ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

Two-Temperature Generalized Thermo-Visco-Elastic Medium with Voids Subjected To Moving Loads under Green-Naghdi Theory

Mohamed I. A. Othman [1,2] , R. A. Mohamed [3] , Montaser Fekry [3]

Faculty of Science, Department of Mathematics, Zagazig University, P.O. Box 44519, Zagazig, Egypt [1]

Department of Mathematics, Faculty of Science, Taif University 888, Saudi Arabia [2]

Faculty of Science, Department of Mathematics, South Valley University, P.O. Box 83523, Qena, Egypt [3]

ABSTRACT: The present chapter is concerned with the investigation of disturbances in a homogeneous, isotropic, generalized thermo-viscoelastic material with voids and two-temperature under the effect of moving loads. The problem formulated in the context of Green-Naghdi theories (G-N II without energy dissipation and G-N III with energy dissipation). The analytical expressions for the physical quantities are obtained in the physical domain by using the normal mode analysis. These expressions are calculated numerically for a specific material and explained graphically. Comparisons are made with the results predicted by G-N II and G-N III theories in the presence and absence of moving initial stress

KEYWORDS: Green-Naghdi, thermo-viscoelasticity, moving loads, two-temperature, voids.

I. INTRODUCTION

The linear viscoelasticity remains an important area of research. Gross [1], Staverman, et al. [2], Alfrey and Gurnee [3] and Ferry [4] investigated the mechanical model representation of linear viscoelastic behavior results. Solution of the boundary value problems for linear viscoelastic materials, including temperature variations in both quasi-static and dynamic problems made great strides in the last decades, in the work of (Biot [5], 6]) and Huilgol and Phan-Thien [7]. Bland [8] linked the solution of linear-viscoelasticity problems with corresponding linear elastic solutions. A notable works in this field were the work of Gurtin and Sternberg [9], and Ilioushin [10] offered an approximation method for the linear thermal viscoelastic problems. One can refer to the book of Ilioushin and Pobedria [11] for a formulation of the mathematical theory of thermal viscoelasticity and the solutions of some boundary value problems, as well as, to the work of Pobedria [12] for the coupled problems in continuum mechanics. Results of important experiments determining the mechanical properties of viscoelastic materials were involved in the book of Koltunov [13]. Othman [14] studied the uniqueness and reciprocity theorems for generalized thermo-viscoelasticity. Othman [15] studied the generalized thermo-viscoelasticity under three theories. Othman [16] studied the thermal relaxation effect on the generalized linear thermo-viscoelasticity. Othman [17] studied the effect of rotation in the case of the 2-D problem of the generalized thermo-viscoelasticity with two relaxation times. The heat conduction equations for the classical linear uncoupled and coupled thermoelasticity theories are of the diffusion type predicting infinite speed of propagation of heat wave contrary to physical observations. To eliminate this paradox inherent in the classical theories, generalized theories of thermoelasticity were developed. The generalized thermo-elasticity theories admit so-called second-sound effects, that is, they predict the finite velocity of propagation for heat field. The first attempt towards the introduction of generalized thermoelasticity was headed by Lord and Shulman [18], who formulated the theory by incorporating a flux-rate term into conventional Fourier's law of heat conduction. The Lord-Shulman theory introduces a new physical concept which called a relaxation time. Since the heat conduction equation of this theory is of the wave-type, it automatically ensures finite speed of propagation of heat wave. The second generalization was developed by Green and Lindsay [19]. This theory contains two constants that act as relaxation times and modifies all the equations of coupled

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

theory, not the heat conduction equation only. Later on, Green and Naghdi [20-22] proposed another three models, which are subsequently referred to as GN-I, GN-II, and GN-III models. The linearized version of model-I corresponds to the classical thermoelastic model-II for which the internal rate of production of entropy is taken to be identically zero, implying no dissipation of thermal energy. This model assumes un-damped thermoelastic waves in a thermoelastic material and is best known as the theory of thermoelasticity without energy dissipation. Model-III includes the previous two models as special cases, and assumes dissipation of energy in general. The theory of elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory is concerned with elastic materials consisting of a distribution of small pores (voids, which contain nothing of mechanical or energetic significance) in which the void volume is included among the kinematic variables. Practically, this theory is useful for investigating various types of geological and biological materials for which elastic theory is inadequate. Nunziato and Cowin [23] have studied a non-linear theory of elastic materials with voids. They showed that the changes in the volume fraction cause an internal dissipation in the material and this internal dissipation leads to a relaxation property in the material. Cowin and Nunziato [24] have developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. This linearized theory of elastic materials with voids is a generalization of the classical theory of elasticity and reduces to it when the dependence of change in volume fraction and its gradient are suppressed. In this theory, the volume fraction corresponding to void volume is taken as an independent kinematic variable. Puri and Cowin [25] have studied the behavior of plane waves in a linear elastic material with voids. Domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [26]. Dhaliwal and Wang [27] also developed a heat-flux dependent theory of thermoelasticity with voids. Cowin [28] studied the viscoelastic behavior of linear elastic materials with voids. Ieşan [29] has developed a linear theory of thermoelastic material with voids by generalizing some ideas of the paper of Cowin and Nunziato [24]. While a nonlinear and linear theory of thermo-viscoelastic materials with voids studied by Ieşan [30].

Chen and Gurtin [31], Chen et al. [32-33] have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature T and the thermo-dynamic temperature θ. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two-temperatures are identical Chen and Williams [32]. For time dependent problems, however, and to wave propagation problems in particular, the two temperatures are in general different regardless of the presences of a heat supply. The two temperatures T ,θ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body Boley and Tolins [34]. The key element that sets the two-temperature thermoelasticity (2TT) apart from the classical theory of thermoelasticity (CTE) is the material parameter $a > 0$, called the temperature discrepancy. Specifically, if $a = 0$, then $T = \theta$, and the field equations of the 2TT are reduced to those of CTE. Warren and Chen [35] investigated the wave propagation in the two-temperature theory of thermoelasticity, but Youssef [36] investigated this theory in the context of the generalized theory of thermoelasticity. The present work is to obtain the physical quantities in a homogenous, isotropic, thermo-viscoelastic material with voids and two-temperature in the case of absence and presence of moving loads. The model is illustrated in the context of GN-II, and GN-III theories. The normal mode analysis is used to obtain the exact expressions for physical quantities. The distributions of considered variables are represented graphically.

II. FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, thermally conducting viscoelastic half-space $z \geq 0$ with voids and two-temperature. For the two-dimensional problem we assume the dynamic displacement vector as $\boldsymbol{u} = (u, 0, w)$. All quantities considered will be as functions of the time variable t and of the coordinates x and z . The whole body is at a constant temperature T_0 . The basic governing equations for a linear generalized visco-thermoelastic media with voids and two-temperature under the effect of moving loads in the absence of body forces are written by Ieşan [30] and Green and Naghdi [22]

$$\mu^* \nabla^2 \boldsymbol{u} + (\lambda^* + \mu^*) \nabla (\nabla \cdot \boldsymbol{u}) - \beta^* \nabla T + b^* \nabla \phi = \rho \ddot{\boldsymbol{u}}, \tag{1}$$

$$A^* \nabla^2 \phi - \xi_1 \phi - \xi_2 \dot{\phi} - B^* (\nabla \cdot \boldsymbol{u}) + (\tau \nabla^2 + m) T = \rho \chi \ddot{\phi}, \tag{2}$$

$$\rho C_e \ddot{T} + \beta^* T_0 \ddot{e} + (m T_0 - \varsigma \nabla^2) \dot{\phi} = K \nabla^2 \theta + K^* \nabla^2 \dot{\theta}, \tag{3}$$

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

$$T = (1 - a\nabla^2)\theta. \tag{4}$$

And the constitutive relations are given by

$$\sigma_{ij} = \mu^* u_{i,j} + \mu^* u_{j,i} + [\lambda^* u_{k,k} - \beta^* T + b^* \phi]\delta_{ij}, \tag{5}$$

The parameters $\lambda^*, \mu^*, \beta^*, A^*, B^*$ and b^* are defined as

$$\lambda^* = \lambda(1+\alpha_0 \frac{\partial}{\partial t}),\ \mu^* = \mu(1+\alpha_1 \frac{\partial}{\partial t}),\ \beta^* = \beta(1+\beta_0 \frac{\partial}{\partial t}),\ A^* = A(1+\alpha_3 \frac{\partial}{\partial t}),\ B^* = b(1+\alpha_4 \frac{\partial}{\partial t}),\ b^* = b(1+\alpha_2 \frac{\partial}{\partial t}). \tag{6}$$

Where, $\beta_0 = \frac{1}{\beta}(3\lambda\alpha_0 + 2\mu\alpha_1)\alpha_t$, $\beta = (3\lambda + 2\mu)\alpha_t$, λ, μ are the Lame's constants, σ_{ij} are the components of stress tensor, ϕ is the volume fraction field, $A, \xi_1, \xi_2, B, \tau, \varsigma, m, \chi$ are the material constants due to presence of voids, T is the thermodynamic temperature, θ is the conductive temperature and the reference temperature is T_0. K, ρ and C_e are the thermal conductivity, density and specific heat at constant strain, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the visco-elastic parameters, α_t is the coefficient of linear thermal expansion, e is the dilatation and δ_{ij} is Kronecker's delta. The dot notation is used to denote time differentiation.

The strain tensor is $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \ i, j = 1,3$

For $x - z$ plane, Eq. (1) gives rise to the following two equations

$$\mu(1+\alpha_1 \frac{\partial}{\partial t})\nabla^2 u + [\lambda(1+\alpha_0 \frac{\partial}{\partial t}) + \mu(1+\alpha_1 \frac{\partial}{\partial t})]\frac{\partial e}{\partial x} - \beta(1+\beta_0 \frac{\partial}{\partial t})(1 - a\nabla^2)\frac{\partial \theta}{\partial x} + b(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$\mu(1+\alpha_1 \frac{\partial}{\partial t})\nabla^2 w + [\lambda(1+\alpha_0 \frac{\partial}{\partial t}) + \mu(1+\alpha_1 \frac{\partial}{\partial t})]\frac{\partial e}{\partial z} - \beta(1+\beta_0 \frac{\partial}{\partial t})(1 - a\nabla^2)\frac{\partial \theta}{\partial z} + b(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}. \tag{7}$$

For simplifications we shall use the following non-dimensional variables:

$$x_i' = \frac{\varpi}{c_1} x_i,\ \ u_i' = \frac{\rho c_1 \varpi}{\beta T_0} u_i,\ \ \{T', \theta'\} = \frac{\{T, \theta\}}{T_0},\ \ \phi' = \frac{\varpi^2 \chi}{c_1^2}\phi,\ \ t' = \varpi t,\ \ \sigma_{ij}' = \frac{\sigma_{ij}}{\beta T_0},$$

$$\{\alpha_0', \alpha_1', \alpha_2', \alpha_3', \alpha_4'\} = \varpi\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4\},\ \ c_1^2 = \frac{\lambda + 2\mu}{\rho},\ \ c_2^2 = \frac{\mu}{\rho},\ \ \varpi = \frac{C_e(\lambda + 2\mu)}{K}. \tag{9}$$

Where, ϖ is the characteristic frequency of the material and c_1, c_2 are the longitudinal and shear wave velocities in the medium, respectively.

Using Eq. (9), then, Eqs.(7), (7), (2), and (3) become respectively (dropping the dashed for convenience).

$$\delta^2(1+\alpha_1 \frac{\partial}{\partial t})\nabla^2 u + [(1-2\delta^2)(1+\alpha_0 \frac{\partial}{\partial t}) + \delta^2(1+\alpha_1 \frac{\partial}{\partial t})]\frac{\partial e}{\partial x} - (1+\beta_0 \frac{\partial}{\partial t})(1 - a_1\nabla^2)\frac{\partial \theta}{\partial x} + a_2(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$\delta^2(1+\alpha_1 \frac{\partial}{\partial t})\nabla^2 w + [(1-2\delta^2)(1+\alpha_0 \frac{\partial}{\partial t}) + \delta^2(1+\alpha_1 \frac{\partial}{\partial t})]\frac{\partial e}{\partial z} - (1+\beta_0 \frac{\partial}{\partial t})(1 - a_1\nabla^2)\frac{\partial \theta}{\partial z} + a_2(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \tag{9}$$

$$(1+\alpha_3 \frac{\partial}{\partial t})\nabla^2 \phi - a_3(\phi + \xi\dot{\phi}) - a_4(1+\alpha_4 \frac{\partial}{\partial t})e + (a_5\nabla^2 + a_6)(1 - a_1\nabla^2)\theta = \frac{\ddot{\phi}}{\delta_1^2}, \tag{10}$$

$$(1 - a_1\nabla^2)\ddot{\theta} + \varepsilon_1(1+\beta_0 \frac{\partial}{\partial t})\ddot{e} + a_7\dot{\phi} - a_8\nabla^2\dot{\phi} = \varepsilon_2\nabla^2\theta + \varepsilon_3\nabla^2\dot{\theta}. \tag{11}$$

Where,

$$a_1 = \frac{a\varpi^2}{c_1^2},\ a_2 = \frac{bc_1^2}{\varpi^2 \chi \beta T_0},\ a_3 = \frac{\xi_1 c_1^2}{A\varpi^2},\ a_4 = \frac{b\chi\beta T_0}{A\rho c_1^2},\ a_5 = \frac{\tau\varpi^2\chi T_0}{Ac_1^2},\ a_6 = \frac{m\chi T_0}{A},\ a_7 = \frac{mc_1^2}{\rho C_e \varpi^3 \chi},$$

$$a_8 = \frac{\varsigma}{\rho C_e \varpi \chi T_0},\ \varepsilon_1 = \frac{\beta^2 T_0}{\rho^2 c_1^2 C_e},\ \varepsilon_2 = \frac{K}{\rho C_e c_1^2},\ \varepsilon_3 = \frac{K^* \varpi}{\rho C_e c_1^2},\ \delta^2 = \frac{c_2^2}{c_1^2},\ c_3^2 = \frac{A}{\rho\chi},\ \delta_1^2 = \frac{c_3^2}{c_1^2},\ \xi = \frac{\xi_2 \varpi}{\xi_1},\ i, j = 1,3.$$

The non-dimensional constitutive relations are given by

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

$$\sigma_{ij}=[\delta^2(1+\alpha_1\frac{\partial}{\partial t})u_{i,j}+\delta^2(1+\alpha_1\frac{\partial}{\partial t})u_{j,i}]+[(1-2\delta^2)(1+\alpha_0\frac{\partial}{\partial t})u_{k,k}-(1+\beta_0\frac{\partial}{\partial t})(1-a_1\nabla^2)\theta+a_2(1+\alpha_2\frac{\partial}{\partial t})\phi]\delta_{ij}. \quad (12)$$

The expressions relating displacement components $u(x,z,t)$, $w(x,z,t)$ to the potentials are

$$u=\Phi_{,x}+\Psi_{,z},\ w=\Phi_{,z}-\Psi_{,x},\ e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=\nabla^2\Phi,\ \ \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\nabla^2\Psi. \quad (13)$$

Substituting from Eq. (13) into Eqs. (8)-(11), we obtain

$$\delta^2(1+\alpha_1\frac{\partial}{\partial t})\nabla^2\Psi=\ddot{\Psi}, \quad (14)$$

$$(1+\delta_0\frac{\partial}{\partial t})\nabla^2\Phi-(1+\beta_0\frac{\partial}{\partial t})(1-a_1\nabla^2)\theta+a_2(1+\alpha_2\frac{\partial}{\partial t})\phi=\ddot{\Phi}, \quad (15)$$

$$(1+\alpha_3\frac{\partial}{\partial t})\nabla^2\phi-a_3(\phi+\xi\dot{\phi})-a_4(1+\alpha_4\frac{\partial}{\partial t})\nabla^2\Phi+(a_5\nabla^2+a_6)T=\frac{\ddot{\phi}}{\delta_1^2}, \quad (16)$$

$$(1-a_1\nabla^2)\ddot{\theta}+\varepsilon_1(1+\beta_0\frac{\partial}{\partial t})\nabla^2\ddot{\Phi}+a_7\dot{\phi}-a_8\nabla^2\dot{\phi}=\varepsilon_2\nabla^2\theta+\varepsilon_3\nabla^2\dot{\theta}. \quad (17)$$

Where, $\delta_0=\alpha_0+2\delta^2(\alpha_1-\alpha_0)$.

III. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[\Phi,\Psi,T,\phi,\sigma_{ij}](x,z,t)=[\bar{\Phi},\bar{\Psi},\bar{T},\bar{\phi},\bar{\sigma}_{ij}](z)\exp[\omega t+icx]. \quad (18)$$

Where, ω is the frequency, c is the wave number in the x - direction and $i=\sqrt{-1}$

Eqs. (14)- (17) with the aid of Eq. (18)become respectively,

$$(D^2-k_1^2)\bar{\Psi}=0, \quad (19)$$

$$(b_1D^2-b_2)\bar{\Phi}+(b_3D^2-b_4)\bar{\theta}+b_5\bar{\phi}=0, \quad (20)$$

$$(b_6D^2-b_7)\bar{\Phi}+[b_8D^4+b_9D^2+b_{10}]\bar{\theta}-(b_{11}D^2-b_{12})\bar{\phi}=0, \quad (21)$$

$$(b_{13}D^2-b_{14})\bar{\Phi}-(b_{15}D^2-b_{16})\bar{\theta}-(b_{17}D^2-b_{18})\bar{\phi}=0. \quad (22)$$

where, $D=\frac{d}{dz},\ k_1^2=c^2+\frac{\omega^2}{\delta^2(1+\alpha_1\omega)},\ b_1=(1+\omega\delta_0),\ b_2=b_1c^2+\omega^2,\ b_3=(1+\omega\beta_0)a_1,\ b_4=b_3c^2+(1+\omega\beta_0),$

$b_5=a_2(1+\omega\alpha_2),\ b_6=a_4(1+\alpha_4\omega),\ b_7=b_6c^2,\ b_8=a_1a_5,\ b_9=a_1a_6-a_5-2b_8c^2,\ b_{10}=b_8c^4-(a_1a_6-a_5)c^2-a_6,$

$b_{11}=(1+\alpha_3\omega),\ b_{12}=b_{11}c^2+a_3(1+\xi\omega)+\frac{\omega^2}{\delta_1^2},\ b_{13}=\varepsilon_1(1+\omega\beta_0)\omega^2,\ b_{14}=b_{13}c^2,\ b_{15}=\varepsilon_2+\varepsilon_3\omega+a_1\omega^2,$

$b_{16}=b_{15}c^2+\omega^2,\ b_{17}=a_8\omega,\ b_{18}=b_{17}c^2+a_7\omega.$

Eliminating $\bar{T},\bar{C}$ and $\bar{\phi}$ between Eqs. (20)-(22) we get the following ordinary differential equation satisfied with $\bar{\Phi}$

$$(D^8-d_1D^6+d_2D^4-d_3D^2+d_4)\bar{\Phi}=0. \quad (23)$$

Where, $d_1=\frac{f_1}{f_0},\ d_2=\frac{f_2}{f_0},\ d_3=\frac{f_3}{f_0},\ d_4=\frac{f_4}{f_0},\ f_0=-b_1b_8b_{17},\ f_1=b_3(b_6b_{17}-b_{11}b_{13})+b_8(b_2b_{17}-b_5b_{13})+b_1(b_8b_{18}-b_9b_{17}-b_{11}b_{15}),$

$f_2=b_1(b_9b_{18}-b_{10}b_{17}+b_{11}b_{16}+b_{12}b_{15})+b_2(b_9b_{17}+b_{11}b_{15}-b_8b_{18})+b_3(b_{11}b_{14}+b_{12}b_{13}-b_6b_{18}-b_7b_{17})+b_4(b_{11}b_{13}-b_6b_{17})$
$\quad +b_5(b_8b_{14}-b_6b_{15}-b_9b_{13}),$

$f_3=b_1(b_{10}b_{18}-b_{12}b_{16})+b_2(b_{10}b_{17}-b_9b_{18}-b_{11}b_{16}-b_{12}b_{15})+b_3(b_7b_{18}-b_{12}b_{14})+b_4(b_6b_{18}+b_7b_{17}-b_{11}b_{14}-b_{12}b_{13})$
$\quad +b_5(b_6b_{16}+b_7b_{15}+b_9b_{14}-b_{10}b_{13}),$

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology
(An ISO 3297: 2007 Certified Organization)
Vol. 4, Issue 4, April 2015

$f_4 = b_2(b_{12}b_{16} - b_{10}b_{18}) + b_4(b_{12}b_{14} - b_7b_{18}) + b_5(b_{10}b_{14} - b_7b_{16})$.

In a similar manner we arrive at

$$(D^8 - d_1D^6 + d_2D^4 - d_3D^2 + d_4)\{\bar{\Phi}, \bar{\theta}, \bar{\phi}\} = 0. \quad (24)$$

Eq. (23) can be factored as:

$$(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)(D^2 - k_5^2)\bar{\Phi} = 0. \quad (25)$$

Where, $k_j^2\ (j = 2,3,4,5)$ are the roots of the characteristic equation of Eq.(23)

The solutions of Eqs.(19), (25) and (24) which are bound as $z \to \infty$, can be written as

$$\bar{\Psi} = R_1 e^{-k_1 z}. \quad (26)$$

$$\bar{\Phi}(z) = \sum_{j=2}^{5} R_j e^{-k_j z}. \quad (27)$$

$$\{\bar{\theta}(z), \bar{\phi}(z)\} = \sum_{j=2}^{5} \{S_{1j}, S_{2j}\} R_j e^{-k_j z}. \quad (28)$$

Where, $$S_{1j} = \frac{b_1b_{17}k_j^4 - (b_1b_{18} + b_2b_{17} - b_5b_{13})k_j^2 + b_2b_{18} - b_5b_{14}}{-b_3b_{17}k_j^4 + (b_5b_{15} + b_3b_{18} + b_4b_{17})k_j^2 - (b_4b_{18} + b_5b_{16})},$$

$$S_{2j} = \frac{(b_3b_{13} + b_1b_{15})k_j^4 - (b_3b_{14} + b_4b_{13} + b_1b_{16} + b_2b_{15})k_j^2 + (b_4b_{14} + b_2b_{16})}{b_3b_{17}k_j^4 - (b_3b_{18} + b_4b_{17} + b_5b_{15})k_j^2 + (b_4b_{18} + b_5b_{16})}, \quad j = 2,3,4,5.$$

Substituting Eqs. (26), (27) and (28) into Eq. (18) we get

$$\Psi = R_1 e^{(\omega t + icx - k_1 z)}, \quad \{\Phi, \theta, \phi\} = \sum_{j=2}^{5} \{1, S_{1j}, S_{2j}\} R_j e^{(\omega t + icx - k_j z)}. \quad (29)$$

Inserting Eq. (29) in Eq.(13), the displacement components u and w, which are bound as $z \to \infty$ are obtained as

$$u = (\sum_{j=2}^{5} icR_j e^{-k_j z} - k_1 R_1 e^{-k_1 z}) e^{(\omega t + icx)}, \quad (30)$$

$$w = -(\sum_{j=2}^{5} k_j R_j e^{-k_j z} + icR_1 e^{-k_1 z}) e^{(\omega t + icx)}. \quad (31)$$

The stress components and the chemical potential are of the form

$$\sigma_{xx} = \{ick_1[(1-2\delta^2)(1+\alpha_0\omega) - b_1]R_1 e^{-k_1 z} + \sum_{j=2}^{5}[(1-2\delta^2)(1+\alpha_0\omega)k_j^2 - b_1c^2 + [b_3(k_j^2 - c^2) - (1+\beta_0\omega)]S_{1j} + b_5S_{2j}]R_j e^{-k_j z}\} e^{(\omega t + icx)}, \quad (32)$$

$$\sigma_{zz} = \{ick_1[b_1 - (1-2\delta^2)(1+\alpha_0\omega)]R_1 e^{-k_1 z} + \sum_{j=2}^{5}[b_1k_j^2 - (1-2\delta^2)(1+\alpha_0\omega)c^2 + [b_3(k_j^2 - c^2) - (1+\beta_0\omega)]S_{1j} + b_5S_{2j}]R_j e^{-k_j z}\} e^{(\omega t + icx)}, \quad (33)$$

$$\sigma_{xz} = \delta^2(1+\alpha_1\omega)[(c^2 + k_1^2)R_1 e^{-k_1 z} - 2ic\sum_{j=2}^{5} k_j R_j e^{-k_j z}] e^{(\omega t + icx)}. \quad (34)$$

IV. The Boundary Conditions

In order to determine the parameters $R_j\ (j = 1,2,3,4,5)$ we need to consider the boundary conditions at $z = 0$ as follows:

$$\sigma_{zz} = -p_1 N(x,t), \quad \sigma_{xx} = 0, \quad \sigma_{xz} = 0, \quad \frac{\partial \phi}{\partial z} = 0, \quad T = 0. \quad (35)$$

A moving load with a constant velocity v_0 in the normal direction is assumed to be acting on the surface $z = 0$ of the medium so $p_1 = p_0(1 + v_0)$. Where, p_0 is the magnitude of the mechanical force, and $N(x,t)$ is known function. Substituting from the expressions of the variables considered into the boundary conditions (35), respectively, we can obtain the following equations:

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

$$h_{11}R_1+\sum_{j=2}^{5}h_{1j}R_j=-p_1, \tag{36}$$

$$h_{21}R_1+\sum_{j=2}^{5}h_{2j}R_j=0, \tag{37}$$

$$h_{31}R_1+\sum_{j=2}^{5}h_{3j}R_j=0 \tag{38}$$

$$\sum_{j=2}^{5}h_{4j}R_j=0, \tag{39}$$

$$\sum_{j=2}^{5}S_{1j}R_j=0. \tag{40}$$

Where, $h_{11}=ick_1[b_1-(1-2\delta^2)(1+\alpha_0\omega)],\quad h_{21}=ick_1[(1-2\delta^2)(1+\alpha_0\omega)-b_1],\quad h_{31}=\delta^2(1+\alpha_1\omega)(k_1^2+c^2),$

$$h_{1j}=[b_1k_j^2-(1-2\delta^2)(1+\alpha_0\omega)c^2+[b_3(k_j^2-c^2)-(1+\beta_0\omega)]S_{1j}+b_5S_{2j}],$$

$$h_{2j}=[(1-2\delta^2)(1+\alpha_0\omega)k_j^2-b_1c^2+[b_3(k_j^2-c^2)-(1+\beta_0\omega)]S_{1j}+b_5S_{2j}],$$

$$h_{3j}=-2ic\delta^2(1+\alpha_1\omega)k_j,\quad h_{4j}=-k_jS_{2j},\quad j=2,3,4,5.$$

Solving Eqs.(36) - (40) for $R_j\ (j=1,2,3,4,5)$ by using the inverse of matrix method as follows:

$$\begin{pmatrix}R_1\\R_2\\R_3\\R_4\\R_5\end{pmatrix}=\begin{pmatrix}h_{11}&h_{12}&h_{13}&h_{14}&h_{15}\\h_{21}&h_{22}&h_{23}&h_{24}&h_{25}\\h_{31}&h_{32}&h_{33}&h_{34}&h_{35}\\0&h_{42}&h_{43}&h_{44}&h_{45}\\0&S_{12}&S_{13}&S_{14}&S_{15}\end{pmatrix}^{-1}\begin{pmatrix}-p_1\\0\\0\\0\\0\end{pmatrix}. \tag{41}$$

V. NUMERICAL RESULTS AND DISCUSSIONS

With an aim to illustrate the problem, we will present some numerical results. The material chosen for the purpose of numerical computation is copper, the physical data for which are given by [37] in SI units:

$\lambda=7.76\times10^{10}\,\mathrm{kg\,m^{-1}\,s^{-2}},\quad \mu=3.86\times10^{10}\,\mathrm{kg\,m^{-1}s^{-2}},\quad K=386\,\mathrm{W\,m^{-1}K^{-1}},\ T_0=293\mathrm{K},\quad \rho=8954\,\mathrm{kg\,m^{-3}},$
$\alpha_t=1.78\times10^{-5}\mathrm{K^{-1}},\ C_e=383.1\,\mathrm{J\,kg^{-1}K^{-1}},\ a=0.15\times10^{-14}.$

The voids parameters are

$A=1.688\times10^{-5}\mathrm{kg\,m\,s^{-2}},\ b=1.139\times10^{10}\mathrm{kg\,m^{-1}s^{-2}},\ m=2\times10^{5}\mathrm{kg\,m^{-1}s^{-2}K^{-1}},\ \chi=1.75\times10^{-15}\mathrm{m^2},$
$\xi_1=1.475\times10^{10}\mathrm{kg\,m^{-1}s^{-2}},\ \xi_2=3.8402\times10^{-4}\mathrm{kg\,m^{-1}\,s^{-3}},\ \tau=0.2\times10^{-5}\mathrm{kg\,m\,s^{-2}K^{-1}},\ \varsigma=0.1\times10^{-5}\mathrm{kg\,m\,s^{-2}}.$

The comparisons were carried out for

$p_0=0.5,\ v_0=0.5,\ t=0.1,\ x=0.2,\ \omega=1.5+1.5i,\ c=1.5,\ 0\le z\le3,\ \alpha_0=3.25\times10^{-2},\ \alpha_1=3.91\times10^{-2},$
$\alpha_2=6.51\times10^{-2},\ \alpha_3=1.02\times10^{4},\ \alpha_4=1.95\times10^{-2}.$

The above numerical technique was used for the distribution of the real parts of the displacement components u and w the conductive temperature θ, the thermodynamic temperature T , the stress components $\sigma_{xx},\sigma_{zz},\sigma_{xz}$ and the change in the volume fraction field ϕ with distance z for (G-N II) and (G-N III) with and without moving load effect which are shown graphically in the 2-D figures 1-8. At $v_0=0$ the solid lines represent the solution in the context of the (G-N II) and the dot lines represent the solution for the (G-N III). In the case of $v_0=0.5$, the dashed lines represent the solution in the context of the (G-N II) and the dot lines with circles represents the solution for the (G-N III).

ISSN(Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 4, April 2015

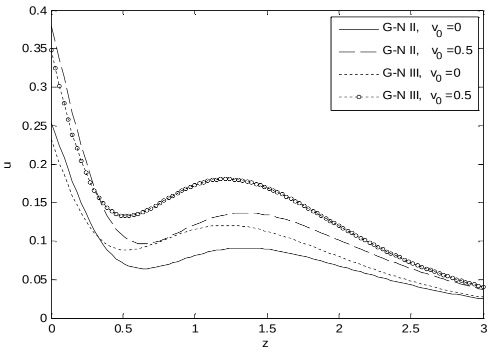

Fig. 1 Variation of the displacement u with horizontal distance z in the presence and absence of moving loads

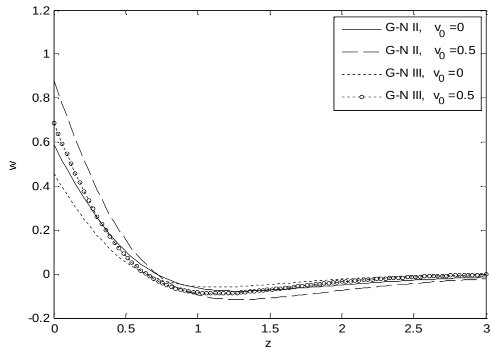

Fig. 2 Variation of the displacement w with horizontal distance z in the presence and absence of moving loads

Fig.1 depicts that the distribution of the horizontal displacement component u, always begins from negative values for $v_0 = 0, v_0 = 0.5$. In the context of (G-N II) and (G-N III) the distribution of u at $v_0 = 0.5$ is higher than that at $v_0 = 0$ for $z > 0$. Fig.2 shows the distribution of the displacement component w in the case of $v_0 = 0, v_0 = 0.5$. In the context of (G-N II) and (G-N III) the distribution of w at $v_0 = 0.5$ is larger than that at $v_0 = 0$ in the range $0 < z < 0.6$ for (G-N II) and in the range $0 < z < 0.7$ for (G-N III), then, conversely in the other ranges for both types.

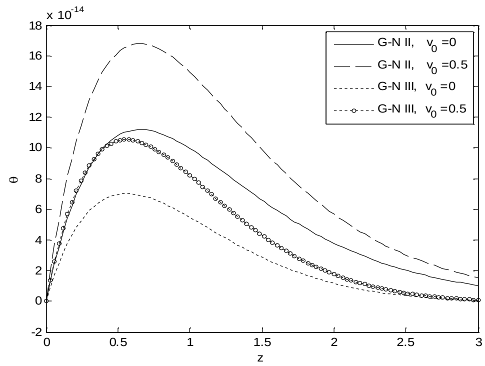

Fig. 3 Variation of the conductive temperature θ with horizontal distance z in the presence and absence of moving loads

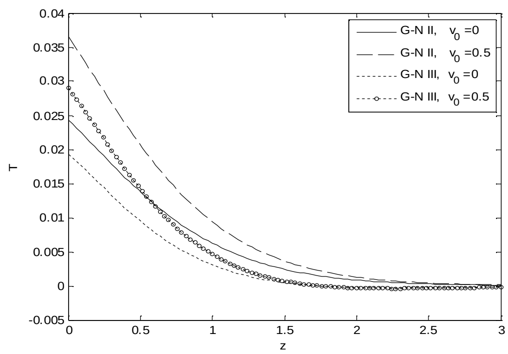

Fig. 4 Variation of the thermodynamic temperature T with horizontal distance z in the presence and absence of moving loads

Figs. 3, 4 explain that the distribution of the conductive temperature θ and the thermodynamic temperature T in the case of $v_0 = 0, v_0 = 0.5$. In the context of (G-N II) and (G-N III) the distribution of θ at $v_0 = 0.5$ is higher than that at $v_0 = 0$ for $z > 0$. Fig. 5 expresses the distribution of the stress component σ_{xx} in the case of $v_0 = 0, v_0 = 0.5$. In the context of (G-N II) and (G-N III) the distribution of σ_{xx} at $v_0 = 0.5$ is larger than that at $v_0 = 0$ for $z > 0$. Fig. 6 expresses the distribution of the stress component σ_{zz} in the case of $v_0 = 0, v_0 = 0.5$. The distribution of σ_{zz} at $v_0 = 0.5$ is larger than that in the range $0 < z < 0.45$ for (G-N II) and in the range $0 < z < 0.4$ for (G-N III), then, conversely in the other ranges for both types. Fig.7 expresses the distribution of the stress component σ_{xz} in the case of $v_0 = 0, v_0 = 0.5$. The distribution of σ_{xz} at $v_0 = 0.5$ is greater than that at $v_0 = 0$ in the range $0.3 < z < 1.3$ for (G-N II) and in the range $0 < z < 1.2$ for (G-N III), then, conversely in the other ranges for both types.

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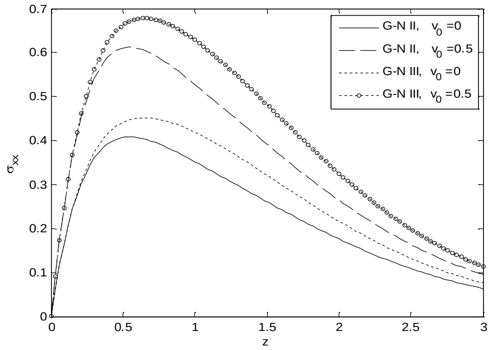

Fig. 5 Variation of the stress σ_{xx} with horizontal distance z in the presence and absence of moving loads

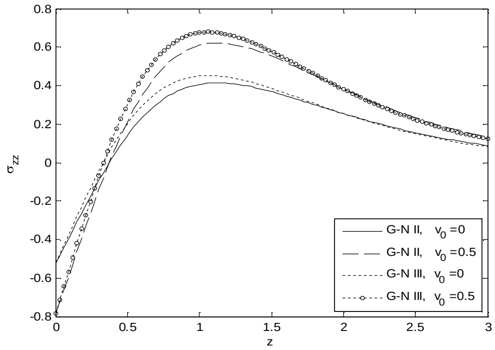

Fig. 6 Variation of the stress σ_{zz} with horizontal distance z in the presence and absence of moving loads

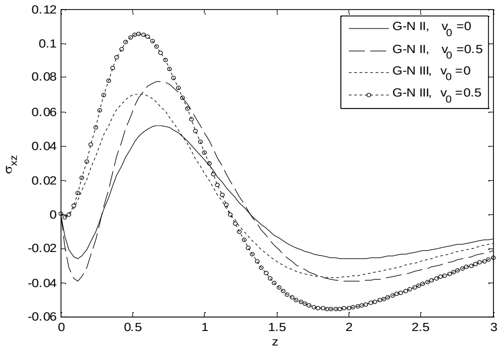

Fig. 7 Variation of the stress σ_{xz} with horizontal distance z in the presence and absence of moving loads

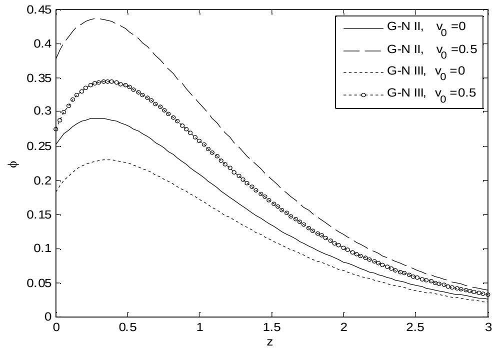

Fig. 8 Variation of the volume fraction field ϕ with horizontal distance z in the presence and absence of moving loads

Fig.8 depicts the distribution of the change in the volume fraction field ϕ for $v_0 = 0, v_0 = 0.5$. In the context of (G-N II) and (G-N III) the distribution of ϕ at $v_0 = 0.5$ is larger than that at $v_0 = 0$ for $z > 0$. It explains that all the curves converge to zero, and the initial stress is significant on the distributions of all physical functions.

VI. CONCLUSIONS

Analysis of the components of displacement, the stresses, the temperature distribution, and the change in the volume action field due to moving loads for thermo-viscoelastic solid with voids and two-temperature is an interesting problem of mechanics. The normal mode analysis has been used which is applicable to a wide range of problems in thermo-viscoelasticity. This method gives exact solutions without any assumed restrictions on the actual physical quantities that appear in the governing equations of the physical problem considered. The value of all physical quantities converges to zero with the increase of distance and all of them are continuous. It noticed that the thermo-viscoelastic materials with voids have an important role in the distribution of the field quantities, also the moving load has a great role in all considered physical quantities since the amplitudes of these quantities is varying (increasing or decreasing) with the increase of the moving loads values. Finally, it deduced that the deformation of a body depends on the nature of the applied forces and the moving loads effect as well as the type of boundary conditions.

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