

# Three Dimensional Fluctuating Slip Couette Flow with Heat Transfer through A Porous Channel

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**ABSTRACT:** In this paper, the work of Guria and Jana [1] is extended to include the slip condition at the lower porous plate. The upper plate is subjected to a constant injection  $-V_0$  and the lower plate to a transverse sinusoidal suction velocity distribution, so that the flow becomes three dimensional. Approximate solutions were found for velocity and temperature using regular perturbation techniques. The effect of various parameters like Reynolds Number, Prandtl Number, Nusselt Number on the flow fields were studied numerically. Influence of thermal diffusion is also taken into consideration. The validation of the theoretical results obtained is also carried out.

**KEY WORDS:** Slip regime, Couette flow, porous medium, heat transfer

## I. INTRODUCTION

Couette flow is important in numerous mechanisms involving the relative motion of two surfaces. It is also used in transpiration cooling. In this process several engines can be protected from the influence of hot gases. This process is used in turbojet and rocket engines, like combustion chamber walls, exhaust nozzles, and gas turbine blades. The channel flows through Porous medium have numerous engineering and geophysical applications, for example, in the field of agriculture engineering to study the underground water resources, in the field of chemical engineering for filtration and purification processes in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs etc.

Pai [3] discussed the velocity and temperature distribution in a plane couette flow. Singh, Sharma and Misra [12] investigated the free convection flow along a vertical porous plate with transverse sinusoidal suction velocity distribution. Vorticity of the velocity field for flow in a porous medium with random inhomogeneities was described by Shvidler [8]. Raptis, Peridikis and Tzivanidis [4] investigated the free convection flow through a porous medium bounded by a vertical surface. In this work the flow characteristics of two dimensional free convection flow through a porous medium bounded by a vertical infinite surface was considered. In [5] Raptis extended the above work to an unsteady two dimensional free convective flow through a porous medium bounded by an infinite vertical plate with the temperature of the plate oscillating with the time about a constant non zero mean value. An analytical solution for the velocity field was derived and the effects of permeability parameter and parameter of frequency on the velocity field are discussed. Raptis [7] made an analytical examination of steady, free convective flow and mass transfer through a porous medium bounded by a vertical porous plate in the case of a rotating fluid with a constant angular velocity and a constant temperature at the plate. Khalid and Vatai [2] investigated the role of porous media in modeling flow and heat transfer in biological tissues. Raptis and Peridikis [6] further studied the problem of free convection flow through a porous medium bounded by a vertical porous plate with constant suction when the free stream velocity oscillates in time with a constant mean value.

Singh *et al* [9] discussed the transient effects on Magneto Hydro Dynamic couette flow with rotation when one of the plates has been set in to uniformly accelerated motion. Singh and Rakesh Sharma [11] studied the three dimensional

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couette flows through a porous medium with heat transfer. Here a theoretical analysis of heat transfer in couette of a viscous incompressible fluid through a porous medium between two infinite horizontal parallel porous flat plates is presented taking into account slip boundary conditions.

## II. FORMULATION OF THE PROBLEM

We consider the unsteady flow of a viscous incompressible fluid between two horizontal flat plates separated by a distance  $d$ . The upper plate moves with a uniform velocity  $U$  in the direction of the flow. We choose a Cartesian coordinates system with its origin on the lower stationary plate,  $x^*$ - axis is in the direction of the flow,  $y^*$ - axis is perpendicular to the plate, and  $z^*$ - axis normal to the  $x^*y^*$ - plane. The upper plate is subjected to a constant injection  $-V_0$  and the lower plate a transverse sinusoidal suction velocity distribution of the form

$$v^* = -V_0 \left[ 1 + \epsilon \cos \left( \frac{\pi z^*}{d} - ct^* \right) \right]$$

Where  $\epsilon \ll 1$  is the amplitude of the suction velocity. Due to periodic suction, the flow becomes three dimensional. In fluid mechanics it is generally accepted that at an interface between two (mobile) fluids the tangential velocities  $v$  have to be continuous. In the past it was also assumed that the boundary condition for fluid flow past a solid wall is a no-slip boundary condition. During the past years, various experiments showed a non-vanishing tangential velocity, the so-called slip velocity. Hence in this problem the velocity profile is sketched with a slip boundary condition.

Denoting velocity components  $u^*, v^*, w^*$  in the direction  $x^*, y^*,$  and  $z^*$ - axes, respectively, the flow is governed by the following equations.

Equation of continuity

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0$$

Equation of motion

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} &= g\beta(T^* - T_0) + \nu \left( \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) \\ \frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} &= -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right) \\ \frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} &= -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left( \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right) \\ \frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} &= \alpha \left( \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) \end{aligned} \tag{2.1}$$

Where

- $\nu$  is the kinematic viscosity
- $\rho$  is the density
- $p^*$  is the fluid pressure.
- $g$  is the gravitational acceleration
- $\beta$  is the coefficient of thermal expansion.
- $\mu$  is the viscosity.
- $\alpha$  is the thermal diffusivity of the fluid.
- $C_p$  is the specific heat at constant pressure

The Boundary conditions are

$$\begin{aligned} u^* = L^* \frac{\partial u^*}{\partial y^*}, \quad v^* = -V_0 \left[ 1 + \epsilon \cos \left( \frac{\pi}{d} z^* - ct^* \right) \right], \quad w^* = 0, \quad T^* = T_0 \quad \text{at } y^* = 0 \\ u^* = U, \quad v^* = -V_0, \quad w^* = 0, \quad T^* = T_1 \quad \text{at } y^* = d \end{aligned}$$

Introducing the following non-dimensional quantities.

$$y = \frac{y^*}{d} \quad z = \frac{z^*}{d} \quad t = ct^* \quad p = \frac{p^*}{\rho U^2} \quad u = \frac{u^*}{U} \quad v = \frac{v^*}{U} \quad w = \frac{w^*}{U} \quad \theta = \frac{T^* - T_0}{T_1 - T_0}$$

in equation (2.1) we get

$$\begin{aligned} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\ \lambda \frac{\partial u}{\partial t} + Re \left[ v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] &= Re Gr \theta + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \end{aligned}$$

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$$\begin{aligned}
 \lambda \frac{\partial v}{\partial t} + Re \left[ v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] &= -Re \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \\
 \lambda \frac{\partial w}{\partial t} + Re \left[ v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] &= -Re \frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \\
 Pr \lambda \frac{\partial \theta}{\partial t} + Re Pr \left( v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) &= \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}
 \end{aligned} \tag{2.2}$$

Where

$\lambda$  is the velocity ratio parameter,

Re is the Reynolds number,

Gr is the Grashof number,

Pr is the Prandtl number

The corresponding boundary conditions in non-dimensional form are

$$\begin{aligned}
 u = h \frac{\partial u}{\partial y}, \quad v = -S[1 + \epsilon \cos(\pi z - t)], \quad w = 0, \quad \theta = 0 \quad \text{at } y = 0 \\
 u = 1, \quad v = -s, \quad w = 0, \quad \theta = 1 \quad \text{at } y = 1
 \end{aligned}$$

### III. SOLUTION OF THE PROBLEM

Assuming the solution to be the following form

$$\begin{aligned}
 u &= u_0(y) + \epsilon u_1(y, z, t) + \epsilon^2 u_2(y, z, t) + \dots \\
 v &= v_0(y) + \epsilon v_1(y, z, t) + \epsilon^2 v_2(y, z, t) + \dots \\
 w &= w_0(y) + \epsilon w_1(y, z, t) + \epsilon^2 w_2(y, z, t) + \dots \\
 \theta &= \theta_0(y) + \epsilon \theta_1(y, z, t) + \epsilon^2 \theta_2(y, z, t) + \dots
 \end{aligned} \tag{3.1}$$

When  $\epsilon = 0$  the flow reduces to two dimensional with constant suction and injection at both the plates. In this case (2.2) reduces to

$$\begin{aligned}
 \frac{\partial v_0}{\partial y} &= 0 \text{ or } v_0 = -s \\
 \frac{\partial^2 u_0}{\partial y^2} + SRe \frac{\partial u_0}{\partial y} + ReGr\theta_0 &= 0 \\
 \frac{\partial p_0}{\partial y} &= 0 \text{ or } p_0 = 1 \\
 \frac{\partial^2 w_0}{\partial y^2} + SRe \frac{\partial w_0}{\partial y} &= 0 \\
 \frac{\partial^2 \theta_0}{\partial y^2} + RePrS \frac{\partial \theta_0}{\partial y} &= 0
 \end{aligned} \tag{3.2}$$

The corresponding boundary conditions are,

$$\begin{aligned}
 u_0 = h \frac{\partial u_0}{\partial y} \quad v_0 = -S \quad w_0 = 0 \quad \theta_0 = 0 \quad \text{at } y = 0 \\
 u_0 = 1 \quad v_0 = -S \quad w_0 = 0 \quad \theta_0 = 1 \quad \text{at } y = 1
 \end{aligned} \tag{3.3}$$

The solution of (3.2) becomes

$$\begin{aligned}
 u_0 &= A_1 + A_2 e^{-SRey} \\
 v_0 &= -S \quad p_0 = 1 \quad w_0 = 0 \quad \theta_0 = A_3(1 - e^{-SPrRey})
 \end{aligned} \tag{3.4}$$

When  $\epsilon \neq 0$ , substituting equations (3.1) and equating the terms of  $o(\epsilon)$  we get,

$$\begin{aligned}
 \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} &= 0 \\
 \lambda \frac{\partial u_1}{\partial t} + Re \left[ -S \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} \right] &= ReGr\theta_1 + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \\
 \lambda \frac{\partial v_1}{\partial t} + Re \left[ -S \frac{\partial v_1}{\partial y} \right] &= -Re \frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \\
 \lambda \frac{\partial w_1}{\partial t} + Re \left[ -S \frac{\partial w_1}{\partial y} \right] &= -Re \frac{\partial p_1}{\partial z} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \\
 Pr \lambda \frac{\partial \theta_1}{\partial t} + RePr \left[ -S \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} \right] &= \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}
 \end{aligned} \tag{3.5}$$

The boundary conditions are,

$$\begin{aligned}
 u_1 = h \frac{\partial u_1}{\partial y} \quad v_1 = -S \cos(\pi z - t) \quad w_1 = 0 \quad \theta_1 = 0 \quad \text{at } y = 0 \\
 u_1 = 0 \quad v_1 = 0 \quad w_1 = 0 \quad \theta_1 = 0 \quad \text{at } y = 1
 \end{aligned} \tag{3.6}$$

These are the linear partial differential equations describing the three dimensional flow. To solve (3.5) we assume  $u_1, v_1, w_1$  and  $p_1$  of the following form

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$$\begin{aligned}
 u_1(y, z, t) &= u_{11}(y)e^{i(\pi z - t)} \\
 v_1(y, z, t) &= v_{11}(y)e^{i(\pi z - t)} \\
 w_1(y, z, t) &= \frac{i}{\pi} v'_{11}(y)e^{i(\pi z - t)} \\
 p_1(y, z, t) &= p_{11}(y)e^{i(\pi z - t)} \\
 \theta_1(y, z, t) &= \theta_{11}(y)e^{i(\pi z - t)}
 \end{aligned} \tag{3.7}$$

The boundary conditions are

$$\begin{aligned}
 u_{11} = h \frac{\partial u_{11}}{\partial y}, \quad v_{11} = -S, \quad v'_{11} = 0, \quad \theta_{11} = 0 \quad \text{at } y = 0 \\
 u_{11} = 0, \quad v_{11} = 0, \quad v'_{11} = 0, \quad \theta_{11} = 0 \quad \text{at } y = 1
 \end{aligned} \tag{3.8}$$

Substituting (3.7) in (3.5), we get

$$\begin{aligned}
 \frac{\partial v_{11}}{\partial y} + i\pi w_{11} &= 0 \\
 (-i\lambda)u_{11} - SRe \frac{\partial u_{11}}{\partial y} + Rev_{11} \frac{\partial u_0}{\partial y} &= ReGr\theta_{11} + \frac{\partial^2 u_{11}}{\partial y^2} - \pi^2 u_{11} \\
 \frac{\partial^2 v_{11}}{\partial y^2} + ReS \frac{\partial v_{11}}{\partial y} - (\pi^2 - i\lambda)v_{11} &= Re \frac{\partial p_{11}}{\partial y} \\
 \frac{\partial^2 v_{11}}{\partial y^2} + ReS \frac{\partial v_{11}}{\partial y} - (\pi^2 - i\lambda)v_{11} &= Re \frac{\partial p_{11}}{\partial y} \\
 \frac{\partial^2 w_{11}}{\partial y^2} + ReS \frac{\partial w_{11}}{\partial y} - (\pi^2 - i\lambda)w_{11} &= i\pi Rep_{11} \\
 \frac{\partial^2 \theta_{11}}{\partial y^2} + ReSpr \frac{\partial \theta_{11}}{\partial y} - (\pi^2 - i\lambda Pr)\theta_{11} &= RePrv_{11} \frac{\partial \theta_0}{\partial y}
 \end{aligned} \tag{3.9}$$

Solving, we get

$$\begin{aligned}
 u_1 &= [A_{17}e^{-r_1 y} + A_{18}e^{-r_2 y} + A_{19}e^{-r_3 y} + A_{20}e^{-r_4 y} + A_{21}e^{(\pi - SprRe)y} + A_{22}e^{-(\pi + SRePr)y} \\
 &\quad + A_{23}e^{-(r_1 + SprRe)y} + A_{24}e^{-(r_2 + SprRe)y} + A_{25}e^{(\pi - SRe)y} + A_{26}e^{-(\pi + SRe)y} + A_{27}e^{-(r_1 + SRe)y} \\
 &\quad + A_{28}e^{-(r_2 + SRe)y}]e^{i(\pi z - t)} \\
 v_1 &= [A_4 e^{\pi y} + A_5 e^{-\pi y} + A_6 e^{-r_1 y} + A_7 e^{-r_2 y}]e^{i(\pi z - t)} \\
 w_1 &= \frac{i}{\pi} [A_4 \pi e^{\pi y} - A_5 \pi e^{-\pi y} - r_1 A_6 e^{-r_1 y} - A_7 r_2 e^{-r_2 y}]e^{i(\pi z - t)} \\
 \theta_1 &= [A_8 e^{-r_3 y} + A_9 e^{-r_4 y} + A_{11} e^{(\pi - SprRe)y} + A_{12} e^{-(\pi + SprRe)y} + A_{13} e^{-(r_1 + SprRe)y} \\
 &\quad + A_{14} e^{-(r_2 + SprRe)y}]e^{i(\pi z - t)}
 \end{aligned}$$

Where

$$\begin{aligned}
 r_1 &= \frac{SRe + \sqrt{S^2 Re^2 + 4(\pi^2 - i\lambda)}}{2}, & r_2 &= \frac{SRe - \sqrt{S^2 Re^2 + 4(\pi^2 - i\lambda)}}{2} \\
 r_3 &= \frac{SRePr + \sqrt{S^2 Re^2 Pr^2 + 4(\pi^2 - i\lambda Pr)}}{2}, & r_4 &= \frac{SRePr - \sqrt{S^2 Re^2 Pr^2 + 4(\pi^2 - i\lambda Pr)}}{2} \\
 A_1 &= \frac{1 + hSRe}{(1 + hSRe) - e^{-SRe}}, & A_2 &= \frac{1}{e^{-SRe} - (1 + hSRe)}, & A_3 &= \frac{1}{1 - e^{-SprRe}} \\
 C &= (r_1 + \pi)(r_2 - \pi)(e^{-(\pi + r_2)} + e^{(\pi - r_1)}) - (r_1 - \pi)(r_2 + \pi)(e^{-(\pi + r_1)} + e^{(\pi - r_2)}) \\
 &\quad - 2\pi(r_2 - r_1)(e^{-(r_1 + r_2)} + 1) \\
 A_4 &= \frac{sr_1(\pi - r_2)e^{-(\pi + r_2)} - sr_2(\pi - r_1)e^{-(\pi + r_1)} - s\pi(r_1 - r_2)e^{-(r_1 + r_2)}}{C} \\
 A_5 &= \frac{sr_1(\pi + r_2)e^{(\pi - r_2)} - sr_2(\pi + r_1)e^{(\pi - r_1)} + s\pi(r_2 - r_1)e^{-(r_1 + r_2)}}{C} \\
 A_6 &= \frac{s\pi(\pi - r_2)e^{-(r_2 + \pi)} - s\pi(r_2 + \pi)e^{(\pi - r_2)} + 2s\pi r_2}{C} \\
 A_7 &= \frac{s\pi(\pi + r_1)e^{(\pi - r_1)} - s\pi(\pi - r_1)e^{-(\pi + r_1)} - 2s\pi r_1}{C}, & A_8 &= \frac{A_{15}e^{-r_4} - A_{16}}{e^{-r_4} - e^{-r_3}} \\
 A_9 &= \frac{A_{16} - A_{15}e^{-r_3}}{e^{-r_4} - e^{-r_3}}, & A_{10} &= A_3 Spr^2 Re^2, & A_{11} &= \frac{A_4 A_{10}}{(\pi - SprRe)^2 + SRePr(\pi - SRePr) - (\pi^2 - i\lambda Pr)}
 \end{aligned}$$

$$\begin{aligned}
 A_{12} &= \frac{A_5 A_{10}}{(\pi + SPrRe)^2 - SRePr(\pi + SRePr) - (\pi^2 - i\lambda Pr)}, & A_{13} &= \frac{A_6 A_{10}}{(r_1 + SPrRe)^2 - SRePr(r_1 + SPrRe) - (\pi^2 - i\lambda Pr)} \\
 A_{14} &= \frac{A_7 A_{10}}{(r_2 + SPrRe)^2 - SRePr(r_2 + SPrRe) - (\pi^2 - i\lambda Pr)}, & A_{15} &= -(A_{11} + A_{12} + A_{13} + A_{14}) \\
 A_{16} &= -(A_{11} e^{(\pi - sRePr)} + A_{12} e^{-(\pi + sRePr)} + A_{13} e^{-(r_1 + sRePr)} + A_{14} e^{-(r_2 + sRePr)}) \\
 A_{17} &= \frac{A_{29} e^{-r_2} - A_{30}(1+r_2h)}{e^{-r_2(1+r_1h)} - e^{-r_1(1+r_2h)}}, & A_{18} &= \frac{A_{30}(1+r_1h) - A_{29} e^{-r_1}}{e^{-r_2(1+r_1h)} - e^{-r_1(1+r_2h)}} \\
 A_{19} &= \frac{-ReGr A_8}{r_3^2 - SRe A_7 - (\pi^2 - i\lambda)}, & A_{20} &= \frac{-ReGr A_9}{r_4^2 - SRe A_8 - (\pi^2 - i\lambda)} \\
 A_{21} &= \frac{-ReGr A_{11}}{(\pi - SPrRe)^2 + SRe(\pi - SPrRe) - (\pi^2 - i\lambda)}, & A_{22} &= \frac{-ReGr A_{12}}{(\pi + SPrRe)^2 - SRe(\pi + SPrRe) - (\pi^2 - i\lambda)} \\
 A_{23} &= \frac{-ReGr A_{13}}{(r_1 + SPrRe)^2 - SRe(r_1 + SPrRe) - (\pi^2 - i\lambda)}, & A_{24} &= \frac{-ReGr A_{14}}{(r_2 + SPrRe)^2 - SRe(r_2 + SPrRe) - (\pi^2 - i\lambda)} \\
 A_{25} &= \frac{-SA_2 Re^2 A_4}{(\pi - SRe)^2 + SRe(\pi - SRe) - (\pi^2 - i\lambda)}, & A_{26} &= \frac{-SA_2 Re^2 A_5}{(\pi + SRe)^2 - SRe(\pi + SRe) - (\pi^2 - i\lambda)} \\
 A_{27} &= \frac{-SA_2 Re^2 A_6}{(r_1 + SRe)^2 - SRe(r_1 + SRe) - (\pi^2 - i\lambda)}, & A_{28} &= \frac{-SA_2 Re^2 A_7}{(r_2 + SRe)^2 - SRe(r_2 + SRe) - (\pi^2 - i\lambda)} \\
 A_{29} &= -A_{19}(hr_3 + 1) - A_{20}(hr_4 + 1) + A_{21}(h(\pi - SPrRe) - 1) - A_{22}(h(\pi + SPrRe) + 1) \\
 &\quad - A_{23}(h(r_1 + SPrRe) + 1) - A_{24}(h(r_2 + SPrRe) + 1) + A_{25}(h(\pi - SRe) - 1) \\
 &\quad - A_{26}(h(\pi + SRe) + 1) - A_{27}(h(r_1 + SRe) + 1) - A_{28}(h(r_2 + SRe) + 1) \\
 A_{30} &= -[A_{19} e^{-r_3} + A_{20} e^{-r_4} + A_{21} e^{\pi - SPrRe} + A_{22} e^{-(\pi + SPrRe)} + A_{23} e^{-(r_1 + SPrRe)} + A_{24} e^{-(r_2 + SPrRe)} + \\
 &\quad A_{25} e^{\pi - SRe} + A_{26} e^{-(\pi + SRe)} + A_{27} e^{-(r_1 + SRe)} + A_{28} e^{-(r_2 + SRe)}]
 \end{aligned}$$

#### IV. RESULT AND DISCUSSION

The study of couette flow through porous plates with heat transfer and slip boundary condition has attracted the attention of number of researchers because of its possible applications in several areas. In the previous section, we have studied the unsteady couette flow and heat transfer between two horizontal parallel porous flat plates with periodic suction at the stationary plate and constant injection at the plate in motion. The periodic suction velocity was assumed to be time dependent and perpendicular to the flow direction. Due to the periodic suction, the flow becomes three dimensional. Theoretical expressions were found to calculate temperature and velocity profile.

In order to understand clearly the influence of non dimensional parameters such as Reynolds Number, Prandtl Number, Suction parameter and Frequency parameter on these profiles, we have calculated velocity and temperature profile, Skin friction and Nusselt Number numerically and plotted the results.

From figures 1,2 and 3, it can be seen that the qualitative behavior of the main flow velocity does not change due to the presence of slip regime. The main flow velocity increases as Re or S increases and decreases with the decrease in the frequency parameter.

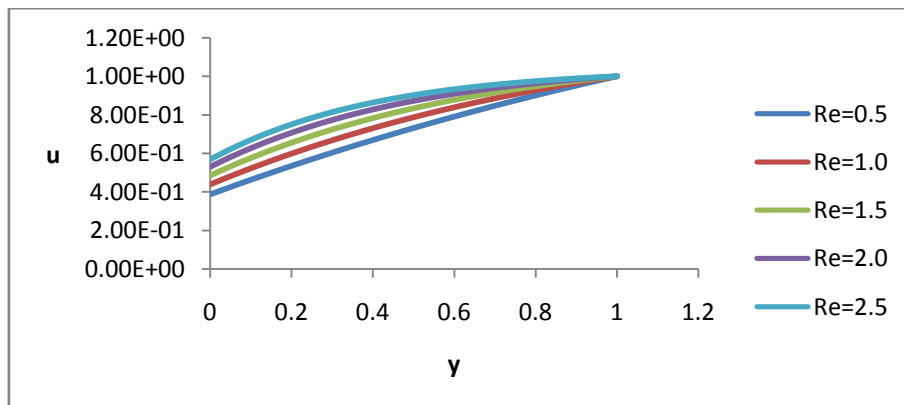


Figure 1. Main velocity u for  $\lambda=6.0$ ,  $S=1.0$ ,  $t=0.2$ ,  $Z=0.0$ ,  $\varepsilon = 0.2$  at  $h=0.5$

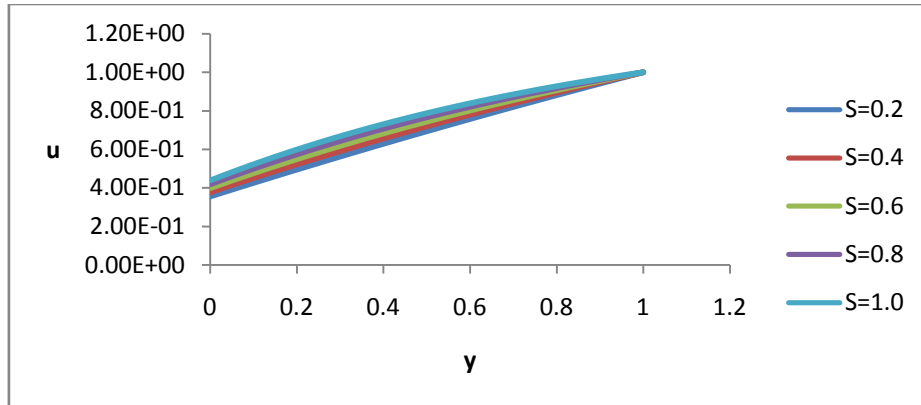


Figure 2. Main velocity  $u$  for  $Re=1.0, \lambda=6.0,$   
 $t=0.2, Z=0.0, \epsilon = 0.2$  at  $h=0.5$

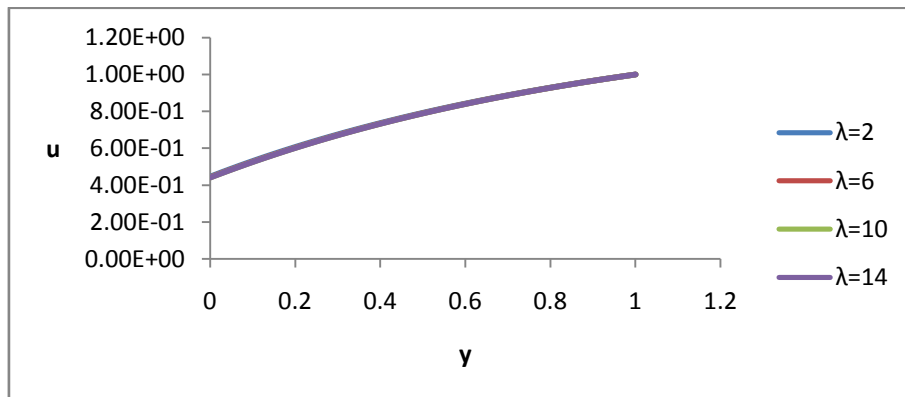


Figure 3. Main velocity  $u$  for  $Re=1.0 S=1.0,$   
 $t=0.2, Z=0.0, \epsilon = 0.2$  at  $h=0.5$

Figures 4 and 5 illustrate the behavior of the cross flow velocity as a function of various non dimensional numbers. It is observed that the magnitude of the cross flow velocity  $w$  increases with the increase in  $S$ , but it increases near the stationary plate and decreases near the moving plate with the increases in  $Re$ . This is due to the fact that suction at the stationary plate and injection at the moving plate are two exactly opposite process.

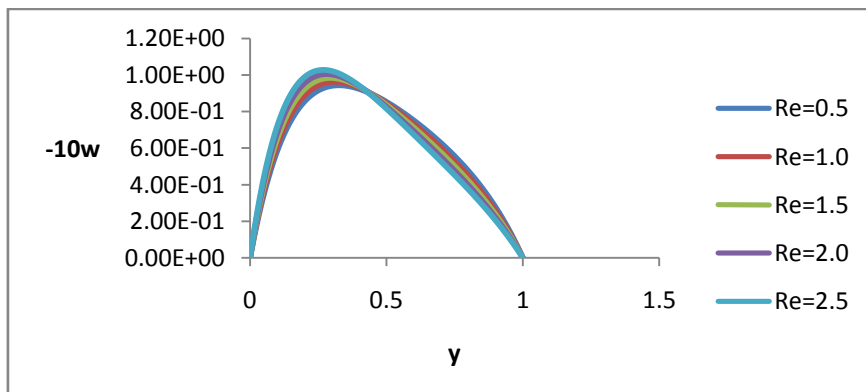


Figure 4. Cross velocity  $-10w$  for  $S=1.0,$   
 $\lambda=6.0, t=0.2, Z=0.5, \epsilon = 0.2$  at  $h=0.5$

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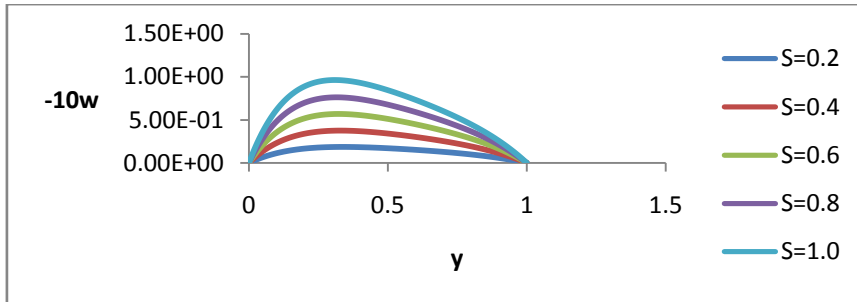


Figure 5. Cross velocity  $-10w$  for  $Re=1.0$ ,  $\lambda=6.0$ ,  $t=0.2$ ,  $Z=0.5$ ,  $\varepsilon = 0.2$  at  $h=0.5$

Figures 6,7,8 and 9 depict the behavior of the temperature profile with respect to the non dimensional parameter  $Re$  and  $S$ . To use the results in practical situations, the Prandtl number is assumed to be 7.0. It is observed that the temperature increases with the increase in frequency parameter. On the other hand, it increases near the stationary plate and decreases away from the stationary plate with the increase in Reynolds number. The wall shear stress at the stationary plate is found to be increasing with increase in the slip parameter. Increase in the Reynolds number increases the skin friction while increase in the frequency parameter decreases the skin friction. These results can be observed from figures 10,11 and 12. From figures 13, 14 and 15 we can observe that the skin friction due to cross flow increases with increase in Reynolds number. The skin friction at the upper plate  $y=1$ , increases with increase in Reynolds number and exhibit an oscillatory behavior with increasing  $S$ .

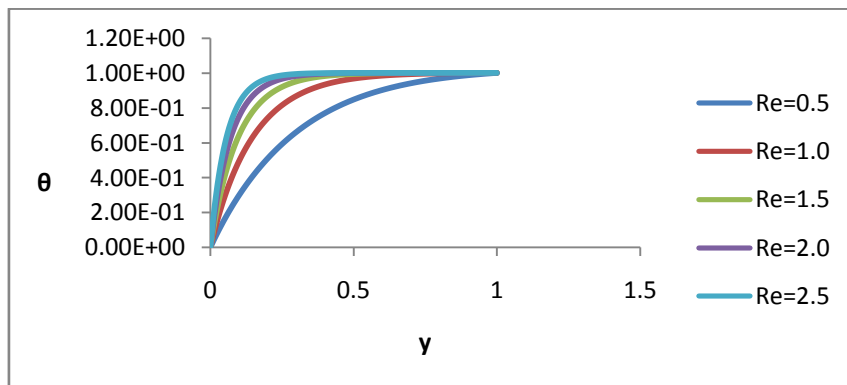


Figure 6. Temperature  $\theta$  for  $S=1.0$ ,  $\lambda=6.0$ ,  $t=0.2$ ,  $Z=0.5$ ,  $\varepsilon = 0.2$  at  $h=0.0$ ,  $Pr=7.0$

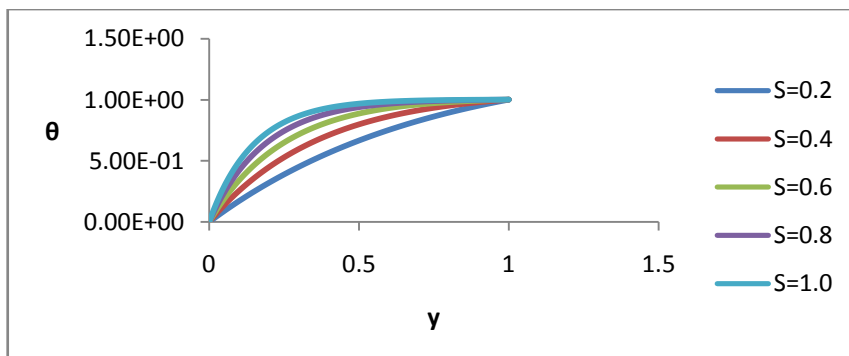


Figure 7. Temperature  $\theta$  for  $Re=1.0$ ,  $\lambda=6.0$ ,

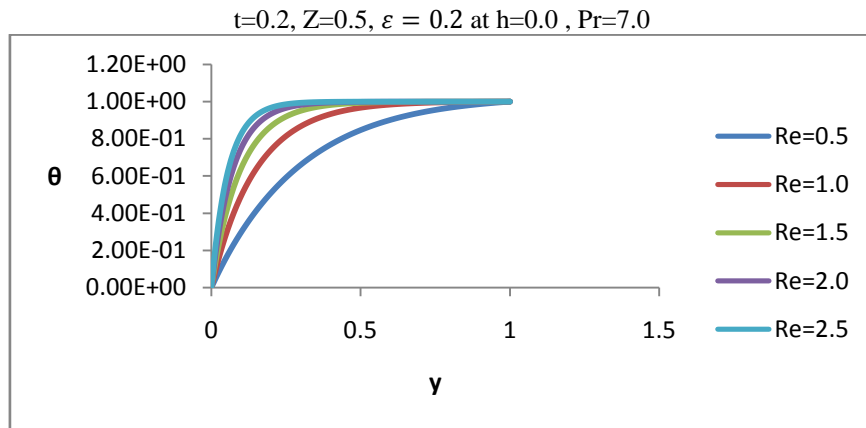


Figure 8. Temperature  $\theta$  for  $S=1.0, \lambda=6.0,$   
 $t=0.2, Z=0.5, \varepsilon = 0.2$  at  $h=0.5, Pr=7.0$

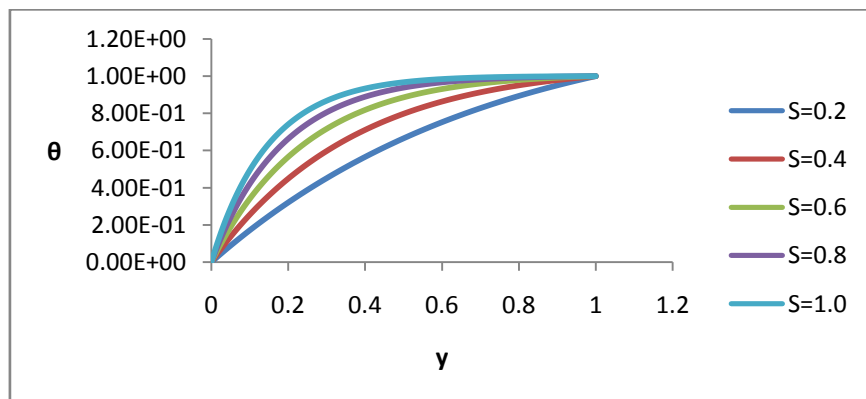


Figure 9. Temperature  $\theta$  for  $Re=1.0, \lambda=6.0,$   
 $t=0.2, Z=0.5, \varepsilon = 0.2$  at  $h=0.5, Pr=7.0$

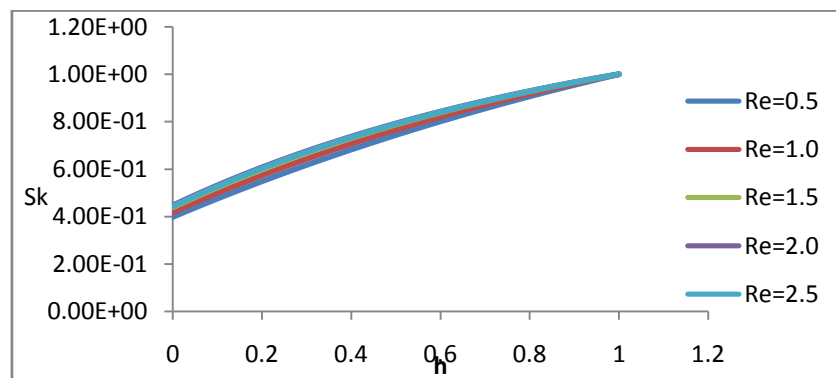


Figure 10. Skin Friction  $Sk$  for  $S=1.0,$   
 $\lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=0$



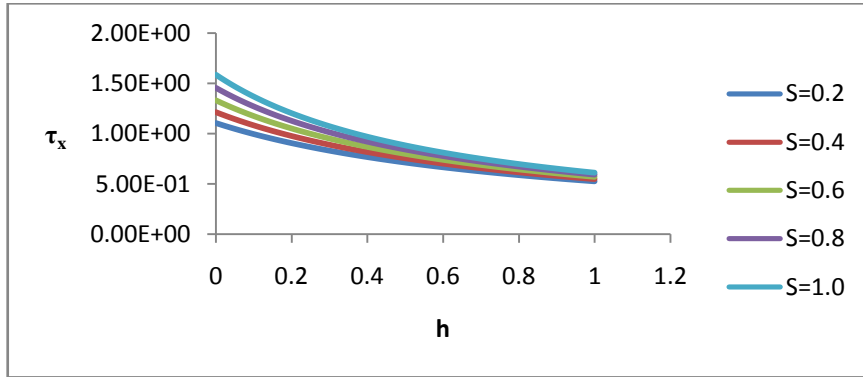


Figure11. Skin Friction due to main flow  $\tau_x$  for  $Re=1.0, \lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=0$

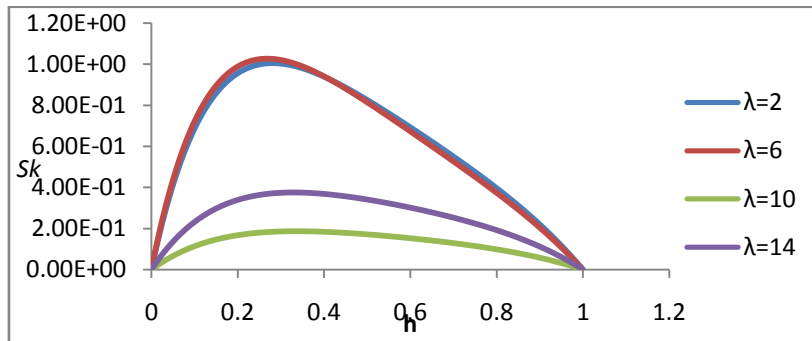


Figure12. Skin Friction due to main flow  $\tau_x$  for  $S=1.0, Re=1.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=0$

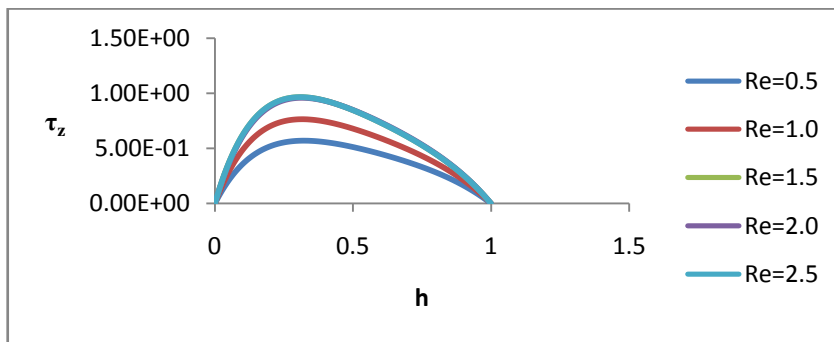


Figure 13. Skin friction due to Cross flow  $\tau_z$  for  $S=1.0, \lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=0$ .

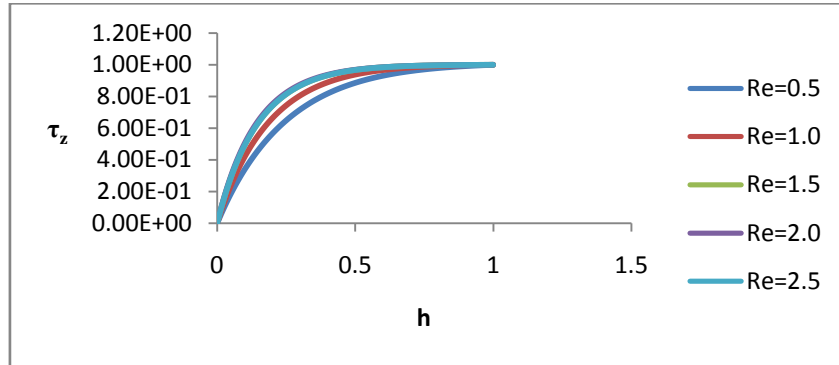


Figure 14.Skin friction due to Cross flow  $\tau_z$  for  $S=1.0, \lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=1$ .

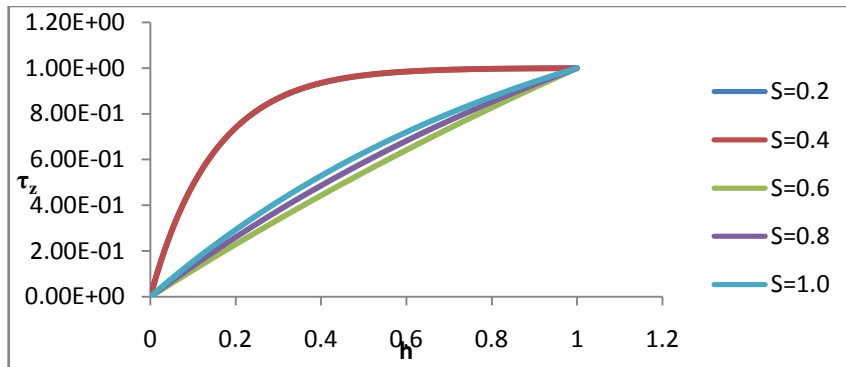


Figure 15.Skin friction due to Cross flow  $\tau_z$  for  $Re=1.0, \lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=1$ .

A very important flow characteristics, the heat flux at the plate is measured in terms of Nusselt number which is shown in figures 16 and 17. Increase in Prandtl number decreases the heat flux at the moving plate. It is also seen that the increase in the Prandtl number increases the rate of heat transfer at the upper plate. Increase in the Suction parameter decreases the rate of heat transfer and the frequency parameter does not influence the rate of heat transfer significantly.

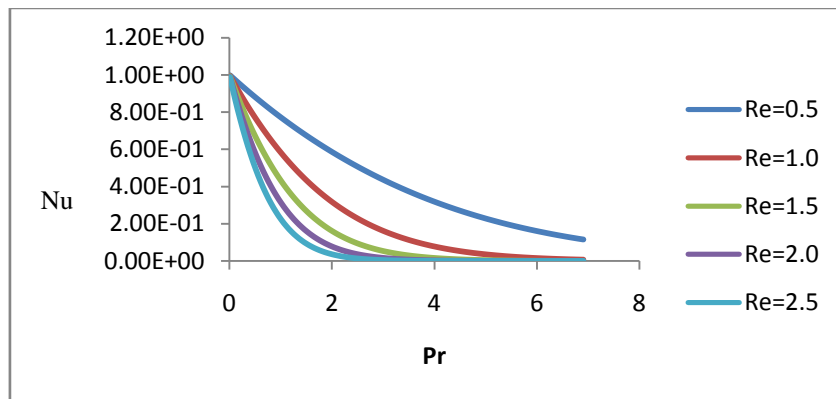


Figure 16.Nusselt Number  $Nu$  for  $S=1.0, \lambda=6.0, t=0.2, Z=0.5, \varepsilon = 0.2$  at  $y=1$ .

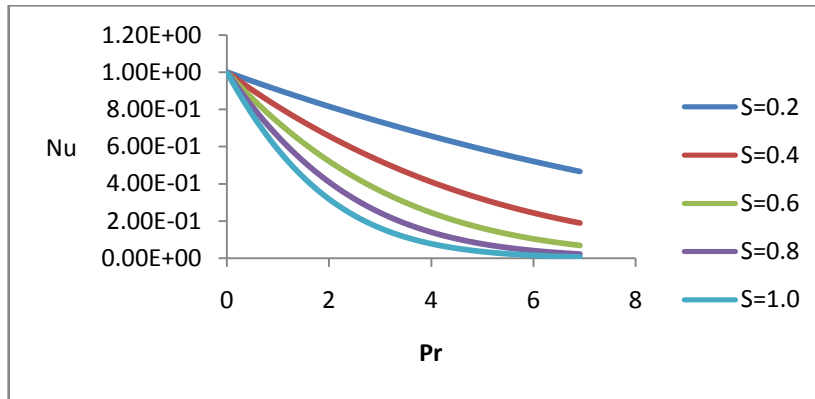


Figure 17. Nusselt Number Nu for  $Re=1.0$ ,  $\lambda=6.0$ ,  $t=0.2$ ,  $Z=0.5$ ,  $\varepsilon = 0.2$  at  $y=1$ .

## V. CONCLUSION

Fluctuating flows are known to result in higher rates of heat and mass transfer. Many studies have been carried out to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors. It can be seen from the literature the studies pertaining to three dimensional, couette flow through porous channels with slip boundary conditions have not received much attention.

Hence, in this paper, we have extended the work of Guria and Jana [1] to investigate the effect of velocity slip of couette flow past an infinite vertical porous channel. We have assumed that the upper plate is subjected to a constant injection  $-V_0$  and the stationary plate is subjected to a transverse sinusoidal suction velocity distribution. Analytical expressions were found to determine velocity and temperature profiles using perturbation techniques.

It can be seen that the qualitative behavior of the main flow velocity does not change due to the presence of slip regime. The main flow velocity increases as  $Re$  or  $S$  increases and decreases with the decrease in the frequency parameter. The wall shear stress at the stationary plate is found to be increasing with increase in the slip parameter.

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