ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Issue 11, November 2017

Variable Viscosity on Unsteady Dissipative Carreau Fluid over a Truncated Cone Filled with Titanium Alloy Nanoparticles

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Abstract: In this study, we proposed a theoretical investigation on the temperature dependent viscosity effect on magnetohydrodynamic dissipative nanofluid over a truncated cone with heat source/sink. The involving set of nonlinear partial differential equations is transforming to set of nonlinear ordinary differential equations by using self-similarity solutions. The transformed governing equations are solved numerically using Runge-Kutta based Newton's technique. The effects of various dimensionless parameters on the skin friction coefficient and the local Nusselt number profiles are discussed and presented with the support of graphs. We also obtained the validation of the current solutions with existing solution under some special cases. The water based titanium alloy have a lesser friction factor coefficient as compared with kerosene based titanium alloy, whereas the rate of heat transfer is higher in water based titanium alloy compared with kerosene based titanium alloy. From this we can highlight that depending on the industrial needs cooling/heating choose the water or kerosene based titanium alloys.

Keywords: Viscous dissipation, Nanofluid, Temperature dependent viscosity, Truncated cone, Heat source/sink, Magneticfield parameter

I. Introduction

Now days, it is well recognized that numerous fluids have their own implication in transpiration structures, drug delivery system, electronics, fuel cells, nuclear reactor heating up and down classifications and biological sensor systems etc. For physical as well as the industrial applications, the geometry of the problem demands vital role. In particular, the flow over a cone has various engineering as well as biomedical applications such as aerosols engines, solar collectors, aeronautics, rotating heat exchangers, geophysics, heartbeat controlling systems and auto mobile industries etc. In view of this the revolution of laminar flow over a cone was started Tien [1] in 1960's. Later on, Lin and Rubin [2] analyzed the three dimensional flow due to cone filled with viscous nature at one incidence. The flow of micropolar fluid over a rotating cone with mixed convective conditions was illustrated by Subba and Gorla [3] and highlighted that rotation parameter improves the heat transfer rate. Chamkha [4] studied the heat and mass transfer characteristics of magnetohydrodynamic flow over a cone with radiation. The variable viscosity and thermal radiation on magnetohydrodynamic flow over a truncated cone in the presence of double-diffusive convection was examined by Mahdy et al. [5]. The water based unsteady nanofluid flow over a rotating vertical cone was investigated by Saleem et al. [6]. The researchers studied the heat and mass transfer characteristics of Newtonian and non-Newtonian fluids with various flow geometries and effects such as cross-diffusion, convective conditions, with rotation or without rotation [7-12]. With that they concluded the rotation have a tendered to encourages the heat transfer rate and cross diffusion is controlled the mass transfer rate.

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A metallic solid or liquid that can be composed from a homogeneous or non-homogeneous combination of two or more non-metals or metals or metalloid nanometer sized particles is called an alloy. Recent days, these can be extensively used for conveying the specific physical characteristics of mixtures. The steel, phosphor bronze, gold, brass and solder are the examples of alloys. Alloys have variety of wide range applications such as aerospace science technology, hip joint replacement processes, advanced powder technology, surgical implantation and various biological treatments. The brief summary of alloys and its application are given [13-15]. Frequently these are used in fabrication handling systems, construction of cold and hot rolling sheets etc. Finally, the combination of α and β alloys has various industrial as well science and engineering applications. These alloys supports the weight savings in place of aluminum and lesser strengthen aerospace type steel model. Later on, the researchers presented the experimental as well as theoretical investigations on titanium alloys for various flow geometries and effects [16-19].

In heat transfer equipment, fluids are often utilized as heat carriers. For instance, a heat transfer fluid plays crucial role in several continuous industrial applications like electronics industry, petroleum, polymer, automotive industry, etc. So the thermal conductivity of heat transfer fluids is important in the heat transfer equipment. But conventional heat transfer fluids like water, ethylene glycol and oil are usually inadequate for heat transfer. We know that solid metals like copper have more thermal conductivity than those conventional fluids like water. Hence metallic liquids have more thermal conductivity equated to nonmetallic liquids. So researchers attempted to dispense solid metal/metallic-liquid particles in conventional fluids to enhance thermal conductivity and heat transfer rate. In 1881, Maxwell [20] discovered that the thermal conductivity of suspensions consist of spherical particles enhanced with the volume fraction of the solid particles. Later several researchers continued this work. But in a small passage, the fluid flows might have clogging problems when mini and micro sized particles mixed with the base fluids. Therefore, very small quantity of particle is needed to dispense in the fluids to get rid of clogging problems. Choi [21] introduced the term nanofluid which refers to the fluid in which nanometer-sized (less than 50 nm) particles disseminated. But current technology is acceptable to manufacture nanoparticle of 10 nm. Later some authors measured the thermal conductivity of nanofluids using different methods [22,23]. Recently, several researchers investigated the nanofluid flow through different channels by considering water as a base fluid and metallic particles like Cu, Cuo, Al_2O_3, TiO_2 as nanoparticles by viewing various parameters [24-30]. Some of their discoveries are mixture of water and Al_2O_3 exhibited more enhancement compare to the mixture of water and Cuo, the Nusselt number raises with the rise in nanoparticle volume fraction and enhancing thermal radiation parameter lessen the temperature of the nanofluid. The magnetohydrodynamic is the study of conducting liquids. It has various applications such as controller in MHD generators, nuclear reactor system, activating the blood cells by applying external magnetic field etc. Keeping view into this the authors studied the magnetic field with various effects of flow controlling parameters like (magnetic Nano properties, cross-diffusion, thermal radiation and non-Darcy porous layer) and different geometries (cone, plate, wavy surface, sheet etc.) [31-35]. With this they highlighted that the magnetic field act as a controller in the flow field. Similarly the heat source/sink is also is have significance in various heat treatment processes such as heat exchanger systems, sugar factories, air-conditioning equipment's, crystallization process etc. Because of this significance the authors investigated the heat source or sink on flow over various geometries (sheet, plate, wedge cone etc.) with various flow controlling parameters (chemical reaction, dissipation, non-Newtonian properties etc.) [36-41]. In that papers they concluded that the heat source/sink is useful in heat treatment controlling processes.

In most of the real time applications combination of fluids are very helpful to enhancing or decreasing the temperature as well as heat transfer rate. It has various applications such as cooling organisms, heating developments and also biomedical as well as aerospace science and technologies. With the inspirations of the above stated applications and studies, this paper address the temperature dependent viscosity effect on dissipative unsteady magnetohydrodynamic nanofluid flow over a truncated cone in the presence of heat source/sink. The set of nonlinear partial differential equations are transforming to ordinary differential equations and then found the solution by using Runge-Kutta based shooting technique.

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II. Formulation of the Problem

In this study, we consider the heat source/sink effect on unsteady dissipative nanofluid flow due to a truncated cone in rotating frame with temperature dependent viscosity. We consider the rectangular curvilinear fixed coordinate system. The geometry of the problem is shown in Fig. 1. Let, x and y be the velocity components along the (tangential), (circumferential or azimuthal) and z (normal) directions, respectively. The half angle of is γ. Both the fluid and the cone are in a rigid body rotation about the axis of cone with timed dependent angular velocity. Their rotation is either in same or in opposite direction, which causes the unsteadiness in the fluid flow. The temperature variations are responsible for the buoyancy forces in the fluid flow. The surface of cone is an electrically-insulate. The flow is considered to be axisymmetric. The magnetic Reynolds number is taken to be small such that the effect of induced magnetic field is negligible. The wall temperature is a function of x. According to above assumptions the boundary layer equations of momentum and energy for an incompressible, unsteady flow Ching Yang Cheng [29], Patralescu et al. [11] and Anil kumar and Roy [28] are given by:

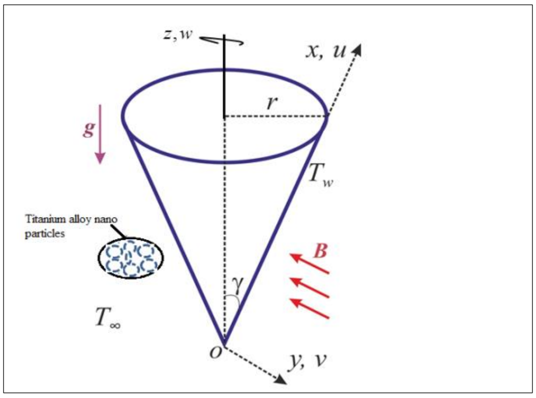

Fig. 1. Physical configuration.

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial z}(rw)=0, \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}\right)=\left(\frac{\partial}{\partial z}\left(\mu_{nf}\frac{\partial u}{\partial z}\right)-\frac{v_e^{\,2}}{x}+g(\rho\beta)_{nf}(T-T_\infty)\sin\gamma-\sigma_f B^2 u+\frac{3(n-1)}{2}\Gamma^2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+\frac{vu}{x}\right)=\left(\frac{\partial}{\partial z}\left(\mu_{nf}\frac{\partial u}{\partial z}\right)-\frac{dv_e}{dt}-\sigma_{nf}B^2 v\right), \tag{3}$$

$$(\rho c_p)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=k_{nf}\frac{\partial^2 T}{\partial z^2}+\mu_{nf}\left(\frac{\partial u}{\partial z}\right)^2-Q_0(T-T_\infty), \tag{4}$$

With the boundary conditions:

$$\left.\begin{aligned} &u=0,\ v=x\Omega_1\sin\gamma(1-\Omega\sin(\gamma)t)^{-1},\ w=0,\ \mathrm{T}=\mathrm{T_w}\ \text{ at }\ z=0,\\ &u=0,\ v=v_e=x\Omega_2\sin(\gamma)(1-\Omega\sin(\gamma)t)^{-1},\ \mathrm{T}\to\mathrm{T_\infty}\ \text{ as }\ \mathrm{z}\to\infty,\end{aligned}\right\} \tag{5}$$

Where u, v are the velocity components along the x, y directions respectively. ρ_{nf} is the density of the nanofluid, g is the acceleration due to gravity, v_ε is free stream velocity, $(\rho\beta)_{nf}$ is the thermal expansion coefficient due to temperature

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difference, σ_f is the electric conductivity of the fluid, σ_s is the electric conductivity of the solid, γ is the semi-vertical angle of the cone, v is the kinematic viscosity. μ_{nf} is the dynamic viscosity coefficient. Where $(\rho c_p)_{nf}$ is the heat capacity of the nanofluid, T is the fluid temperature, T_w, T_∞ are the fluid temperatures near and the far away from the boundary, K_{nf} is the thermal conductivity of the nanofluid, c_p is the specific heat capacitance at constant pressure, Q_0 is the heat source/sink parameter. Here we assume that the viscosity changes with temperature in the form as given by Ching Yang Cheng [29].

$$\mu_f = \frac{\mu_\infty}{1+\omega(T-T_\infty)} \tag{6}$$

Where μ_∞ is viscosity of the ambient fluid, ω is the constant, $E=\omega(T_w-T_\infty)$ is the viscosity variation parameter. To convert the nonlinear partial differential equations into ordinary nonlinear differential equations we introduce the self-similarity transformations are given by:

$$\left.\begin{aligned} &v_e = x\Omega_2 \sin\gamma(1-St\Omega\sin\gamma)^{-1}, \eta = v^{-1/2}(\Omega\sin\gamma)^{1/2}(1-St\Omega\sin\gamma)^{-1/2}z,\\ &u = -2^{-1}x\Omega\sin\gamma(1-St\Omega\sin\gamma)^{-1} f'(\eta), v = x\Omega\sin\gamma(1-St\Omega\sin\gamma)^{-1} g(\eta),\\ &w = (v\Omega\sin\gamma)^{1/2}(1-St\Omega\sin\gamma)^{-1/2} f(\eta),\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \end{aligned}\right\} \tag{7}$$

Here S is the unsteady parameter, Ω_1 and Ω_2 are the angular velocities of the cone and free stream fluid respectively, $\Omega = \Omega_1 + \Omega_2$ is the composite angular velocity, Here in equation (5), u and v are automatically satisfying the continuity equation. The nanofluid constants Buongiorno [30] are given by:

$$\left.\begin{aligned} &\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\\ &\frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}, \sigma_{nf} = \sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right], \sigma = \frac{\sigma_s}{\sigma_f} \end{aligned}\right\} \tag{8}$$

By using equation (5) and (6), the equations (1) to (4) are transformed as follows:

$$\begin{gathered} \left(\left(\frac{1}{1+E\theta}\right)\frac{1}{(1-\phi)^{2.5}}\right) f''' + \left(ff'' - \frac{f'^2}{2} - 2g^2 - Sf' - \frac{1}{2}S\eta f''\right)\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right) + 2(1-\alpha_1)^2 + 3\frac{(n-1)}{2}Wef''' f''^2 - \\ 2Mf' - \frac{1}{(1+E\theta)^2}\theta'^2 f'' + \left((1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)2\sin(\gamma)\lambda\theta = 0, \end{gathered} \tag{9}$$

$$\left(\left(\frac{1}{1+E\theta}\right)\frac{1}{(1-\phi)^{2.5}}\right) g'' - \left(S(g+\frac{\eta}{2}g' + fg' - f'g)\right)\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right) + S - S\alpha_1 - 2Mf' - \frac{1}{(1+E\theta)^2}\theta'^2 g' = 0, \tag{10}$$

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$$\frac{k_{nf}}{k_f}\theta''+Q_H\theta\Pr-\Pr\left((1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(S(2\theta+\frac{\eta}{2}\theta'-\frac{f'\theta}{2}+f\theta')\right)+\frac{1}{(1-\phi)^{2.5}}\frac{1}{(1+E\theta)}Ecf''^2=0, \quad (11)$$

The transformed boundary conditions are:

$$f(0)=0=f'(0),\ g(0)=\alpha_1,\theta(0)=1,\quad f'(\infty)=0, g(\infty)=1-\alpha_1,\theta(\infty)=0, \quad (12)$$

$$\left.\begin{aligned}&Gr=\frac{g\beta_T\cos\gamma(T_w-T_\infty)L^3}{{v_f}^2},\mathrm{Re}_L=\frac{\Omega L^2\sin\gamma}{v},\alpha_1=\frac{\Omega_1}{\Omega},\Pr=k_f/(\mu c_p)_f,Q_H=Q_0(1-st\sin\gamma)/\Omega\sin\gamma,\\&We=\frac{\Gamma^2x^2}{\upsilon}\left(\frac{\Omega\sin\gamma}{(1-st\sin\gamma)}\right)^3,\lambda=\frac{Gr}{{\mathrm{Re}_L}^2},M=\frac{\sigma {B_0}^2}{\rho}(1-st\sin\gamma)/\Omega\sin\gamma\end{aligned}\right\}$$

Where α_1 is ratio of the angular velocity to composite velocity, Gr is the grashof number, Re_L is the Reynolds number, λ is the buoyancy force parameter, ρ_f, ρ_s are the densities of the fluid and solid particles respectively, $T_w-T_\infty>0$ is for heated cone and $T_w-T_\infty<0$ is for cooled cone.
Where $\Pr = k_f/(\mu c_p)_f$ is the Prandtl number, S is the unsteadiness parameter and $Q_H = Q_0(1\text{-}st\sin\gamma)/\Omega\sin\gamma$ is the heat source/sink parameters.
For physical quantities of interest, the friction factor coefficient and the rate of heat transfer are given by:

$$C_{fx}Gr^{1/4}=\frac{1}{(1+E\theta)(1-\phi)^{2.5}}f''(0), \quad (13)$$

$$C_{fy}Gr^{1/4}=\frac{1}{(1+E\theta)(1-\phi)^{2.5}}g'(0), \quad (14)$$

$$Gr^{-1/4}Nu_x=-\frac{k_{nf}}{k_f}\theta'(0). \quad (15)$$

III. Method of Solution

The nonlinear differential equations (9), (10) and (11) with the boundary conditions (12) are solved numerically using Runge-Kutta and Newton's methods. Initially, the set of nonlinear ordinary differential equations converted to first order differential equations, by using the following procedure:

$$G'=y2, G''=y3, H'=y5, G=y1, H=y4, \theta=y6, \theta'=y7,$$

$$A=\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right), B=\left(1-\phi+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right),$$

$$C=\left(1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right), D=\left(1-\phi+\phi\left(\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\right), \quad (16)$$

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$$G''' = \left(1/\frac{1}{(1-\phi)^{2.5}(1+Ey_6)} + \frac{3(n-1)}{2}Wef''^2\right)\left\{\begin{array}{l} A\left(\begin{array}{l}1/2y_2^2 - y_1y_3 - 2y_4^2 - S(y_2+1/2\eta y_3) - \\ 2(y_4^2-(1-\alpha_1)^2)\end{array}\right) + \\ \frac{y_3y_7^2}{(1-\phi)^{2.5}(1+y_6E)^2} + 2\lambda\sin(\gamma)By_6 - 2My_2 \end{array}\right\} \quad (17)$$

$$H'' = (1-\phi)^{2.5}(1+Ey_6)\left\{A\left(y_4 + \frac{\eta}{2}y_5 + y_1y_5 - y_2y_4 - S + S\alpha_1\right) + \frac{y_5y_7^2}{(1-\phi)^{2.5}(1+y_6E)^2} - MCy_4\right\} \quad (18)$$

$$\theta'' = \left(\frac{k_f}{k_{nf}}\right)\left\{\Pr DS\left(2y_6 + \frac{\eta}{2}y_7 - \frac{y_2y_6}{2} + y_1y_7\right) - Q_H \Pr y_6 - \frac{1}{(1-\phi)^{2.5}}\frac{1}{(1+E\theta)}Ecy_3^2\right\} \quad (19)$$

With boundary conditions as:

$$y1 = y2 = 0, y_4 = \alpha, y_6 = 1, \text{ at } \eta \to 0$$
$$y_2 = 0, y_4 = 1-\alpha, y_6 = 0 \text{ at } \eta \to \infty \quad (20)$$

We guess the values of $y_3(0), y_5(0), y_7(0)$ which are not given at the initial conditions. The equations (17)-(19) are integrated by taking the help of Runge-Kutta method with the successive iterative step length is 0.01. For this we used ODE45 MATLAB solver to solve the first order nonlinear coupled differential equations. The correctness of the supposed values is checked by equating the calculated values y_2, y_4, y_6, y_8 at $\eta = \eta_{max}$ with their given values at $\eta = \eta_{max}$. If there is any difference exist the process is continued upto the required good values. Alternatively, we are using the Newton's Raphson method to get the accurately found the initial values of $y_3(0), y_5(0), y_7(0)$ and then integrate Eq. (17)-(19) by using the Runge-Kutta Newton's Raphson method. This process is repeated until the settlement between the designed value and the condition given at is within the specified degree of accuracy 10^{-5}. In order to validates the precision of the present solutions with Raju et al. [7], Anikumar and Roy [28] solutions. We found worthy agreement with their solutions.

IV. Results and Discussion

The nonlinearordinary differential equations (9)-(11) together with boundary conditions (12) are solved numerically by using Runge-Kutta based shooting technique. The numerical computations have been carried out for different values of involving non-dimensional governing parameters. Results describe the effect of various non-dimensional governing parameters on the friction factor coefficient and local Nusselt number for with viscous variation and without viscous variation cases. For numerical solutions we have chosen the non-dimensional parameter values as M=0.2, $\phi=0.1$, $Q_H=0.1$, $Ec=0.2$, $\alpha_1=0.2$, $\gamma=\Pi/4$, $We=0.5$, $n=2$, $\lambda=5$, $\eta=2$ these values are kept as fixed in entire study except the variations in the corresponding figures and tables.

Skin Friction Factor Coefficients Distribution

The variations of the non-dimensional governing parameters on friction factor coefficient for both the E=0 and E=2 cases were observed in Figs. 2-5. Fig. 2 displays the buoyancy force, volume fraction of nanoparticle and Eckert number on friction factor coefficient for the E=0 and E=2 cases. It is noted that the rising values of buoyancy and volume fraction of nanoparticles are improves friction factor coefficient ($Cf_xGr^{1/4}$) monotonically, whereas there is no change in with improving values of Eckert number. Generally, increasing values of buoyancy force generates high pressure forces in the flow. Due to this we saw improvement in friction factor coefficient. Similarly the rising value of volume fraction of nanoparticles encourages the size of the particles, which can lead to improve the friction factor coefficient ($Cf_xGr^{1/4}$) for the E=0 and E=2 cases. Fig. 3 shows the effects of W_e, γ and Q_H on friction factor coefficient

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for the *E*=0and *E*=2 cases. It is noted that the Weissenberg number and half angle parameters are improves the friction factor coefficients ($Cf_xGr^{1/4}$), but heat source/sink parameter depreciates the friction factor coefficient. It is found that the friction factor coefficient ($Cf_xGr^{1/4}$) is more in E=0 case when compared with E=2 case. From these results we can able to say that viscous variation parameter modulating the friction factor coefficients depending on the industrial needs. The effect of λ, *Ec and We* on friction factor coefficient ($Cf_yGr^{1/4}$) is plotted in Fig. 4. The buoyancy parameter and weissenberg number are increases the friction factor coefficient ($Cf_yGr^{1/4}$). But, the Eckert number minimizes the friction factor coefficient for the *E*=0and *E*=2cases. The variations of ϕ, γ and Q_H on ($Cf_yGr^{1/4}$) is shown in Fig. 5. It is clear that due to the dominance of rotation we have seen depreciation in azimuthal friction factor coefficient with rising values of ϕ .

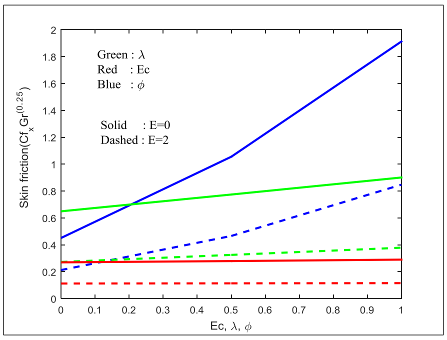

Fig. 2. The variations of λ, *Ec* and ϕ.

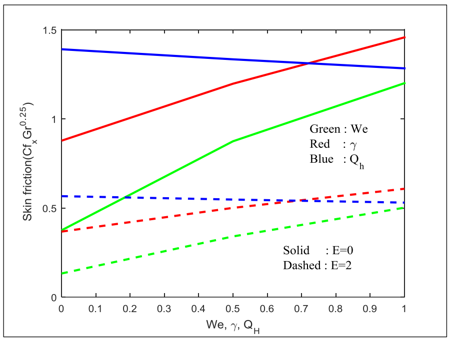

Fig. 3. The variations of *We*,γ and Q_H.

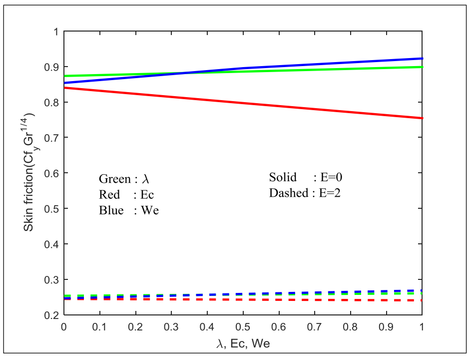

Fig. 4. The Variations of λ, *Ec* and *We*.

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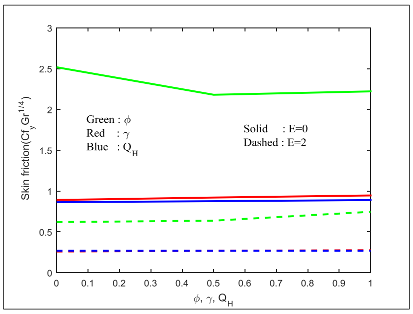

Fig. 5. The variations of ϕ, γ and Q_H.

Heat Transfer Rate Distribution

The variation of local Nusselt number on various non-dimensional governing parameters for E=0 and E=2cases was shown in Figs. 6 and 7. Fig. 6 displays the effects of λ, ϕ *and Ec and We* on local Nusselt number for the E=0 and E=2 cases. With improving values of volume fraction of nanoparticle encourage the heat transfer rate profiles, whereas depreciates the Nusselt number increasing values of Eckert number. Physically, by mixing of nanoparticles into the base fluid improves the interaction between the particles, this can leads to improves the heat transfer rate. The variations of W_e, γ and Q_H on the heat transfer rate profiles is plotted in Fig. 7. It is noted that the heat transfer rate is rises with increasing values of heat source/sink parameter, but minimizes with half angle and weissenberg number. It is interesting to mention that the heat transfer rate is higher in E=0 case compared with E=2 case, this can help to highlight that the viscous variation parameter control the heat transfer rate. Generally, higher viscosity variations lead to improve the boundary layer due to this we seen depreciation in the heat transfer rate.

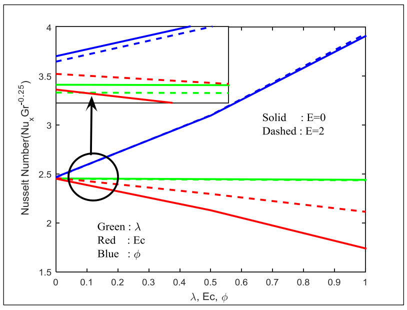

Fig. 6. The variations of λ, *Ec* and ϕ.

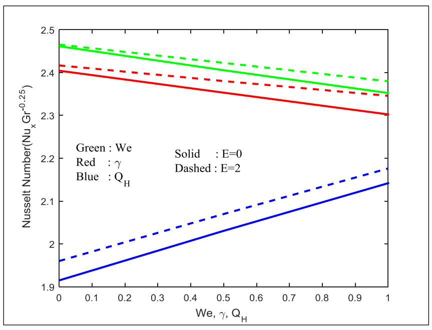

Fig. 7. The variations of *We*, γ and Q_H.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

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Special Cases

Table 1 depicts the thermo physical properties of nanofluids and base fluid. Table 2 displays the comparison of the new solutions with the existed literature under some special restricted cases. We found better coincidence with the existed and available literature. This authenticates the current results and the numerical procedure we used in the current study. Table 3 demonstrates the variations of friction factor and local Nusselt number with varied values of Eckert number and magnetic field parameter with S=0, S=0.8 and ϕ=0, ϕ=0.1 cases for two type of Ti6Al4V+water and Ti6Al4V+kerosene. The titanium alloy has higher heat transfer rate in unsteady case compared with steady case; however the friction factor coefficient is higher in steady case as compared with unsteady case. Similarly, the kerosene based titanium alloy has higher friction factor coefficients as compared with water based titanium alloy, whereas water based titanium alloy has greater heat transfer rate comparing kerosene based titanium alloy. From this we can say depending on the societal needs we can use steady, unsteady and base, nanofluids.

Thermo Physical Properties	**Water**	**Kerosene**	**Ti6A14V**
$\rho(K\gamma/\mu^3)$	997.1	783	4420
C_p(J/KgK)	4179	2090	0.56
k(W/m K)	0.613	0.15	7.2
$\beta\times10^{-5}(1/K)$	21	9.6	5.8

Table 1. Thermo physical properties of water and Ti6A14V.

	Pr=0.7			**Pr=1**			**Pr=10**		
λ	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$
↓	**Anilkumar and Roy [28]**	**Raju et al. [7]**	**present**	**Anilkumar and Roy [28]**	**Raju et al. [7]**	**Present**	**Anilkumar and Roy [28]**	**Raju et al. [7]**	**Present**
0	0.4305	0.476975	0.4305	0.557294	0.51808	0.557	1.4042	1.6359	1.4952
1	0.6127	0.60041	0.6004	0.721982	0.7005	0.7005	1.5885	2.0911	2.0912
10	1.0175	0.995235	1.0175	1.170983	1.1494	1.1709	2.3528	2.7734	2.773

Table 2. The validation of the current results with already existed literature some limited case M= We=Ec=E= Q_H =ϕ=0, n=1with different values of prandtl number.

Velocity and Temperature Distribution

Fig. 8 displays the variations of volume fraction of titanium alloy nanoparticles on temperature field. This proves physically behavior ofϕ. The increasing values of ϕ improve the particle to particle distribution; this can helps to encourages the temperature field. The similar performance was seen in Ec. The increasing value of Ec advances the distribution of particles. This is displayed in Fig. 9. The effects of γ on velocity and temperature fields are plotted in Figs. 10 and 11. It is noted that both the velocity and temperature fields are encouraged with higher values of half angle parameter. This may happen due to the rising values of half angle parameter changes the channel position vertical to horizontal, which helps to boost up the velocity and temperature fields.

The variation in temperature fields against different values of heat source/sink and weissennberg parameter (Q_H and W_e) is plotted in Figs. 12 and 13. It is clear that the temperature field is enhanced with improving values of Q_H and W_e. Generally, the rising values of Q_H and we generate a high pressure forces in the flow. Because of this cause we seen enhancement in temperature field. Figs. 14 and 15 show the influence of λ on velocity and temperature fields. Due to the domination of rotation and unsteadiness in the flow, the velocity and temperature fields are improved with increasing values ofλ.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

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Nanofluid type	*Ec*	*M*	*S*=0		*S*=0.8	
			ϕ=0	ϕ=0.1	ϕ=0	ϕ=0.1
Skin friction coefficient						
	0		0.73064	1.265622	0.77362	1.182953
Ti6A14V+water	1		0.77854	1.432778	0.81319	1.282107
	10		0.91015	0.940187	0.72803	1245.541
		0	0.90392	1.481182	0.8648	1.294856
		1	0.14043	0.653	0.46684	0.886539
		10	0.38047	0.827561	0.49033	0.753029
	0		0.87152	1.576737	0.85323	1.394211
Ti6A14V+kerosene	1		0.92554	1.769537	0.89405	1.509979
	10		0.99298	-358.272	0.71646	-4.004604
		0	1.06107	1.80556	0.95513	1.523035
		1	0.22962	0.922032	0.52251	1.074321
		10	0.67219	1.023025	0.51997	0.837356
Local Nusselt number						
	0		1.87083	2.19028	1.91913	2.434341
	1		1.55535	1.292239	1.70971	1.965837
Ti6A14V+water	10		-4.94976	-9.07624	1.19199	11992.9
		0	1.76844	1.975721	1.86525	2.329107
		1	1.82053	2.135257	1.92383	2.415891
		10	1.88394	2.149879	1.92015	2.434527
	0		1.41425	1.710884	1.67329	2.19532
	1		1.12158	0.811304	1.48141	1.721079
Ti6A14V+kerosene	10		-4.83458	2749.655	2.65637	23.51849
		0	1.32475	1.509045	1.62094	2.086256
		1	1.37705	1.652965	1.68413	2.181869
		10	1.39973	1.670359	1.68246	2.215139

Table 3. Variations of friction factor and nusselt number with various values of viscous dissipation parameter and magnetic field parameter for the steady, unsteady cases and base fluid, nanofluid cases.

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International Journal of Innovative Research in Science, Engineering and Technology

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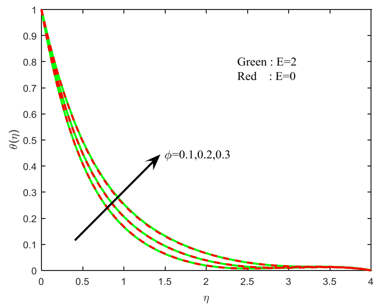

Fig. 8. The variations of ϕ temperature profiles.

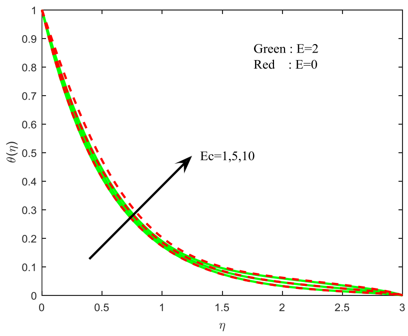

Fig. 9. The variations of *Ec* on temperature field.

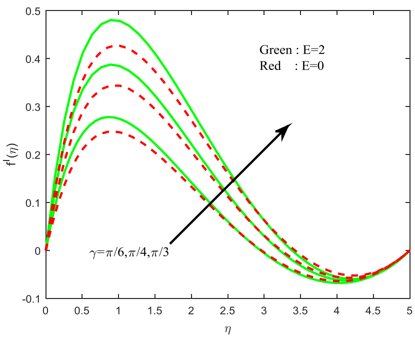

Fig. 10. The variations of γ velocity field.

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ISSN (Print): 2347-6710

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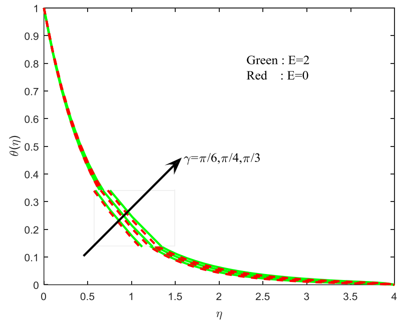

Fig. 11. The variations of γ on temperature field.

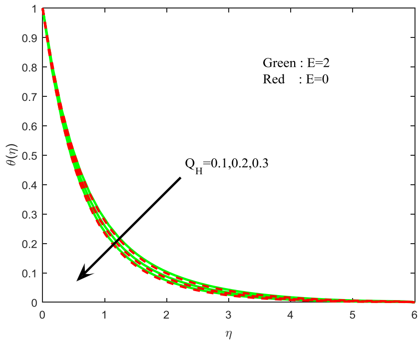

Fig. 12. The variations of Q_H on temperature field.

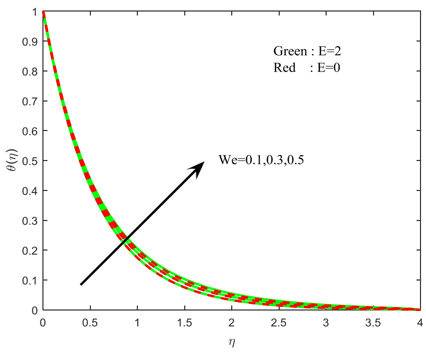

Fig. 13. The variations of *We* on temperature field.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Issue 11, November 2017

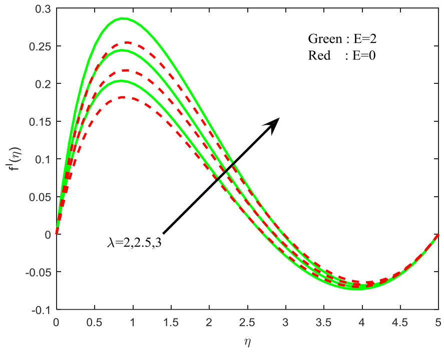

Fig. 14. The variations of λ on velocity field.

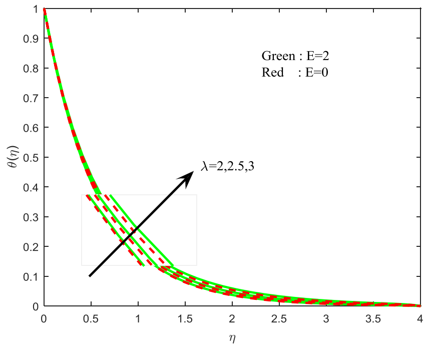

Fig. 15. The variations of λ on temperature field.

V. CONCLUSION

Quantifying the viscosity is a significant effect in state of the flow property. It plays an important role in the quality controlling in various industrial applications such as food, petrochemical, chemical, pharmaceutical, coatings, paints, cosmetics and auto motives. Keeping view into this we analyzed the heat source/sink and temperature dependent viscosity effect on titanium alloy nanofluid due to heated cone in the presence of viscous dissipation. Arising set of governing differential equations are solved numerically using Runge-Kutta and Newton's method. The effects of various dimensionless parameters on the friction coefficient and the local Nusselt number are deliberated with the help of graphs.

The conclusions are as follows:

- The viscous dissipation parameter have tendency to minimizing the rate of heat transfer and improves the temperature field.
- The heat source/sinkparameter and volume fraction of nanoparticleare boost up the heat transfer rate.
- The water based titanium alloy has lesser friction factor compared with kerosene based titanium alloy.
- The water based titanium alloy has higher local Nusselt number compared with kerosene based titanium alloy.
- The dispersion nanoparticle interaction is very high in unsteady case with base and nanofluid cases.
- The magneticfield parameter have higher heat transfer rate compared with Eckert number.

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