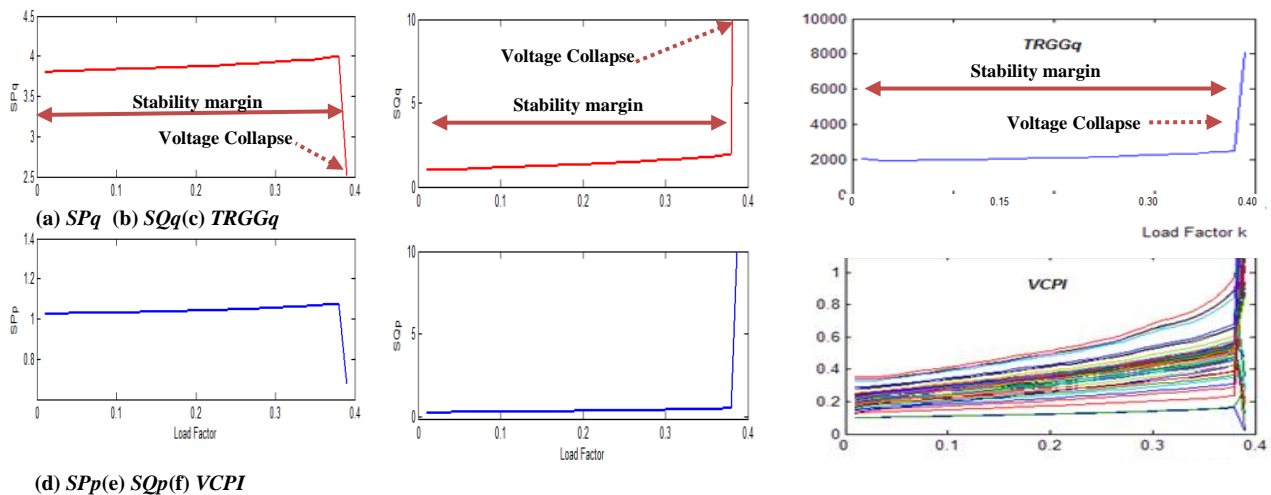


Figure 4 1st scenario on IEEE 57-bus system SPp , SPq , SQp , SQq , $VCPI$ and $TRGGq$ vs. load factor k

B. IEEE 57-bus system:

Figures (4) and (5) show the performance of S_{Pp} , S_{Pq} , S_{Op} , and S_{Oq} on IEEE 57-bus system. The results of performed indices were compared to alternative methods to verify their accuracy for two loading scenarios. In each of these figures, S_{Pp} , S_{Pq} , S_{Op} , and S_{Oq} are shown in sub-figures (a), (b), (d), (e) respectively, while $TRGGQq$ and $VCPI$ are shown in subfigures (c) and (f) respectively.

Figure 4 represents the performance of S_{Pp} , S_{Pq} , S_{Op} , and S_{Oq} indices for the first scenario in which the IEEE 57-bus system was subjected to load increase. All indices started with system initial state and ended at the same voltage collapse point at loading rate $k = 0.39$, where S_{Op} , S_{Oq} and $TRGGQq$ went to infinity and S_{Pp} , S_{Pq} dropped sharply

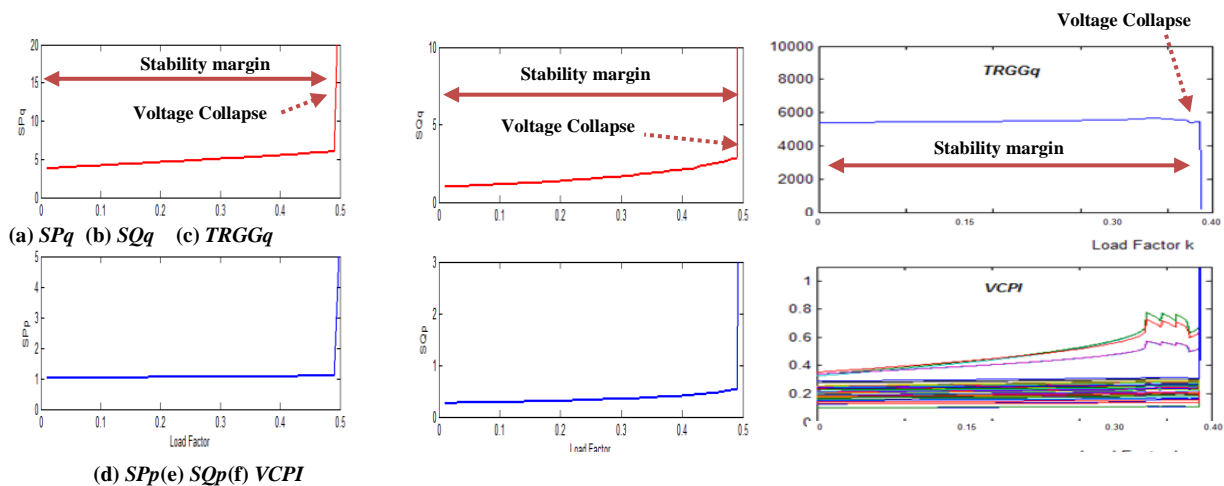


Figure 5 2nd scenario on IEEE 57-bus system, SPp , SPq , SQp , SQq , $VCPI$ and $TRGGq$ vs. load factor k

approaching zero. At the same point, $VCPI$ exceeded its stability boundaries passing its unity approaching infinity. The



VCPI has an advantage of providing voltage stability indications at each individual bus while the proposed sensitive indices along with $TRGGq$ have a great sensitivity to system voltage collapse based on active and reactive powers which can be used in load-generation balance.

The generated powers, active and reactive powers, were clearly demonstrated load-generation sensitivity in IEEE 57-bus system producing system stability indications along with load increase, estimating load-generation stability margin, and approximate the system to the point of its voltage collapse based on its load-generation sensitivity.

Figure 5 illustrates the second loading scenario where the real loads only at all bus were incrementally increased with identical loading rate k until the IEEE 57-bus system collapsed. The results showed that S_{Pp} , S_{Pq} , S_{Qp} , and S_{Qq} indices accurately projected the point of voltage collapse at loading rate $k = 0.49$ while $TRGGq$ and VCPI predicted system collapse earlier by 0.02 loading rate k difference. At the point of system collapse, $TRGGq$ went sharply to zero and VCPI passed its stability limits while S_{Pp} , S_{Pq} , S_{Qp} , and S_{Qq} indices approached infinity.

The proposed indices responded instantly to any load change indicating how the system regains its load-generation equilibrium. S_{Pp} and S_{Qp} express a direct sensitivity to load active power increase while S_{Pq} and S_{Qq} indicate the system impact to any increase of load reactive powers.

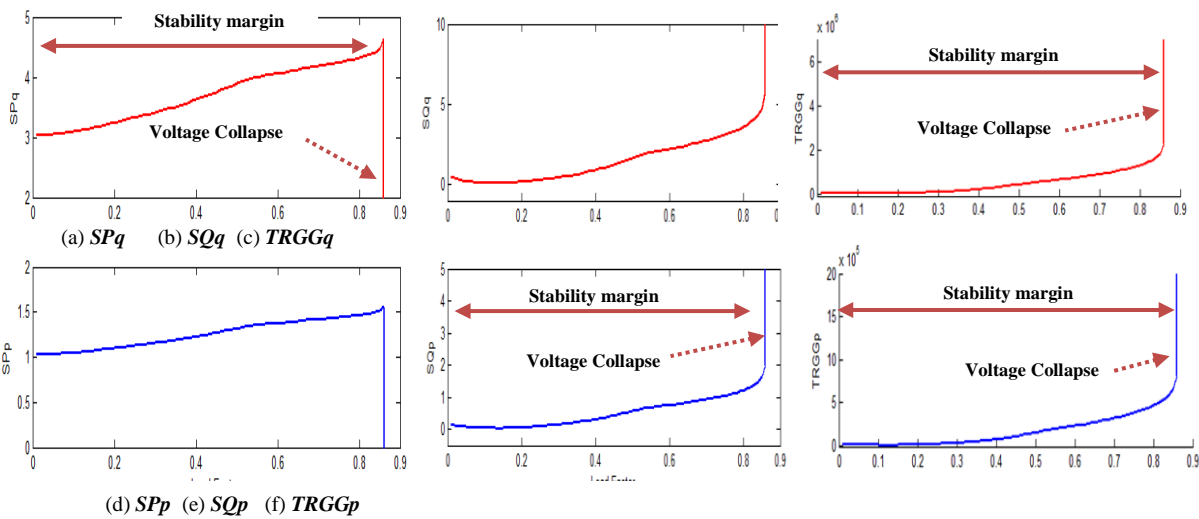


Figure 6 1st scenario on IEEE 118-bus system, SPp , SPq , SQp , SQq , $TRGG$ and $TRGGq$, vs. load factor k

C. IEEE 118-bus system:

The proposed indices are also implemented in large IEEE 118-bus system to check their performance in large power systems. Figures (6) and (7) illustrate their performance for two scenarios comparing their results with $TRGGQp$ and $TRGGQq$ as an alternative method. In each of these figures, S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} are shown in sub-figures (a), (b), (d), (e) respectively, while $TRGGq$ and $TRGGQp$ are shown in subfigures (c) and (f) respectively.

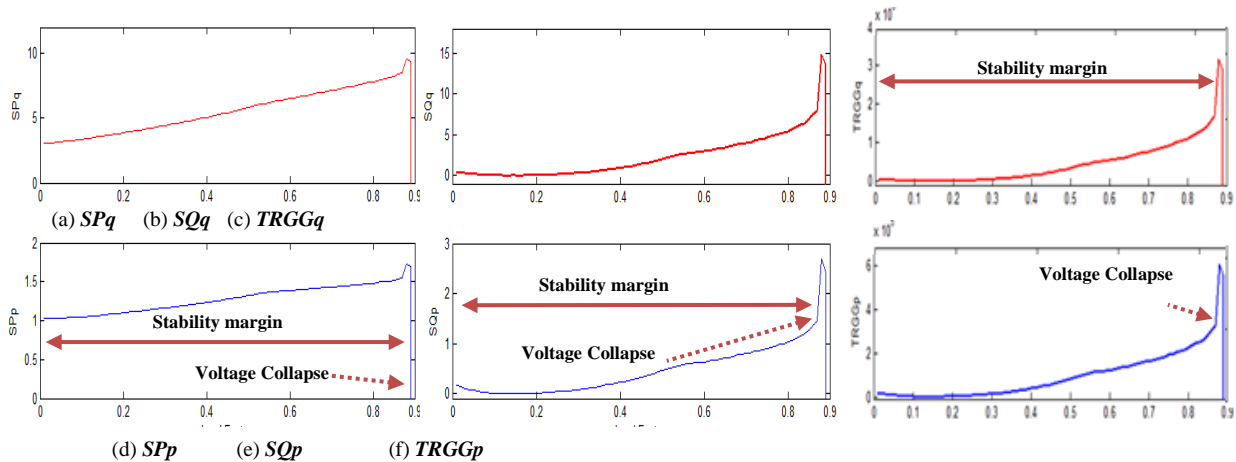


Figure 7 2nd scenario on IEEE 118-bus system, SPp , SPq , SQp , SQq , $TRGG$ and $TRGGq$, vs. load factor k



At figure 6, the results showed that S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} accurately projected the point of voltage collapse as $TRGGp$ and $TRGGq$ did at loading rate $k = 0.86$. At this point, all compared indices increased gradually along with load increase reaching a point where all indices went sharply to infinity except S_{Pp} and S_{Pq} dropped severely to zero. Between initial state and collapse point, all indices shared an identical voltage stability margin.

At a second scenario shown in figure 7 where IEEE 118-bus system was subjected only to real load increase, all indices collapsed at the same voltage collapse point at loading rate $k = 0.89$, where all indices rose with load increase to higher values depending their sensitive characteristics and then dropped sharply approaching zero. Voltage stability margins also estimated equally by S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} accurately indices and verified with $TRGGp$ and $TRGGq$ indices measuring how far the system is from its collapse. Here, the S_{Pq} and S_{Qp} reacted sensitively as real load was changed with clear and readable indications while S_{Pq} and S_{Qq} measured system sensitivity to any demanded reactive powers. This describes clearly the dynamics of power transfer through the transmission systems and how the system responses to load changes.

D. Indices Computation time on IEEE 118-bus system:

Low computation time in voltage stability analysis is considerably critical, because some power systems operate daily perhaps hourly near to transmission capability limits. Conducting a fast voltage stability analysis is a significant factor to prevent such system instability. Thus, S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} indices were demonstrated their speediness in voltage stability analysis on IEEE 118-bus system compared to alternative methods as shown in Table.1. The results showed that the proposed indices had the highest speed records in conducting voltage stability analysis recording 0.9832sec, 0.9948 sec, 0.9986sec, and 1.0355sec for S_{Qq} , S_{Qp} , S_{Pq} , and S_{Pp} respectively while Modal indices recorded the highest computation time estimated at 1.4635sec and 1.5154sec for dV/dQ and dV/dP respectively. A successful avoidance of voltage collapse is based on method accuracy with low computation time.

TABLE1:
 A Computation Time Comparison among Methods Implemented in IEEE 118-Bus System

Methods	Time/Sec	N. Iterations
S_{Qq}	0.9832	10
S_{Qp}	0.9948	10
S_{Pq}	0.9986	10
S_{Pp}	1.0355	10
VCPI	1.3360	10
TRGGq	1.4630	10
TRGGp	1.4633	10
dV/dQ	1.4635	10
dV/dP	1.5154	10

E. Overall Results:

The overall results show that S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} indices accurately predicted voltage collapse point as the alternative methods did showing that they all shared similar voltage stability margin. Our results also show that the proposed indices had the lowest computation time recording 0.9832sec. to 1.3360sec.

VCPI, TRGG, and modal analysis are voltage stability analyses; where VCPI is a ratio between the sending-receiving voltages, and TRGGQ calculates the system distance to voltage collapse based on load power components, while Modal analysis, dV/dP and dV/dQ , computes the eigenvalues and eigenvectors as to provide a proximity measures to Jacobian matrix singularity. Despite the fact that both proposed and alternative methods have different characteristics, they all share system maximum power transfer and voltage stability margin.

However, although modal analysis and TRGGQ are powerful voltage stability analysis, they are complex consuming high computational time. It is unnecessarily to compute the minimum eigenvalues and system mode for a large power system with thousands of busses attempting to extract voltage stability at each individual bus or system line while voltage collapse sensitivity for the system as a whole is not enough to prevent voltage collapse.

VCPI is simple voltage stability analysis with low computational time, yet its index relays on the ratio between the sending and receiving voltages which may fail to detect accurate indications near P-V buses with no sing of remedial actions.

S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} are the gradient of generated powers with respect to load components producing sensitive voltage stability indications. The S_{Pq} and S_{Qp} response sensitively when real load power changes with clear and readable indications while S_{Pq} and S_{Qq} measured system sensitivity to any demanded reactive power changes. This describes



clearly the dynamics of power transfer through the transmission systems and how the system responses to load changes indicating how the system regain its load-generation equilibrium when load rate changes. S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} accurately project the point of voltage collapse as the alternative method predicted measuring system sensitivity margin to system collapse.

Thus, the proposed indices, S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} are superior in their simplicity, accuracy, and speed calculations indicating powerful tools to approximate power system to its collapse point. With such simplicity, accuracy and speeds of data readability operators may act faster than before particularly when the system subjected to a sudden disturbance.

V. CONCLUSION

This paper presents a new approach of voltage stability analysis projecting successfully the point of voltage collapse. Four sensitive indices were proposed: S_{Pp} and S_{Qp} response sensitively when real load power changes with clear and readable indications while S_{Pq} and S_{Qq} measured system sensitivity to any demanded reactive power changes. S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} generate sensitive indications to voltage collapse for any load change indicating how far the system is from its collapse point.

The proposed indices accurately projected the point of voltage collapse as the alternative method predicted measuring system sensitivity margin to system collapse and indicating how the system regain its load-generation equilibrium when load rate changes. The results also showed that the system capability of compensating the reactive power demands was indicated by S_{Pq} and S_{Qq} while S_{Pp} and S_{Qp} reflect the system capability of supplying real power demands. Our results also showed that the proposed indices had the lowest computation time comparing to alternatives.

The proposed indices, S_{Pq} , S_{Qq} , S_{Pp} and S_{Qp} are superior in their simplicity, accuracy, and speed calculations indicating powerful tools to approximate power system to its collapse point. With such simplicity, accuracy and speeds of data readability operators may act faster than before particularly when the system subjected to a sudden disturbance

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