

# Uncertainty Analysis of Water Distribution Networks Using linked EPANET-Vertex Method

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**ABSTRACT** The analysis of hydraulic behaviour of the water distribution network (WDN) is forefront part of the planning and augmentation of any water supply projects. The desired output parameters such as pipe discharges, hydraulic gradient level (HGL) of nodes, nodal concentrations etc., are normally crisp values assuming crisp input parameters. There are many uncertainties in nodal demands, roughness, length, diameters of pipes, valve operations, water levels in reservoirs, head-discharge characteristics of pumps etc. So the results obtained by traditional method keeping the input parameters as crisp may not give satisfactory performance in practice. Hence taking care of this fact, pipe roughness has been considered as fuzzy parameter in this study. The vertex method has been used to obtain the ordered pair values of pipe roughness at each  $\alpha$ -cut level. The hydraulic simulations are done by using EPANET 2 in MATLAB environment. The maximum and minimum values of pipe discharges, nodal HGLs are obtained in single simulation run for each  $\alpha$ -cut level. The obtained results are compared with-past studies and it is found that current method is effective to analyse the uncertainty problem. This study would help to the decision maker to identify the condition of pipes and consequently the corrective action needed.

**KEYWORDS:**  $\alpha$ -cut, EPANET 2, Vertex method, uncertainty analysis, water distribution network (WDN).

## I. INTRODUCTION

The analysis of water distribution network (WDN) determines the estimation of pipe discharges, hydraulic gradient levels (HGL), nodal concentrations etc., to fulfil the requirements of the population. In the conventional method of analysis, unique value of the pipe discharges and hydraulic heads are obtained. The results so obtained may not give satisfactory performance due to many uncertainties in nodal demands, pipe roughness, lengths, diameters of pipes, water levels in reservoirs, head- discharge characteristics of pumps etc., Due to the complex behaviour of WDN, the reliable measurements is not normally possible in each and every node and links of the network. While augmenting the existing network, the length and diameters of pipe are assumed to be consistent even if the network has been used for many years. The diameter and roughness coefficient of the pipe may vary due to scale formation on inside surface of pipes and aging process; length of pipe also vary due to introduction of joints or removal of a pipe line during the normal course of operation. However, the same have not been considered in the conventional method of analysis and hence do not give the expected result. Thus in the present study, pipe roughness is taken as fuzzy parameter, while all other parameters are taken as crisp. Only roughness is taken as uncertain in vertex method of fuzzy set theory in order to avoid the excessive computational requirements. The brief description of the EPANET 2, Fuzzy sets and Vertex methods are given below.

EPANET 2 [1] is a public domain software package that performs the extended period simulation (EPS) of hydraulic and water quality behaviour within the pressurized pipe networks which is based on the principle of gradient

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Vol. 3, Issue 12, December 2014

method [2]. The output of the analysis of WDN such as pipe discharges, nodal heads, nodal concentrations, etc., can be readily calculated with available input parameters such as pipe roughness coefficient, length, diameter of pipe etc.

The fuzzy set theory was introduced by Zadeh [3], it provides a tool for dealing with imprecision due to uncertainty and vagueness, which is essential to many engineering problems. It resembles human decision making with its ability to work from approximate data and imprecise solutions. Since its inception, it has been used to describe imprecision in ground water ([4], [5], [6]); and in the pipe network problems ([7], [8], [9], [10]). Further, Zadeh [3] was developed an extension principle which is generally used in fuzzy arithmetic to deal with uncertainties. Though it is easy to implement ([11], [12]); computationally expensive for complex problems ([11], [5]); because a large number of function evaluations are required.

Dong and Shah [13] suggested the vertex method is an approximate method used in fuzzy arithmetic based on  $\alpha$  level cut concept and standard interval analysis which is alternative to the extension principle. In this method, the membership function is discretized, rather than discretizing the variable domain. The discretization of the membership domain is accomplished by dividing the membership domain into a series of equally spaced cuts, called  $\alpha$ -cuts;  $\alpha$  represents the possibility. For each  $\alpha$ -cut, the maximum and minimum value of the fuzzy variable is selected. Further, Guan and Aral [14] compared the extension principle with vertex method and ensure that the vertex method is a competent method to obtain the membership function of the unknown dependent parameter. Also it reduces considerable computer time to evaluate the objective function.

Revelli and Ridolfi [7] analysed a pipe network by fuzzy method through optimization based methodology considering uncertain parameters such as pipe roughness coefficient; nodal demands; reservoir head levels. They need two fuzzy optimization algorithms to find the minimum and maximum pipe discharges for each  $\alpha$ -cut and further to obtain the membership function of the nodal HGL values at all nodes of a network. Xu and Goulter [15] and Bhavne and Gupta [8] optimized the water distribution networks with fuzzy demands using linear programming. Gupta and Bhavne [9] reanalysed the network of Revelli and Ridolfi [7] by direct method using impact table to obtain the dependent parameters of pipe discharge and nodal heads. They concluded that their proposed methodology requires less computational effort and time as compared to that of Revelli and Ridolfi [7].

Shibu and Reddy [10] developed Fuzzy-Cross Entropy method wherein fuzzy set theory was used to give uncertain input parameters; and cross entropy method is an evolutionary iterative technique used to optimize the membership functions of uncertain output parameters such as pipe discharges and nodal heads for each  $\alpha$ -cut level. They concluded that method proposed by Bhavne and Gupta [16] are difficult as it consume considerable time even for small networks.

In this study, the Hazen-Williams coefficient of pipe roughness is taken as fuzzy and simulations of network is done using EPANET 2 programmer's toolkit [17] in the MATLAB environment. The uncertain dependent parameters of each pipe discharge and nodal HGL of all nodes including maximum and minimum value of each pipe discharge and nodal HGL for each  $\alpha$ -cut level is obtained in single simulation run. Hence it is effective as compared to past studies ([7], [9]).

## II. FUZZY SETS AND VERTEX METHOD

Fuzzy set is a set of objects without clear boundaries or one without well-defined characteristics [18]. The membership function establishes how much the element "belongs" to the set and is included in the interval [0, 1]; the convention being that the closer it is to 1, the more the element belongs to the set and vice versa. Fuzzy sets, membership functions, and  $\alpha$  level cuts are defined mathematically by Ross [11]. Thus, if  $X$  class of objects denoted by  $x$ , then a fuzzy set in  $X$  is a set of ordered pairs  $A = \{x, \mu_A(x) | x \in X\}$ , where  $\mu_A(x)$  represents the membership function for the fuzzy set  $A$ . The value representing the degree for an element  $x$  that belongs to the fuzzy set  $A$  is defined as the degree of membership for  $x$ , which is evaluated by the membership function. The  $\alpha$  level cut of fuzzy set  $A$  is the set of those elements which have a membership value greater than or equal to  $\alpha$  [11]. When  $\alpha = 0$ , the corresponding interval is called the "support" of the fuzzy member with extreme boundaries of the "minimum" and "maximum" values

**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

**Vol. 3, Issue 12, December 2014**

respectively. Similarly, with a triangular function when  $\alpha = 1$ , the interval comes down to a crisp value, or the “most likely value” [12].

Two most common types of membership function for fuzzy numbers are: (1) Triangular; and (2) Trapezoidal. But, former one is preferred by some researchers ([19], [4], [20]) because of its simple shape. The triangular membership function is a special case of trapezoidal membership function, because in this case, at  $\alpha = 1$ , there is a single point rather than a flat line as in the trapezoidal function. It is observed from the study of [14], the width of the support base of the membership function is vital factor rather than the shape of the membership function. Because the width of the support base may reflect the precision of the field information on the parameter obtained. However, in this study triangular membership function has been used.

R. Moore [21] has given a solution in interval computation, and found the basic problem as given a function  $f(x_1, \dots, x_n)$  and  $n$  intervals  $[x_i^-, x_i^+]$  find the interval range of the variable  $y=f(X)$  such that  $x \in x_i [x_i^-, x_i^+]$ . Yang et al. [22] defined the goal of the interval computation is to find the minimum and the maximum of the function when the different possible values of the variables  $x_i$  range in their intervals  $[x_i^-, x_i^+]$ . Some methods are based on finding a finite set of points (called configurations or poles) on which this minimum and maximum is attained. For each  $\alpha$ -cut, the minimum and maximum value of the fuzzy variable is selected as shown in Fig. 1. Prasad and Mathur [7] compared the vertex method with the ANN-GA method in their study of contaminant transport in groundwater flow. The formulation of vertex method [20] is as follows. In the vertex method, fuzzy numbers  $A_1, A_2, A_3, \dots, A_N$  are defined on the real line  $L$  and the elements of  $A_i$  are denoted by  $x_i, i = 1, 2, \dots, N$ . If  $x_1, x_2, \dots, x_N$  are related to real number  $y$  by the mapping  $y=f(x_1, x_2, \dots, x_N)$ , the solution of fuzzy number  $B$  in  $y$  that would correspond to fuzzy numbers  $A_1$  in  $x_1, A_2$  in  $x_2, \dots, A_N$  in  $x_N$  can be obtained in the following procedure:

1. The range of membership  $[0, 1]$  is discretized into a finite number of values, called  $\alpha_1, \alpha_2, \dots, \alpha_M$ . The refinement in discretization depends on the degree of accuracy desired.
2. For each membership value  $\alpha_j$ , the corresponding intervals for  $A_i$  in  $x_i, i = 1, 2, \dots, N$  are determined. These are the supports of the  $\alpha_j$  cuts of  $A_1, A_2, A_3, \dots, A_N$ . The end points of these intervals are represented by  $[a_1, b_1], [a_2, b_2], \dots, [a_N, b_N]$ . Also  $a_i$  may be equal to  $b_i$  in which case the interval would reduce to a point.
3. Taking one end point from each of the intervals, the end points can be combined into  $2^N$  distinct permutations, giving  $2^N$  combinations for the vector  $(x_1, x_2, \dots, x_N)$ . Thus for 2 pipe network having the uncertain parameters as  $(a_1, b_1), (a_2, b_2)$  can be combined in ordered pair as follows:  $[(a_1, a_2), (a_1, b_2), (b_1, a_2), (b_1, b_2)]$ . Similarly, for 3 pipe network having the uncertain parameter as  $(a_1, b_1), (a_2, b_2), (a_3, b_3)$  can be combined as  $[(a_1, a_2), (a_1, b_2), (b_1, a_2), (b_1, b_2)] [a_3, b_3]$  can be combined as  $[(a_3, a_1, a_2), (a_3, a_1, b_2), (a_3, b_1, a_2), (a_3, b_1, b_2), (b_3, a_1, a_2), (b_3, a_1, b_2), (b_3, b_1, a_2), (b_3, b_1, b_2)]$ .
4. The function  $f(x_1, x_2, \dots, x_N)$  is evaluated for each of the  $2^N$  combinations to obtain  $2^N$  for  $y$ , denoted as  $y_1, y_2, \dots, y_N$ . The desired interval for  $y$  is given by  $[\wedge_k y_k, \vee_k y_k]$ , which define support of the  $\alpha_j$  - cut of  $B$ .
5. The process is repeated for other  $\alpha$  - cuts to obtain additional  $\alpha$  - cuts of  $B$  and the solution to the fuzzy number  $B$ .

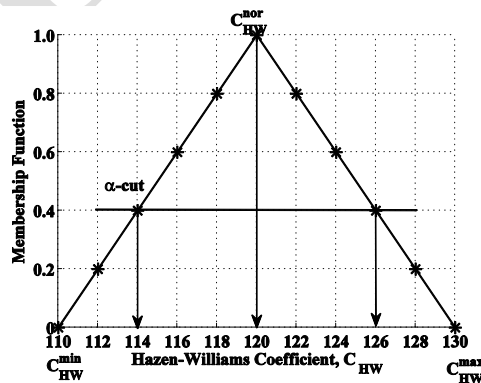


Fig. 1 Triangular membership function for Hazen-Williams coefficient

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Engineering and Technology**  
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**Vol. 3, Issue 12, December 2014**

**III. METHODOLOGY**

**Hazen-Williams formula for head loss**

The head loss due to pipe friction,  $h_f$  in metres (Bhave and Gupta 2006) is given by

$$h_f = \frac{xLQ^{1.852}}{C_{HW}^{1.852} D^{4.87}} \tag{1}$$

Where  $L$ =length of the pipe in metres;  $D$ = diameter of the pipe in metres;  $Q$ = pipe discharge in cubic metres per second;  $C_{HW}$  =Hazen-Williams coefficient and  $x$  = constant in Hazen-Williams formula which depends on the units used in discharge and pipe diameter and with exponent of  $Q$  and  $C_{HW}$  is equal to 1.85 or 1.852. Here in the value of  $x$  is 10.679.

**Fuzzy analysis through vertex method**

Fig. 1 shows Hazen-Williams coefficient,  $C_{HW}$  as a fuzzy parameter in a triangular membership function, having extreme boundary values  $C_{HW}^{min}$  and  $C_{HW}^{max}$  for each  $\alpha$ - cuts (i.e., 0.0, 0.2, 0.4, 0.6, 0.8). Where,  $C_{HW}^{min}$ ,  $C_{HW}^{max}$  is the minimum and maximum value of  $C_{HW}$ . For  $\alpha = 1$ , it represents  $C_{HW}^{nor}$ , a single point value known as crisp value, i.e., most likely or precise or normal value of  $C_{HW}$ . Hence for the triangular fuzzy number  $A$  ( $C_{HW}^{min}$ ,  $C_{HW}^{nor}$ ,  $C_{HW}^{max}$ ), the membership function  $\mu_A$  is given by

$$\mu_A(C_{HW}) = 0, C_{HW} \leq C_{HW}^{min} \tag{2}$$

$$\mu_A(C_{HW}) = \frac{C_{HW} - C_{HW}^{min}}{C_{HW}^{nor} - C_{HW}^{min}}, C_{HW}^{min} \leq C_{HW} \leq C_{HW}^{nor} \tag{3}$$

$$\mu_A(C_{HW}) = \frac{C_{HW} - C_{HW}^{max}}{C_{HW}^{nor} - C_{HW}^{max}}, C_{HW}^{nor} \leq C_{HW} \leq C_{HW}^{max} \tag{4}$$

$$\mu_A(C_{HW}) = 0, C_{HW} \geq C_{HW}^{max} \tag{5}$$

Here  $\alpha$  - cut is represented by  $\alpha^*$ . If  $\alpha^* = 0$ ,  $C_{HW}$  lies between 110 and 130; for  $\alpha^* = 0.4$ ,  $C_{HW}$  lies between 114 and 126;  $\alpha^* = 1$ , normal value of  $C_{HW}$  is 120. The width of the interval represents the imprecision in the input parameters.

In the vertex method, for  $N$  piped network, the order pair of the roughness coefficient of each membership would be  $2^N$ . These roughness coefficients are formulated in ordered pair and output parameters like pipe discharges and nodal heads are obtained using EPANET 2. The output parameters can be grouped for pipe discharges ( $Q_x^{min}$ ,  $Q_x^{max}$ ) and nodal heads ( $H_j^{min}$ ,  $H_j^{max}$ ) for each membership function. Then the membership functions of discharges and nodal heads are plotted at each  $\alpha$  level cut.

**Proposed method**

The detail of the proposed method in the present study is explained in the following steps. The uncertain input parameters of pipe roughness are evaluated in ordered pair according to the vertex method for each  $\alpha$ -cut interval. The hydraulic simulation of the network is run by using EPANET 2 tool kit in the MATLAB environment. The coding of the programme is developed in the MATLAB editor file.

1. Select a network and prepare an input file according to the EPANET 2 programmer's toolkit.
2. Divide the six different  $\alpha$ -cut levels (0, 0.2, 0.4, 0.6, 0.8, and 1) and define the boundary values for each  $\alpha$ -cut interval.
3. Obtain the ordered pair values of uncertain input parameters i.e., pipe roughness coefficient for each  $\alpha$ -cut interval such as  $\alpha^* = 0; 0.2; 0.4; 0.6; 0.8$ . To reduce the computational time for  $\alpha^* = 1$ , crisp value may be used instead of using  $2^N$  combinations of ordered pair values.
4. The obtained roughness coefficient is assigned to each pipe of the network in  $2^N$  times.
5. Run the hydraulic simulation of the network using EPANET 2 tool kit in the MATLAB environment.

**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 12, December 2014

6. Acquire the maximum and minimum values of desired uncertain output parameters like pipe discharges and nodal hydraulic gradient levels (HGL) of each node for each  $\alpha$ -cut level.
7. Plot the above get results of each node and pipe of network for each membership function.  
A detailed MATLAB code has been developed to execute the methodology based on the flow chart as shown in Fig 2.

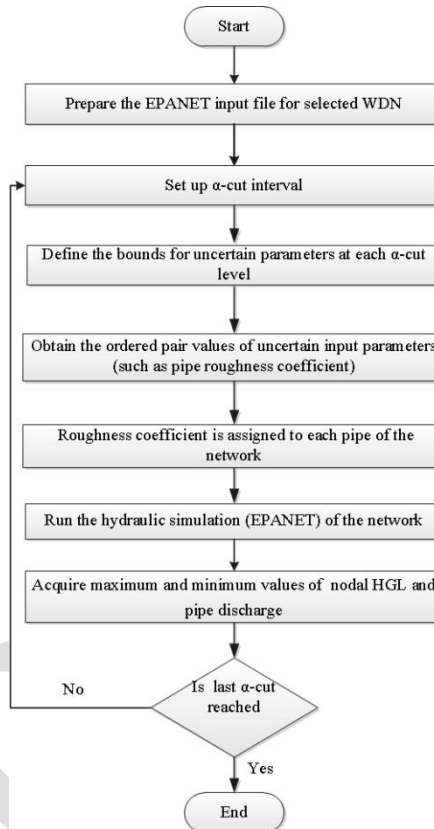


Fig. 2 Flow chart of proposed EPANET linked Vertex method

**IV. APPLICATIONS OF THE PROPOSED METHOD**

**Case study 1 (5 Pipe Network)**

A two loop with five pipe network ([7], [9], [10]) as shown in Fig. 3 has been analysed to explain the proposed methodology. Revelli and Ridolfi [7] used Strickler coefficient of pipe roughness as fuzzy membership function from 55 to 65 with crisp value of 60 where as Bhawe and Gupta [16] Shibu and Reddy [10] have considering Hazen-Williams coefficient of pipe roughness as fuzzy membership function from 110 to 130 with crisp value of 120.

The network has one source node, three demand nodes and five pipes as shown in Fig. 3. Node 1 is a source node with fixed pressure head 100 m and nodes 2, 3, and 4 are demand nodes with demands of 0.150, 0.300 and 0.230 m<sup>3</sup>/s, respectively. The pipe length in metre and diameter in millimetre for different pipes are given in parentheses as pipe 1 (1200, 500); pipe 2 (1100, 500); pipe 3 (1500, 500); pipe 4 (900, 350); and pipe 5 (1000, 350) respectively.

**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

**Vol. 3, Issue 12, December 2014**

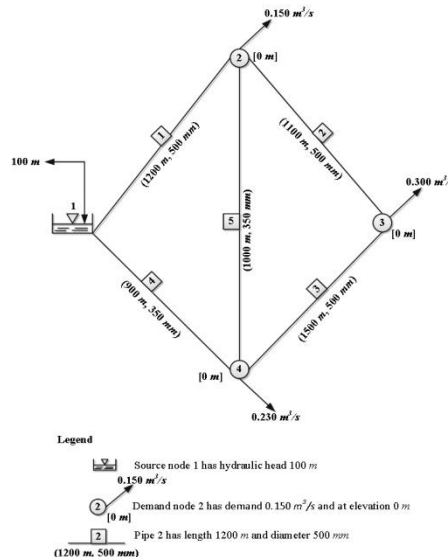


Fig. 3 A two loop with five pipe network

In this analysis, the source pressure head and nodal demands are precise, but uncertainty exists in Hazen-Williams coefficient,  $C_{HW}$  of all pipes. The most likely value of  $C_{HW}$  for all pipes is 120 with minimum and maximum values of 110 and 130 respectively (Fig. 1). In this network, there are 5 pipes with one uncertain parameter (i.e., Hazen-Williams coefficient,  $C_{HW}$ ) for which the ordered pair becomes  $2^5 = 32$  for each  $\alpha$ -cut. Hence the EPANET 2 model has been executed total 161 times including  $\alpha^* = 1.0$  and the minimum and maximum values of pipe discharges and nodal HGL are obtained. In the present analysis, it takes about 2 seconds using laptop having system configuration of Intel(R) Core(TM) i5-2450M CPU @ 2.50GHz processor with 4.00 GB RAM for all minimum and maximum values of pipe discharges and nodal HGLs of each  $\alpha$ -cut in single simulation run except  $\alpha^* = 1.0$ .

In the analysis of pipe network, taking all independent fuzzy parameters at their normal values ( $\alpha^* = 1$ ) of pipe roughness for all pipes i.e.,  $C_{HW1} = C_{HW2} = C_{HW3} = C_{HW4} = C_{HW5} = 120$ , the discharge value of pipe (1) is  $Q_1 = 0.452316 \text{ m}^3/\text{s}$ .

The maximum value of pipe discharge;  $Q_1^{max} = 0.475832 \text{ m}^3/\text{s}$  is obtained when  $C_{HW1} = C_{HW2} = 130$ ;  $C_{HW3} = C_{HW4} = 110$ ;  $C_{HW5} = 130$ . The minimum value of pipe discharge  $Q_1^{min} = 0.427614 \text{ m}^3/\text{s}$  is obtained as when  $C_{HW1} = C_{HW2} = 110$ ;  $C_{HW3} = C_{HW4} = 130$ ;  $C_{HW5} = 110$  in the membership function for  $\alpha^* = 0.0$ .

The maximum value of pipe discharge  $Q_1^{max} = 0.471219 \text{ m}^3/\text{s}$  is obtained when  $C_{HW1} = C_{HW2} = 128$ ;  $C_{HW3} = C_{HW4} = 112$ ;  $C_{HW5} = 128$ . The minimum value of pipe discharge  $Q_1^{min} = 0.432655 \text{ m}^3/\text{s}$  is obtained as when  $C_{HW1} = C_{HW2} = 112$ ;  $C_{HW3} = C_{HW4} = 128$ ;  $C_{HW5} = 112$  in the membership function for  $\alpha^* = 0.2$ .

For,  $\alpha^* = 0$  the ordered pairs of  $C_{HW}$  are obtained using vertex method (Dong and Shah 1987) is given below.

[110 110 110 110 110];	[130 110 110 110 110];
[110 110 110 110 130];	[130 110 110 110 130];
[110 110 110 130 110];	[130 110 110 130 110];
[110 110 110 130 130];	[130 110 110 130 130];
[110 110 130 110 110];	[130 110 130 110 110];
[110 110 130 110 130];	[130 110 130 110 130];
[110 110 130 130 110];	[130 110 130 130 110];
[110 110 130 130 130];	[130 110 130 130 130];
[110 130 110 110 110];	[130 130 110 110 110];
[110 130 110 110 130];	[130 130 110 110 130];

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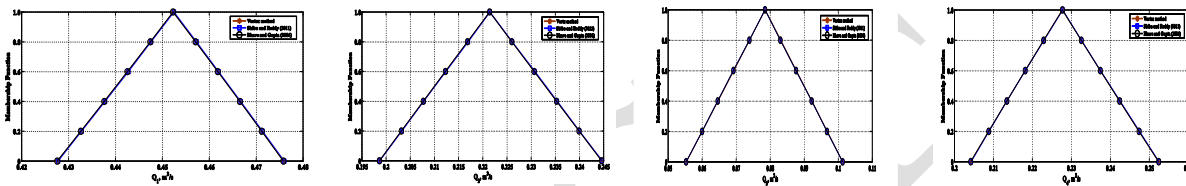
**Vol. 3, Issue 12, December 2014**

[110 130 110 130 110];	[130 130 110 130 110];
[110 130 110 130 130];	[130 130 110 130 130];
[110 130 130 110 110];	[130 130 130 110 110];
[110 130 130 110 130];	[130 130 130 110 130];
[110 130 130 130 110];	[130 130 130 130 110];
[110 130 130 130 130];	[130 130 130 130 130];

The results obtained by proposed vertex method and those of [10], [16] are identical as shown in Fig. 4.

The results of pipe discharges are found to vary about maximum 30% (same as [7]) for pipe 3 whereas the hydraulic heads at nodes vary about 2.7 m for nodal head 3 differ from the crisp value when the uncertainty of about 8% (when  $\alpha^* = 0.0$ ) in Hazen-Williams coefficient of pipe roughness.

Refer from the Table 1, pipe discharges are not much varied but in nodal HGL at node 3 have difference of 20.13 m which is about 31% of variation with respect to [7].



Analyzed by	Roughness coefficient	Crisp value	Unit	Pipe discharge (m <sup>3</sup> /s)					Nodal HGL (m)		
				Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>
Revelli and Ridolfi (2002)	Strickler	60	m <sup>1/3</sup> /s	0.458	0.224	0.076	0.222	0.083	71.11	64.75	65.75
Present Study	Hazen-Williams	120	Unitless	0.452	0.221	0.079	0.228	0.081	87.84	84.88	85.47

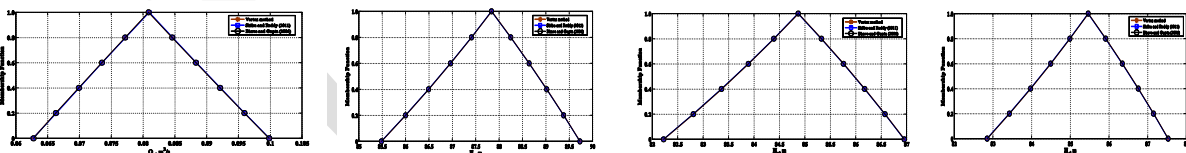


Fig. 4 Membership functions of unknown pipe discharges (m<sup>3</sup>/s) and nodal HGL (m) for the network of Fig. 3

Table 1. Comparison of dependent parameters of Revelli and Ridolfi [7] with present study for crisp value of roughness parameter in all pipes for Example 1

**Case study 2 (8 Pipe Network)**

A two loop with eight pipe network is shown in Fig. 5. This bench mark problem is adopted from the literature which has been used by many researchers [23], [24], [25] for evaluating the least-cost design by using various traditional and non- traditional optimization algorithm.

The network has one source node, six demand nodes and eight pipes as shown in Fig. 5. Node 1 is a source node with fixed pressure head 210 m and nodes 2 to 6 are the demand nodes with demands of 100, 100, 120, 270, 330

**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

**Vol. 3, Issue 12, December 2014**

and 200 m<sup>3</sup>/h, respectively. The lengths of all pipes are 1000 m and diameters for different pipes 1 to 8 are 457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4 mm respectively. These are the diameters obtained by several researchers for least cost of the network satisfying the minimum pressure requirements and assuming a constant value of C<sub>HW</sub> =130 for all pipes.

In this analysis, the source pressure head and nodal demands are precise, but uncertainty exists in Hazen-Williams coefficient, C<sub>HW</sub> of all pipes. The most likely value of C<sub>HW</sub> for all pipes is 130 with minimum and maximum values of 120 and 140 respectively. In this network, there are 8 pipes with one uncertain parameter (i.e., Hazen-Williams coefficient, C<sub>HW</sub>) for which the ordered pair becomes 2<sup>8</sup>= 256 for each α-cut. Hence the EPANET 2 model need to executed 1281 times including α<sup>\*</sup>=1.0 to evaluate the minimum and maximum values of pipe discharges and nodal HGL. It takes about 10 seconds for each α- cut in single simulation run except α<sup>\*</sup>=1.0. In this network, only one pipe is supplying water from the source reservoir so that pipe discharge Q<sub>1</sub> is 1120 m<sup>3</sup>/h irrespective of the α-cut value.

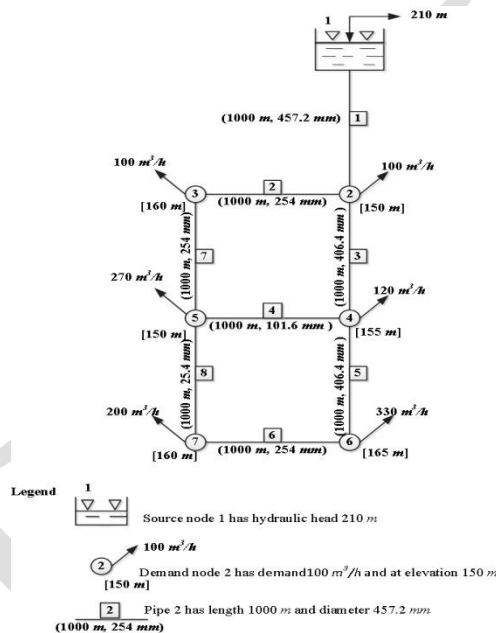
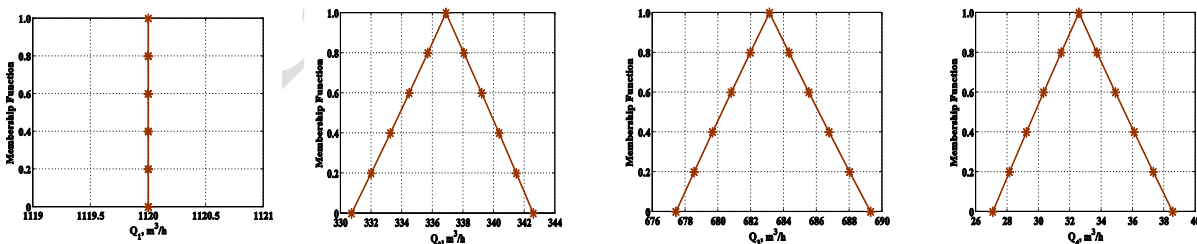


Fig. 5 A two loop with eight pipe network

The results of pipe discharges are found to vary about maximum 47% whereas the hydraulic heads at nodes vary about 4.2 m differ from the crisp value when the uncertainty of about 8 % (when α-cut value is 0.0) in Hazen-Williams coefficient of pipe roughness. Further, the nodal heads are less than the minimum pressure head requirements of 30 m at nodes 3 and 7 for all α- cut level except α<sup>\*</sup>=1.0). Similar kind of results obtained only in α<sup>\*</sup>=0.0 for node 5 and from α<sup>\*</sup>=0.0 to α<sup>\*</sup>=0.6 for node 6. The membership functions of unknown dependent parameters are as shown in Fig. 6. This network has slightly higher value of pipe roughness and pipe length is lesser than example 1.





**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 12, December 2014

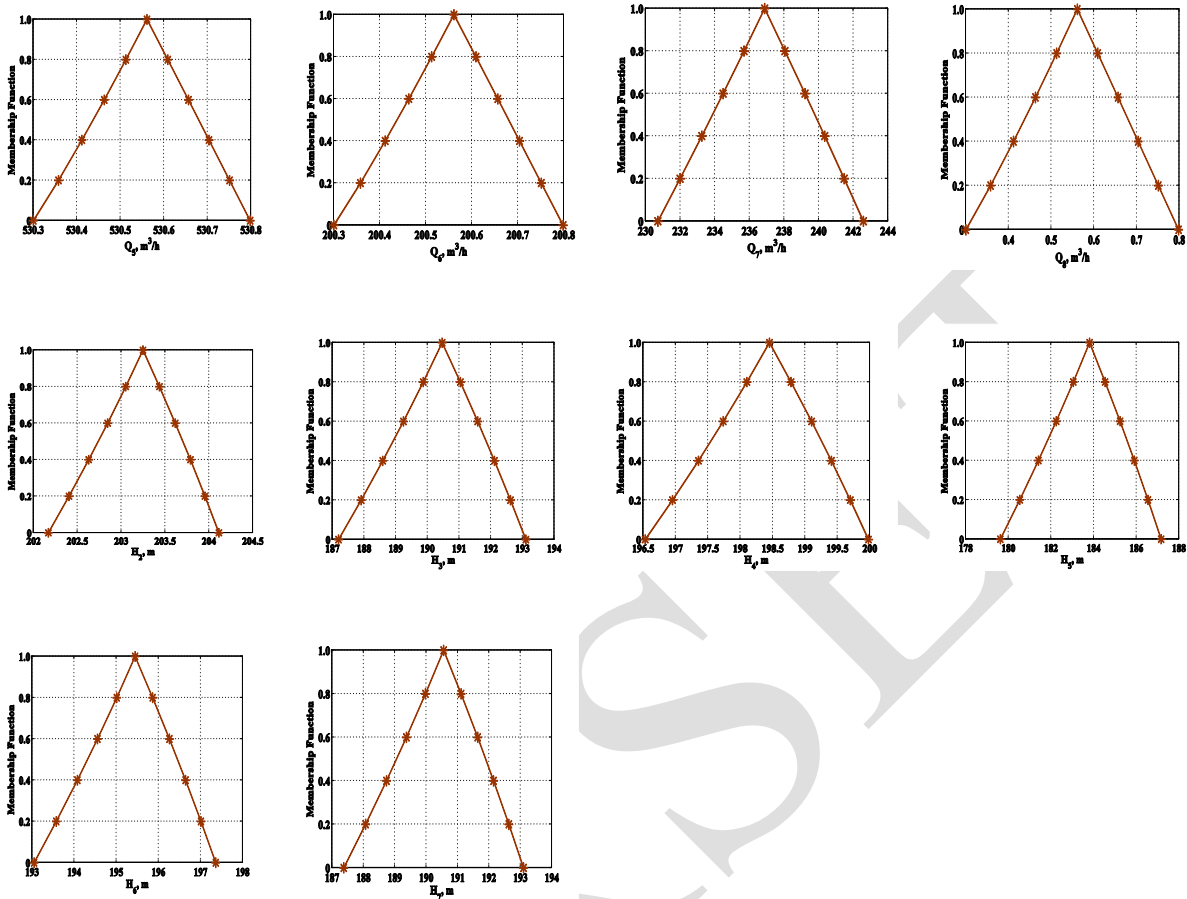


Fig. 6 Membership functions of unknown pipe discharges ( $m^3/h$ ) and nodal HGL (m) for the network of Fig. 5

**Case study 3 (12 Pipe Network)**

Further four loop with twelve pipe network as shown in Fig. 7. This complex bench mark problem introduced by Marchi and Rubatta [26] and the same network were analysed for uncertainty problem by Revelli and Ridolfi [7]. They assumed Strickler coefficient of pipe roughness as fuzzy parameter of membership function from 55 to 65 with 60 as crisp value.

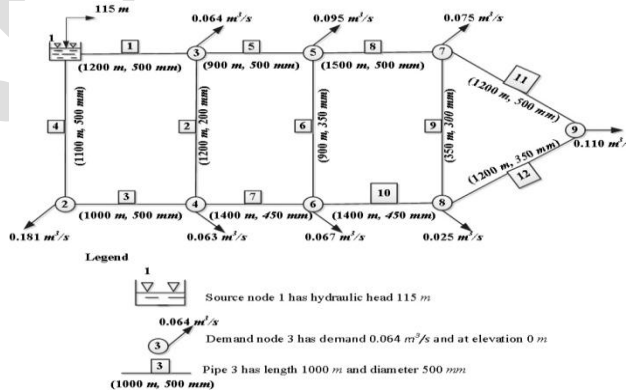


Fig. 7 A four loop with twelve pipe network

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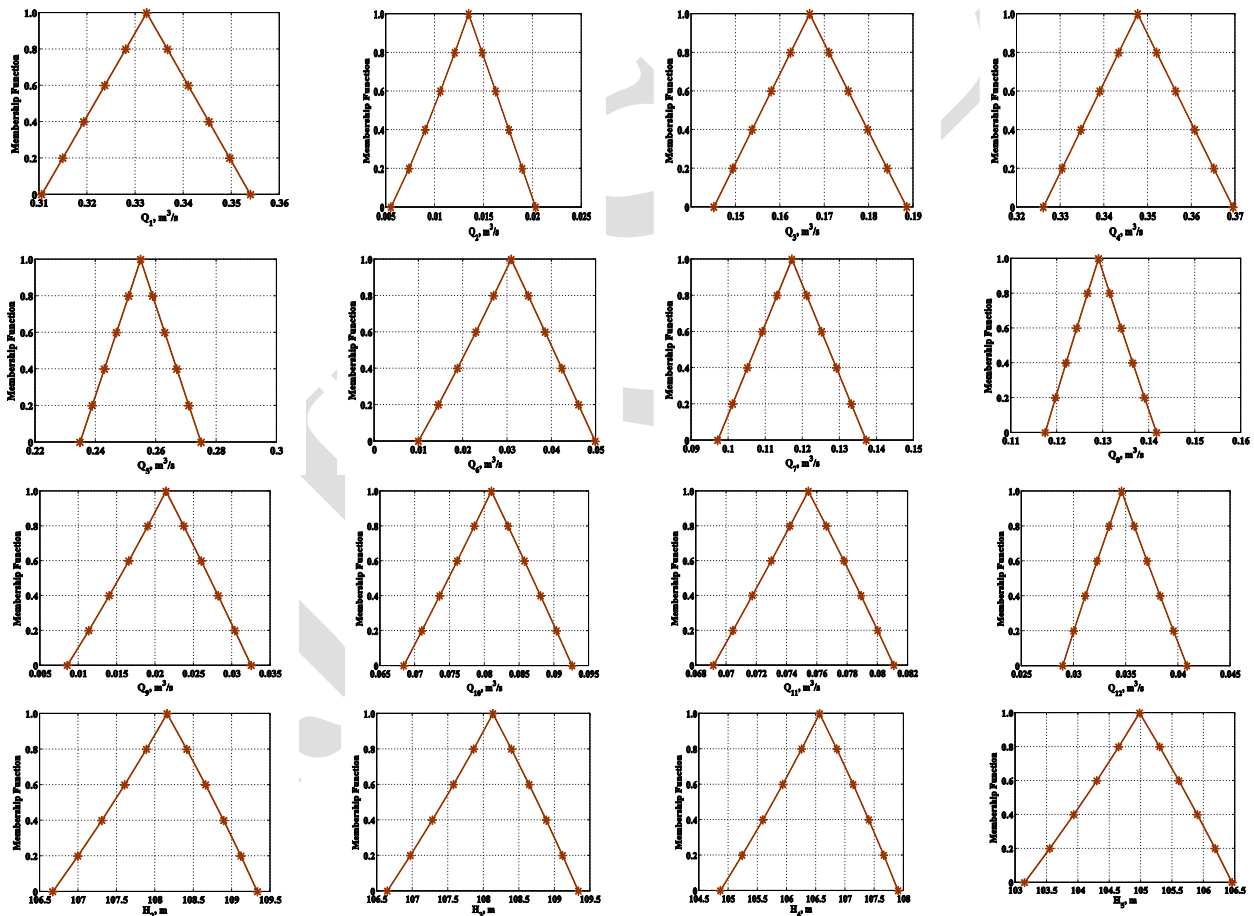
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Vol. 3, Issue 12, December 2014

The network has one source node with fixed pressure head 115 m, eight demand nodes from 2-9 have nodal demands in  $m^3/s$  with zero elevation level as well as twelve pipes from 1-12 with pipe length in m and diameter in mm given in parentheses as shown in Fig. 7.

As explained previous Examples, the source pressure head and nodal demands are precise, but uncertainty exists in Hazen-Williams coefficient,  $C_{HW}$  of all pipes. The most likely value of  $C_{HW}$  for all pipes is 120 with minimum and maximum values of 110 and 130 respectively. In this network, there are 12 pipes with one uncertain parameter (i.e., Hazen-Williams coefficient,  $C_{HW}$ ) for which the ordered pair becomes  $2^{12} = 4096$  for each  $\alpha$ -cut. Hence the EPANET 2 model needed to executed 20481 times including  $\alpha^* = 1.0$  to evaluate the minimum and maximum values of pipe discharges and nodal HGL. It takes elapsed time to run for each  $\alpha$ -cut except  $\alpha^* = 1.0$  varying from about 169 seconds in single simulation run.

The results of pipe discharges are found to vary about maximum 68% (at pipe 6) whereas the hydraulic heads at nodes vary about 2.1 m (at node 9) differ from the crisp value when the uncertainty of about 8% (when  $\alpha$ -cut value is 0.0) in Hazen-Williams coefficient of pipe roughness. The maximum and minimum values of discharge and nodal HGL for each  $\alpha$ -cut and the corresponding membership function are plotted in Fig. 8.



**International Journal of Innovative Research in Science,  
Engineering and Technology**  
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Vol. 3, Issue 12, December 2014

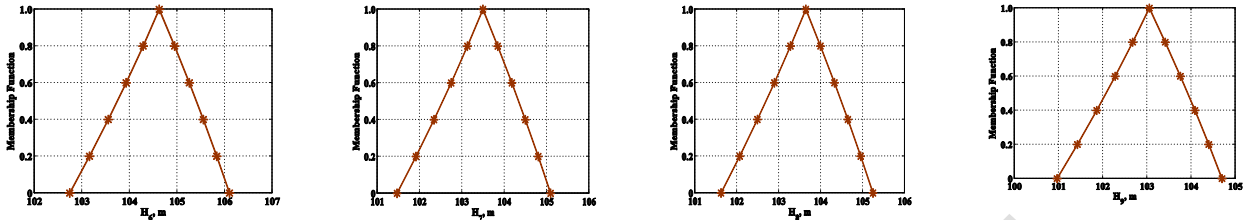


Fig. 8 Membership functions of unknown pipe discharges ( $m^3/s$ ) and nodal HGL (m) for the network of Fig. 7

Table 2. Comparison of dependent parameters of Revelli and Ridolfi [7] with present study for crisp value of roughness parameter in all pipes for Example 3

Row	Pipe discharge ( $m^3/s$ )											
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
1	0.333	0.013	0.166	0.347	0.256	0.011	0.116	0.15	0.004	0.06	0.08	0.031
2	0.332	0.013	0.167	0.348	0.255	0.031	0.117	0.129	0.021	0.081	0.075	0.035

Row	Nodal HGL (m)								
	H2	H3	H4	H5	H6	H7	H8	H9	
1	99.78	99.69	96.61	92.91	92.83	89.04	89.05	88.17	
2.00	108.16	108.13	106.56	104.98	104.62	103.49	103.64	103.05	

In Table 2, resulted values of Row 1 analysed by Revelli and Ridolfi [7] using Strickler coefficient of pipe roughness with crisp value 60 and Row 2 is present study using Hazen-Williams coefficient of pipe roughness with crisp value 120 as given in Table 1. While with this crisp value of pipe roughness parameter analysed by different methods considerable variations in pipe discharges and nodal HGL. The variation of pipe discharges in pipe 6, 8, 9, 10, 11 and 12 have about 182, 14, 425, 35, 5 and 12 % respectively. Similarly, the nodal HGL at node 9 has maximum variation of head difference is 14.88 m (about 17%).

**V. CONCLUSIONS**

Three water distribution network problems have been analysed to determine the membership function of unknown pipe discharges and hydraulic heads by assuming pipe roughness of each pipe as uncertain. In this study, shape of the membership function of pipe roughness is assumed to be triangular. Further, EPANET 2 is used to find the pipe discharges and hydraulic heads of each node by giving required input parameters. The resulted shape of the membership functions of output discharges and hydraulic heads are found to be triangular in nature. Further, the results obtained by proposed method and those of Shibu and Reddy [10], Bhave and Gupta [16] are same for pipe discharges and nodal HGL respectively. Consequently the membership functions for vertex method and earlier studies are identical. Thus, the proposed methodology using vertex method linked with EPANET 2 by programmer's toolkit in MATLAB environment has been successful in quantifying uncertainty in pipe discharges and nodal HGLs. The results of pipe discharges are found to vary 30, 47 and 68% whereas the hydraulic heads at nodes vary 2.7, 4.2 and 2.1 m, which differs from the crisp value when uncertainty of about 8% in Hazen-Williams coefficient of pipe roughness is introduced in 5, 8 and 12 pipe networks respectively. The maximum uncertainty in pipe discharges and hydraulic heads for all the networks are found to be at  $\alpha^* = 0$ . This may be due to maximum uncertainty in roughness coefficient of the pipe. Moreover, some of the nodes are getting less than the minimum pressure head requirements in the selected benchmark problem. To obtain the further accuracy of the result of uncertain output parameters, the refinement of  $\alpha$ -cut level i.e., increase the number of  $\alpha$ -cut with less equal interval may adopted.

Furthermore, EPANET 2 required to run 32, 256 and 4096 times with elapsed time about 2, 10 and 169 seconds for 5, 8 and 12 pipe network problem respectively for each  $\alpha$ -cut level except  $\alpha^* = 1$ . All these networks

**International Journal of Innovative Research in Science,  
Engineering and Technology**  
(An ISO 3297: 2007 Certified Organization)

**Vol. 3, Issue 12, December 2014**

computations were carried out using laptop having system configuration of Intel(R) Core(TM) i5-2450M CPU @ 2.50GHz processor with 4.00 GB RAM. The computational time may vary depends on the type processor used in PC. Further, the variations of results are obtained if the problem analysed by different formulae, range of coefficients used in that formula and number of independent uncertainty parameters encounter in the real network . Moreover, different roughness coefficient and methods of analysis may also lead to uncertainty in output parameters.

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