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The Effect of Magnetic Field and Thermal Relaxation Time on Three Dimensional Thermal Shock Problem in Generalized Thermoelasticity

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ABSTRACT: The purpose of this paper is to scrutinize the effect of magnetic field and thermal relaxation time on a general three-dimensional model of the equations of generalized magneto-thermoelasticity in a homogeneous, isotropic, thermally and perfectly conducting elastic half-space solid whose surface is subjected to a thermal shock in the context of the Lord-Shulman theory. The normal mode analysis is used to obtain the expressions for the displacement, thermal stress, strain and temperature. Some comparisons displayed graphically to illustrate the effects of magnetic field and relaxation time on the physical quantities. Such problems are very important in many dynamical systems and some particular cases of special interest have been deduced from the present investigation.

KEYWORDS: Magneto-thermoelasticity, Three-dimensional modeling, Thermal shock problem, Normal mode method, Lord-Shulman theory.

I. INTRODUCTION

The theory of thermoelasticity deals with the effect of mechanical and thermal disturbances on an elastic body. The theory of uncoupled thermoelasticity consists of the heat equation, which is independent of mechanical effects, and the equation of motion, which contains the temperature as a known function. There are two defects in this theory. First is that the equation of heat conduction of this theory dose not contain any elastic terms . Second, the heat equation, which is parabolic, implies that the speed of propagation of the temperature is infinite, which contradicts physical experiments.

Biot [1] introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic. To overcome this drawback, two generalizations to the coupled theory were introduced.

The first is due to Lord and Shulman [2], who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. This new law contains the heat flux vector as well as its time derivative. It contains also a new constant that act as a relaxation time. Since the heat equation of this theory is of the wave-type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories.

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Mullar [3], in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green

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and Laws [4]. Green and Lindsay [5] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [6]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry.

In the 1990's Green and Naghdi [7]-[12], proposed three new thermoelastic theories based on entropy balance rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories which are called thermoelasticity of type I, of type II and of type III.

When the type I theory is linearized we obtain the classical system of thermoelasticity. The type II theory (is limiting case of type III) does not admit energy dissipation.

Now, there is a great deal of interest in the study of interaction between electromagnetism and thermoelasticity due to its wide use in nuclear industry. Increasing attention is being devoted to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics, and related topics. The foundations of magnetoelasticity were presented by Knopoff [13] and Chadwick [14] and developed by Kaliski and Petykiewicz [15].

In all papers quoted above it was assumed that the interactions between the two fields take place by means of the Lorentz forces appearing in the equations of motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field.

Many authors have considered the propagation of electromagneto-thermoelastic waves in an electrically and thermally conducting solid. Paria [16] discussed the propagation of plane magneto-thermoelastic waves in an isotropic unbounded medium under the influence of a uniform thermal field and with a magnetic field acting transversely to the direction of the propagation. Paria used the classical Fourier law of heat conduction, and neglected the electric displacement. Wilson [17] extended Paria's results by introducing a component of the magnetic field parallel to the direction of the propagation. A comprehensive review of the earlier contributions to the subject can be found in Paria [18]. Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Namat-Nasser [19] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. Choudhuri [20] extended these results to rotating media. Elnaggar et al. [21] discussed The influence of magnetic field, initial stress and gravity field in an isotropic material. Abd-Alla et al. [22] introduced the effect of the stress, temperature and magnetic fields in an isotropic elastic cylinder in a primary magnetic field. Baksia el al. [23] studied the effect of rotation and relaxation time in generalized magneto-thermo-viscoelastic medium in one dimension. Othman and Song [24] proposed the effect of rotation on plane waves of the generalized electromagneto-thermo-viscoelasticity with two relaxation times. Othman et al. [25] presented the effect of rotation on the generalized magneto-thermo-viscoelastic plane waves without energy dissipation. Aouadi [26], discussed some theorems in the generalized theory of thermo-magneto-electroelasticity under Green–Lindsay's model. Othman and Kumar [27] studied the reflection plane harmonic waves in magneto generalized thermoelasticity theories. Abd-Alla et al. [28] discussed the influences of rotation, magnetic field, initial stress, and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Othman and Atwa [29,30] studied the effect of magnetic field on 2-D problem of generalized thermoelasticity with energy dissipation. Deswal and Kalkal [31] introduced a two-dimensional generalized electro-magneto-thermoviscoelastic problem for a half-space with diffusion. Xiong and Tian [32] proposed a two-dimensional thermoelastic problem of an infinite magneto-microstretch homogeneous isotropic plate. Singh et al. [33] solved The governing equations of generalized magneto-thermoelasticity with hydrostatic initial stress for surface wave solutions. Othman et al. [34] discussed the effect of fractional parameter on plane waves of generalized magneto–thermoelastic diffusion with reference temperature-dependent elastic medium and also discussed the effect of magnetic field and rotation on generalized thermo-microstretch elastic solid with mode-I crack under the Green Naghdi theory [35].

Recently, Zakaria [36] depicted the effect of hall current on generalized magneto-thermoelasticity micropolar solid subjected to ramp-type heating. Atwa [37] studied the problem of generalized magneto-thermoelasticity with two temperature and initial stress under Green–Naghdi theory. Abo-Dahab et al. [38] discussed the effect of magnetic field, initial stress and thermal relaxation times on SV-waves incidence at interface between solid-liquid media. Othman et al. [39] presented the effect of magnetic field on wave propagation in generalized thermo micro-stretch for a homogeneous

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isotropic elastic half space solid with mode-I crack. Othman et al. [40] discussed the effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation.
The purpose of the present paper is to study the effect of magnetic field and thermal relaxation time on the general three-dimensional model of the equations of the generalized thermoelasticity for a homogeneous isotropic elastic half-space solid in the context of Lord-Shulman theory without any body forces or heat sources. The effect of magnetic field and thermal relaxation time on different characteristics is shown graphically for generalized thermoelasticity.

II. GOVERNING EQUATIONS AND FORMULATION OF THE PROBLEM

We consider an isotropic, homogeneous, thermally and perfectly conducting elastic medium. The whole body is at constant temperature T_0 and is acted on throughout by a constant initial magnetic field $\boldsymbol{H}=\left(0,H_0,0\right)$ oriented in the positive direction of the *y*-axis (as shown in Fig. 1) and due to the application of this initial magnetic field, there results an induced magnetic field $\boldsymbol{h}=\left(h_x,h_y,h_z\right)$ and an induced electric field $\boldsymbol{E}=\left(E_x,0,E_z\right)$. Maxwell 's equations for homogeneous isotropic material (Strictly speaking, when the material is subjected to magnetic fields and thermal field the material will not remain homogeneous and isotropic; this variation is ignored in this investigation) are given by:

$$curl\ \boldsymbol{h}=\boldsymbol{J}+\varepsilon_0\dot{\boldsymbol{E}}, \tag{1}$$

$$curl\ \boldsymbol{E}=-\mu_0\dot{\boldsymbol{h}}, \tag{2}$$

$$div\ \boldsymbol{h}=0, \tag{3}$$

$$\boldsymbol{E}=-\mu_0(\dot{\boldsymbol{u}}\times\boldsymbol{H}). \tag{4}$$

Where $\boldsymbol{E}$, $\boldsymbol{h}$, $\boldsymbol{J}$, μ_0, ε_0, $\boldsymbol{u}$, t are the electric intensity, the magnetic intensity, the current density vector, the magnetic permeability, the electric permeability, the displacement vector and the time.
The basic equations for electro-magneto-thermoelasticity for a homogeneous isotropic solid in the context of the generalized thermoelasticity of the Lord-Shulman theory are:
i. Equation of motion:
The dynamical equations of motion for the three-dimensional problem taking into consideration the Lorentz force

$$\rho\ddot{u}=\sigma_{ij,j}+F_i, \tag{5}$$

where F_i is the Lorentz force and take the form

$$F_i=\mu_0(\boldsymbol{J}\times\boldsymbol{H})_i. \tag{6}$$

ii. Heat conduction equation:

$$KT_{,ii}=\rho C_E(\dot{T}+\tau_0\ddot{T})+\gamma T_0(\dot{e}+\tau_0\ddot{e}). \tag{7}$$

iii. Stress-displacement-temperature relation:

$$\sigma_{ij}=\lambda e_{kk}\delta_{ij}+2\mu e_{ij}-\gamma T\delta_{ij}. \tag{8}$$

iv. Strain-displacement relation

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}). \tag{9}$$

In the preceding equations, λ and μ are Lame's constant, ρ is the density, σ_{ij} are the components of the stress tensor, $u_i=(u,v,w)$ are the components of the displacement vector, t is the time variable, T is the absolute temperature, γ is a material constant given by $\gamma=(3\lambda+2\mu)\alpha_T$ where α_T is the coefficient of linear thermal expansion, K is the thermal conductivity, C_E is the specific heat at constant strain, τ_0 is the thermal relaxation time, T_0 is the temperature of the medium in its natural state, assumed to be such that $\left|(T-T_0)/T_0\right|<<1$, and

$$e=e_{xx}+e_{yy}+e_{zz}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}. \tag{10}$$

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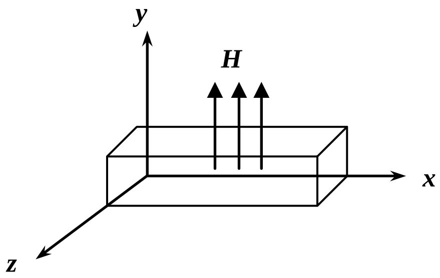

Fig. 1. Geometry of the problem

Here we consider plane waves propagating in plane such that all particles on a line parallel to *y*-axis are equally displaced. Therefore, all the field quantities will be independent of *y* coordinate, i.e. all partial derivatives with respect to y vanish.

From Eqs. (1)-(4) , and with taking into consideration the above assumption we can take the induced magnetic field as $\boldsymbol{h} = (0, h, 0)$ where

$$h = -H_0 e, \tag{11}$$

By using Eqs. (1)-(4) and (6), we get

$$\boldsymbol{E} = \mu_0 H_0 (\dot{w}, 0, -\dot{u}), \tag{12}$$

$$\boldsymbol{J} = (-\frac{\partial h}{\partial z} - \varepsilon_0 \mu_0 H_0 \ddot{w},\ 0\ , \frac{\partial h}{\partial x} + \varepsilon_0 \mu_0 H_0 \ddot{u}), \tag{13}$$

and the external force can be represented as

$$\boldsymbol{F} = -\mu_0 H_0 (\frac{\partial h}{\partial x} + \varepsilon_0 \mu_0 H_0 \ddot{u},\ 0\ , \frac{\partial h}{\partial z} + \varepsilon_0 \mu_0 H_0 \ddot{w}), \tag{14}$$

from Eqs. (5)- (8) and (13) we get

$$\mu \nabla^2 u + (\lambda + \mu)\frac{\partial e}{\partial x} - \gamma \frac{\partial T}{\partial x} - \mu_0 H_0 \frac{\partial h}{\partial x} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{u} = \rho \ddot{u}, \tag{15}$$

$$\mu \nabla^2 v + (\lambda + \mu)\frac{\partial e}{\partial y} - \gamma \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{16}$$

$$\mu \nabla^2 w + (\lambda + \mu)\frac{\partial e}{\partial z} - \gamma \frac{\partial T}{\partial z} - \mu_0 H_0 \frac{\partial h}{\partial z} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{w} = \rho \ddot{w}, \tag{17}$$

$$K \nabla^2 T = \rho C_E (\dot{T} + \tau_0 \ddot{T}) + \gamma T_0 (\dot{e} + \tau_0 \ddot{e}) . \tag{18}$$

The constitutive relations can be written as

$$\sigma_{xx} = \lambda e + 2\mu \frac{\partial u}{\partial x} - \gamma T, \tag{19}$$

$$\sigma_{yy} = \lambda e + 2\mu \frac{\partial v}{\partial y} - \gamma T, \tag{20}$$

$$\sigma_{zz} = \lambda e + 2\mu \frac{\partial w}{\partial z} - \gamma T, \tag{21}$$

$$\sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{22}$$

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$$\sigma_{xz} = \mu \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \tag{23}$$

$$\sigma_{yz} = \mu \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \tag{24}$$

For convenience, we will transform the above equations in non-dimensional forms, so the following non-dimensional variables are used:

$$(x',y',z')=\frac{\varpi}{C_1}(x,y,z), \quad (u',v',w') = \frac{\rho C_1 \varpi}{\gamma T_0}(u,v,w), \quad \{t',\tau_0'\}=\varpi\{t,\tau_0\},$$
$$T'=\frac{T}{T_0}, \quad \sigma'_{ij}=\frac{\sigma_{ij}}{\gamma T_0}, \quad h'=\frac{h}{H_0}, \quad C_1^2=\frac{(\lambda+2\mu)}{\rho}, \quad \varpi=\frac{C_E(\lambda+2\mu)}{K}, \tag{25}$$

Eqs. (15)–(24) in the non-dimensional forms (after suppressing the primes) reduce to

$$\beta\nabla^2 u+\varepsilon_2\frac{\partial e}{\partial x}-\frac{\partial T}{\partial x}=\varepsilon_1\ddot{u}, \tag{26}$$

$$\beta\nabla^2 v+(1-\beta)\frac{\partial e}{\partial y}-\frac{\partial T}{\partial y}=\frac{\partial^2 v}{\partial t^2}, \tag{27}$$

$$\beta\nabla^2 w+\varepsilon_2\frac{\partial e}{\partial z}-\frac{\partial T}{\partial z}=\varepsilon_1\ddot{w}, \tag{28}$$

$$\nabla^2 T=(1+\tau_0\frac{\partial}{\partial t})\frac{\partial T}{\partial t}+\varepsilon_3(1+\tau_0\frac{\partial}{\partial t})\frac{\partial e}{\partial t}. \tag{29}$$

$$\sigma_{xx}=2\beta\frac{\partial u}{\partial x}+(1-2\beta)\,e-T\ , \tag{30}$$

$$\sigma_{yy}=2\beta\frac{\partial v}{\partial y}+(1-2\beta)\,e-T\ , \tag{31}$$

$$\sigma_{zz}=2\beta\frac{\partial w}{\partial z}+(1-2\beta)\,e-T\ , \tag{32}$$

$$\sigma_{xy}=\beta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{33}$$

$$\sigma_{xz}=\beta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \tag{34}$$

$$\sigma_{yz}=\beta\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \tag{35}$$

where

$$\beta=\frac{\mu}{\lambda+2\mu}, \quad \varepsilon_1=1+\frac{\varepsilon_0\mu_0^2H_0^2}{\rho}, \quad \varepsilon_2=1-\beta+R_H\ , \quad \varepsilon_3=\frac{\gamma^2T_0}{\rho^2C_1^2C_E}, \quad R_H=\frac{\mu_0H_0^2}{\rho C_1^2}.$$

From Eqs. (30) – (32) by addition, we get

$$\sigma=\alpha\,e-T, \tag{36}$$

where $\sigma=\dfrac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3}, \qquad \alpha=\dfrac{3-4\beta}{3}.$

We may separate out the purely dilatational and purely rotational disturbances associated with the components *u*, *v* and *w* by introducing the two displacement potentials φ and ψ, which are functions of *x, y, z* and *t,* in the form

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$$\mathbf{u}(u,v,w)=\nabla\varphi+\nabla\wedge\psi,\quad \text{i.e.}\quad u=\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial z}\quad,\quad w=\frac{\partial\varphi}{\partial z}-\frac{\partial\psi}{\partial x}, \tag{37}$$

whence $\nabla^2\varphi=\nabla.\mathbf{u}=e$; e is the dilatation, and $\nabla^2\psi=\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}$.

From Eqs. (37) we can rewrite Eq. (11) as

$$h=-H_0\nabla^2\varphi. \tag{38}$$

By using Eqs. (37) into Eqs. (26)-(29), we obtain

$$(\varepsilon_4\nabla^2-\varepsilon_1\frac{\partial^2}{\partial t^2})\varphi-T=0, \tag{39}$$

$$(\beta\nabla^2-\varepsilon_1\frac{\partial^2}{\partial t^2})\psi=0, \tag{40}$$

$$(\beta\nabla^2-\frac{\partial^2}{\partial t^2})v=0, \tag{41}$$

$$\left[\nabla^2-(1+\tau_0\frac{\partial}{\partial t})\frac{\partial}{\partial t}\right]T-\left[\varepsilon_3(1+\tau_0\frac{\partial}{\partial t})\frac{\partial}{\partial t}\right]\nabla^2\varphi=0. \tag{42}$$

where $\varepsilon_4=1+R_H$.

III. THE SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of normal modes as in the following form

$$(\varphi,\psi,v,T,e,\sigma_{ij})(x,y,z,t)=(\varphi^*,\psi^*,v^*,e^*,T^*,\sigma_{ij}^*)(z)\,e^{\omega t-i(ax+by)}, \tag{43}$$

where $i=\sqrt{-1}$, ω is the angular frequency and a, b are the wave numbers in the x and y-directions respectively and $\varphi^*,\psi^*,v^*,T^*,e^*$ and σ_{ij}^* are the amplitudes of the field quantities.

Using Eq. (33), Eqs. (29) - (32) take the form

$$\left(D^2-A_1\right)\varphi^*-A_2T^*=0, \tag{44}$$

$$\left(D^2-A_3\right)\psi^*=0, \tag{45}$$

$$\left(D^2-A_4\right)v^*=0, \tag{46}$$

$$\left(D^2-A_5\right)T^*-A_6\left(D^2-a^2\right)\varphi^*=0, \tag{47}$$

where $D=\frac{d}{dz}$, $A_1=a^2+\frac{\varepsilon_1\omega^2}{\varepsilon_4}$, $A_2=\frac{1}{\varepsilon_4}$, $A_3=a^2+\frac{\varepsilon_1\omega^2}{\beta}$, $A_4=a^2+\frac{\omega^2}{\beta}$, $A_5=a^2+\omega(1+\tau_0\omega)$, $A_6=\varepsilon_3\omega(1+\tau_0\omega)$.

Eliminating T^* between Eqs. (44) , (47) we get the following fourth order differential equation for $\varphi^*(z)$

$$[D^6-AD^4+BD^2-C]\varphi^*(z)=0. \tag{48}$$

In a similar manner, we can show that $T^*(z)$ satisfy the equation

$$[D^6-AD^4+BD^2-C]T^*(z)=0, \tag{49}$$

where

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$A = A_1 + A_5 + A_2 A_6$, $\quad B = A_1 A_5 + a^2 A_2 A_6$,

Eq. (48) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)\varphi^*(z) = 0, \tag{50}$$

where $k_j^2, (j = 1, 2)$ are the roots of the characteristic equation of Eq. (50) which satisfy that

$k_1^2 + k_2^2 = A$, and $k_1^2 k_2^2 = B$.

The solution of Eqs. (45) and (50), which are bounded as $z \to \infty$, can be written as

$$\psi^*(z) = G(a,b,\omega)\, e^{-A_7 z} \tag{51}$$

$$\varphi^*(z) = \sum_{j=1}^{2} M_j(a,b,\omega) e^{-k_j z}, \tag{52}$$

similarly

$$T^*(z) = \sum_{j=1}^{2} H_j\, M_j(a,b,\omega)\, e^{-k_j z}, \tag{53}$$

where $A_7 = \sqrt{A_3}$, G and M_j are some parameters depending on *a*, *b* and *ω*, and $H_j = \dfrac{k_j^2 - A_1}{A_2}$.

From Eqs. (51)-(53) and (36) we can obtain

$$u^*(z) = -ia \sum_{j=1}^{2} M_j\, e^{-k_j z} - A_7\, G\, e^{-A_7 z}, \tag{54}$$

$$w^*(z) = -\sum_{j=1}^{2} k_j M_j\, e^{-k_j z} + ia\, G\, e^{-A_7 z}, \tag{55}$$

$$e^*(z) = \sum_{j=1}^{2} (k_j^2 - a^2)\, M_j\, e^{-k_j z}, \tag{56}$$

$$\sigma^*(z) = \sum_{j=1}^{2} (\alpha k_j^2 - \alpha a^2 - H_j)\, M_j e^{-k_j z}. \tag{57}$$

IV. Thermal Shock Application

In order to complete the solution we need to know the parameters M_j and G, so we will consider the following non-dimensional boundary conditions at the surface *z = 0*:

(i) The thermal boundary condition is that the surface of the half space is subjected to a thermal shock:

$$T(x,y,0,t) = f\,(x,y,0,t) = f^{\,*} e^{\omega t - i(ax+by)}, \tag{58}$$

(ii) The mechanical boundary condition is that the surface of the half space is traction free:

$$\sigma_{mz}(x,y,0,t) = 0 \qquad (m = x,y,z), \tag{59}$$

where $f\,(x,y,t)$ is an arbitrary function and $f^{\,*}$ is a constant.

Using Eqs. (53), (32) and (34) and substituting the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by parameters:

$$\sum_{j=1}^{2} H_j M_j = f^{\,*}, \tag{60}$$

$$\sum_{j=1}^{2} 2iak_j M_j + (A_3 + a^2) G = 0. \tag{61}$$

$$\sum_{j=1}^{2} (k_j^2 - a^2 + 2a^2\beta - H_j) M_j - 2ia\beta\, A_7\, G = 0, \tag{62}$$

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Solving the system of Eqs. (60) - (62), we get

$$M_1 = \frac{\Delta_1}{\Delta}, \qquad M_2 = \frac{\Delta_2}{\Delta}, \qquad G = \frac{\Delta_3}{\Delta}, \tag{63}$$

where

$$\Delta = H_1\left[4a^2\beta\, k_2 A_7 - \left(A_3 + a^2\right)\left(k_2^2 - a^2 + 2a^2\beta - H_2\right)\right] - H_2\left[4a^2\beta\, k_1 A_7 - \left(A_3 + a^2\right)\left(k_1^2 - a^2 + 2a^2\beta - H_1\right)\right],$$

$$\Delta_1 = f^*\left[4a^2\beta k_2 A_7 - \left(A_3 + a^2\right)\left(k_2^2 - a^2 + 2a^2\beta - H_2\right)\right],$$

$$\Delta_2 = -f^*\left[4a^2\beta k_1 A_7 - \left(A_3 + a^2\right)\left(k_1^2 - a^2 + 2a^2\beta - H_1\right)\right],$$

$$\Delta_3 = f^*\left[2iak_1\left(k_2^2 - a^2 + 2a^2\beta - H_2\right) - 2iak_2\left(k_1^2 - a^2 + 2a^2\beta - H_1\right)\right].$$

V. NUMERICAL RESULTS AND DISCUSSIONS

In order to illustrate the theoretical results obtained in the preceding section, we now present some numerical results. In the calculation process, we take the case of copper material. Since ω is complex, we take $\omega = \omega_0 + i\zeta$, where i is the imaginary number. The numerical constants of the problem were taken as:

$\lambda = 7.76\times10^{10}\, kg\ m^{-1}s^{-2}$, $\mu = 3.86\times10^{10}\, kg\ m^{-1}s^{-2}$, $\rho = 8.954\times10^{3}\, kg\ m^{-3}$, $C_E = 0.3831\times10^{3}\ m^2 k^{-1}s^{-2}$, $\varepsilon_0 = 10^{-9}/36\pi\ Fm^{-1}$, $\mu_0 = 4\pi\times10^{-7}\ Hm^{-1}$, $T_0 = 293k$, $K = 386\ kg\ m\ k^{-1}s^{-3}$, $\alpha_T = 1.78\times10^{-5}\, k^{-1}$, $\omega_0 = 2.5$, $\zeta = 0.1$, $a = 0.8$, $b = 1.2$, $f^* = 1$.

Figures 2-5 represented 2D curves for the distributions of the real part of the displacement component u, stress σ, strain e and the temperature T against the distance z with different values of $H_0 = 0,\ 10^9,\ 10^{10}$ (with a fixed value of $\tau_0 = 0.02$). In these figures, the solid line, dashed line and dotted line correspond for $H_0 = 0,\ 10^9,\ 10^{10}$ respectively, which is furthermore precisely explained in each figure in the legend.

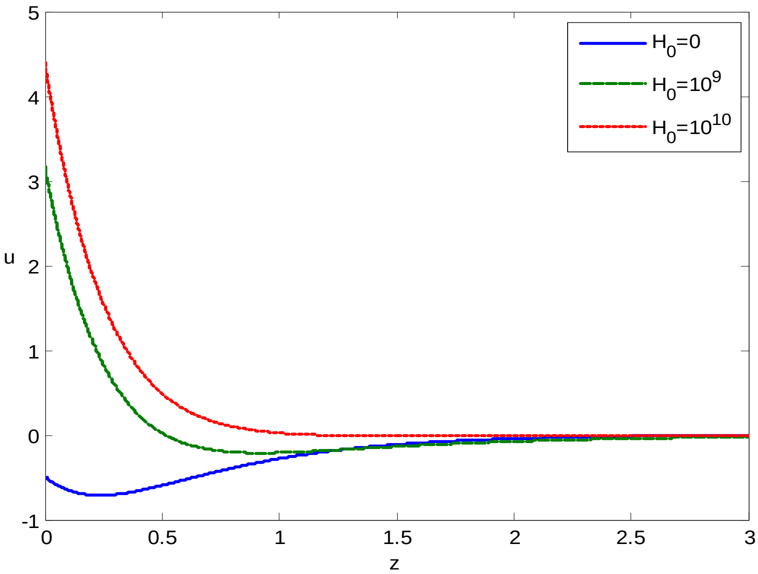

Fig. 2. Displacement distribution at $x = y = 0.1$, $t = 0.1$ and $\tau_0 = 0.02$

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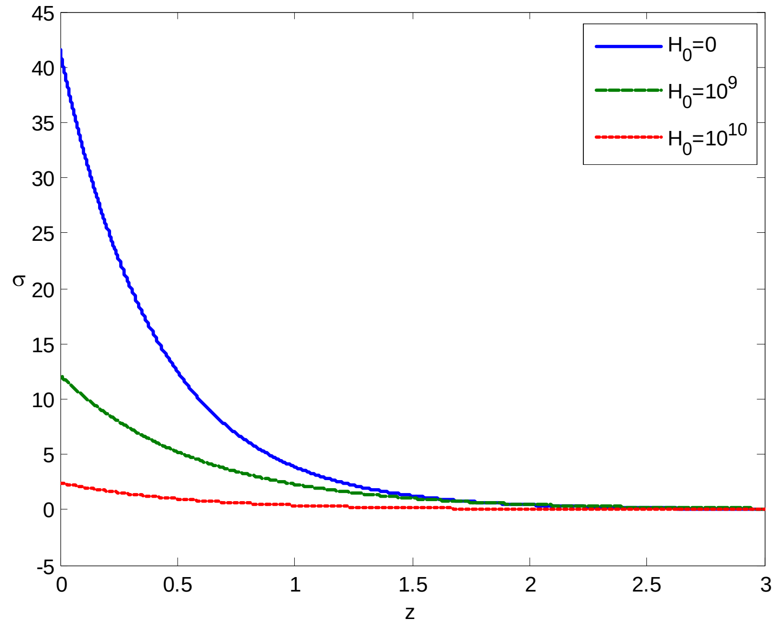

Fig. 3. Stress distribution at $x = y = 0.1, \quad t = 0.1$ and $\tau_0 = 0.02$

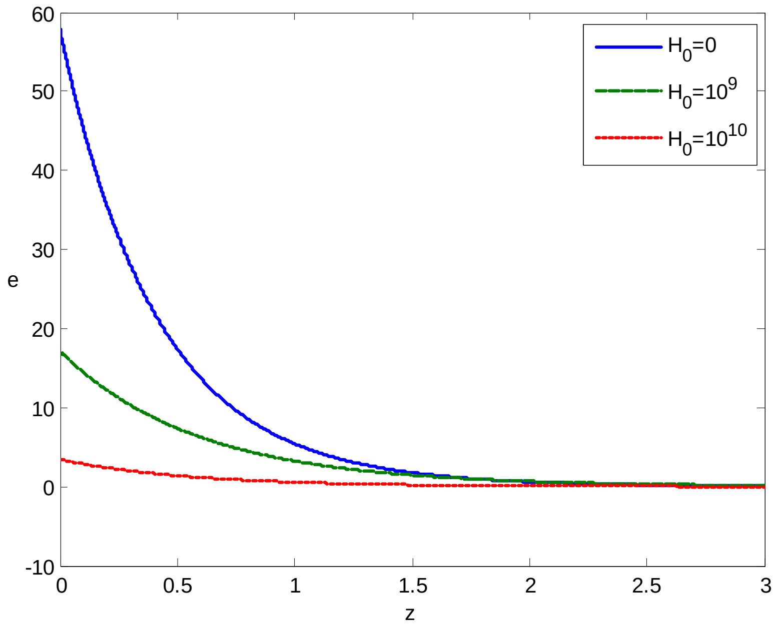

Fig. 4. Strain distribution at $x = y = 0.1, \quad t = 0.1$ and $\tau_0 = 0.02$

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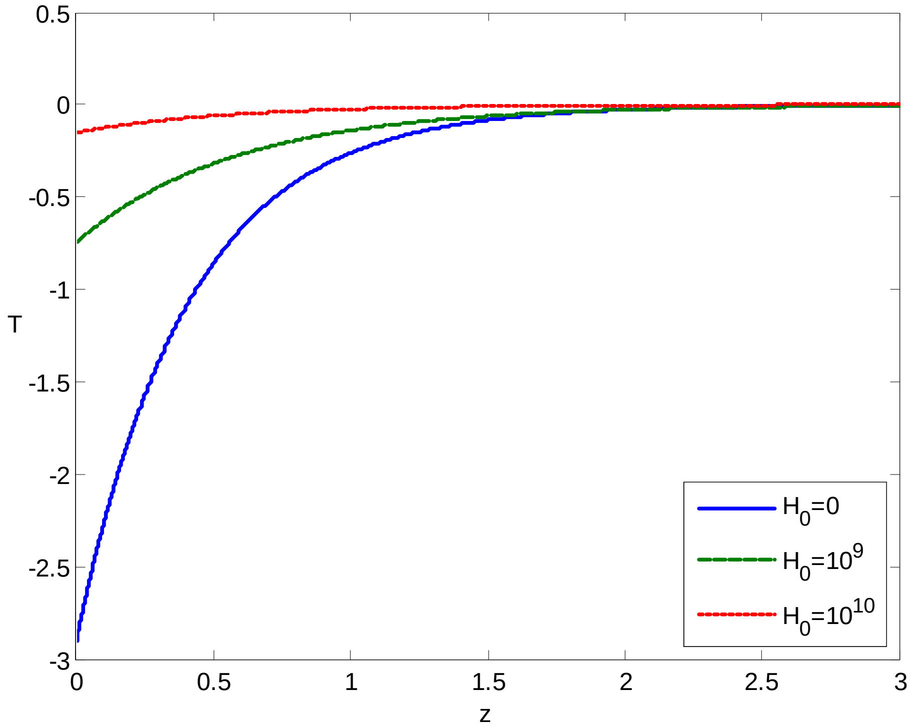

Fig. 5. Temperature distribution at $x = y = 0.1$, $t = 0.1$ and $\tau_0 = 0.02$

Figure 2 illustrates the variations of the displacement component u with a distance z at $x = y = 0.1$ and $t = 0.1$. This figure shows that the displacement component u starts from negative value when the magnetic field is absence ($H_0 = 0$) and then decreases with the increase of the distance z and has a minimum value at $z = 0.25$, and then increases tending to zero for sufficiently large values of distance z. But in the presence of the magnetic field ($H_0 = 10^9,\ 10^{10}$) the displacement component u decreases with the increase of the distance z and finally all curves terminate at the zero value at $z > 3$ approximately . It can be observed from this figure that magnetic field has an increasing effect on the displacement component.

Figures 3, 4 exhibit the variations of the stress σ and the strain e with distance z at $x = y = 0.1$ and $t = 0.1$. These figures show that for the three values of $H_0 = 0,\ 10^9,\ 10^{10}$, the stress σ and the strain e decrease with the increase of the distance z and finally all curves tending to zero at $z > 3$ approximately. It is clear from these figures that the magnetic field has a decreasing effect on the stress σ and strain e. Figure 5 describes the variations of the temperature T with the distance z at $x = y = 0.1$ and $t = 0.1$. This figure shows that the temperature T increases with the increase of the distance z and finally all curves terminate at the zero value at $z > 3$ approximately. It is clearly observed that the magnetic field has an increasing effect on the temperature distribution.

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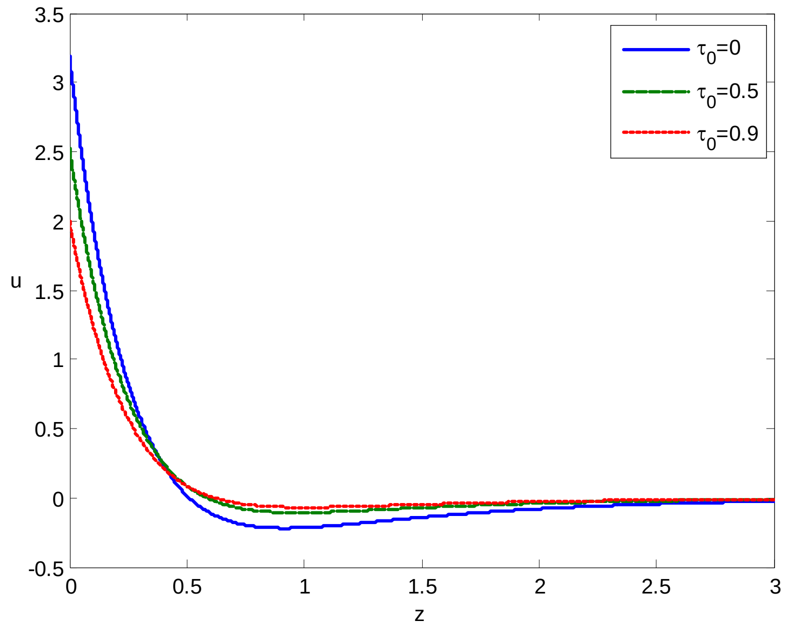

Fig. 6. Displacement distribution at $x = y = 0.1, \quad t = 0.1$ and $H_0 = 10^9$

Figures 6-9 represented 2D curves for the distributions of the real part of the displacement component u, stress σ, strain e and the temperature T against distance z and the dimensionless time t with different values of $\tau_0 = 0,\ 0.5,\ 0.9$ (with a fixed value of $H_0 = 10^9$). In these figures, the solid line, dashed line and dotted line correspond for $\tau_0 = 0,\ 0.5,\ 0.9$ respectively, which is furthermore precisely explained in each figure in the legend. Figure 6 displays the variations of the displacement component u with the distance z and shows that the displacement component u decreases with the increase of the distance z for all values of τ_0 in the range $(0 \le z \le 0.95)$ approximately and then increases with the increase of the distance z in the range $(0.95 \le z \le 3)$ approximately and finally all curves converge to zero for $z > 3$ approximately. It is clearly observed that the relaxation time τ_0 has a decreasing effect on the displacement distribution in the range $(0 \le z \le 0.43)$ while it has an increasing effect in the range $(0.43 \le z \le 3)$ approximately.

Figures 7, 8 depict the variations of the stress σ and the strain e with the distance z at $x = y = 0.1$ and $t = 0.1$. These figures show that for the three values of $\tau_0 = 0,\ 0.5,\ 0.9$, the stress σ and the strain e decrease with the increase of the distance z and finally all curves tending to zero at $z > 3$ approximately. From these figures it is clear that the relaxation

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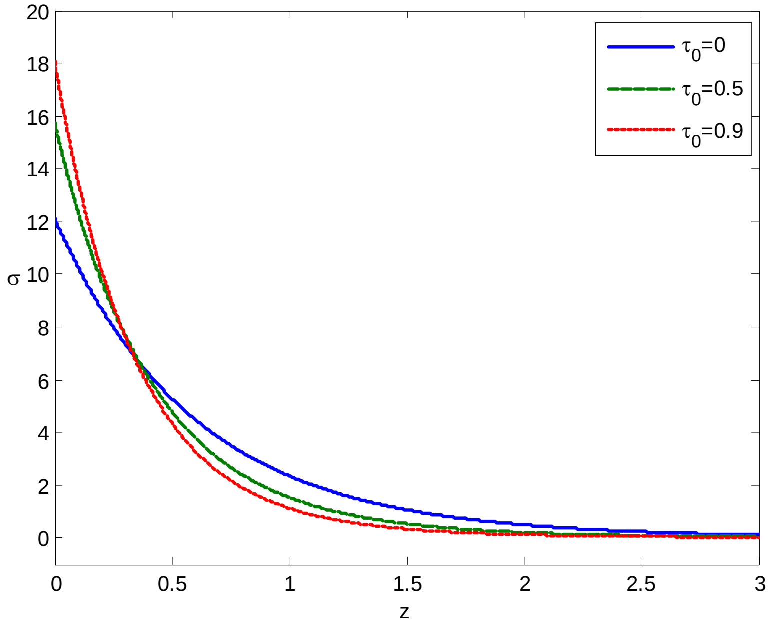

Fig. 7. Stress distribution at $x = y = 0.1$, $t = 0.1$ and $H_0 = 10^9$

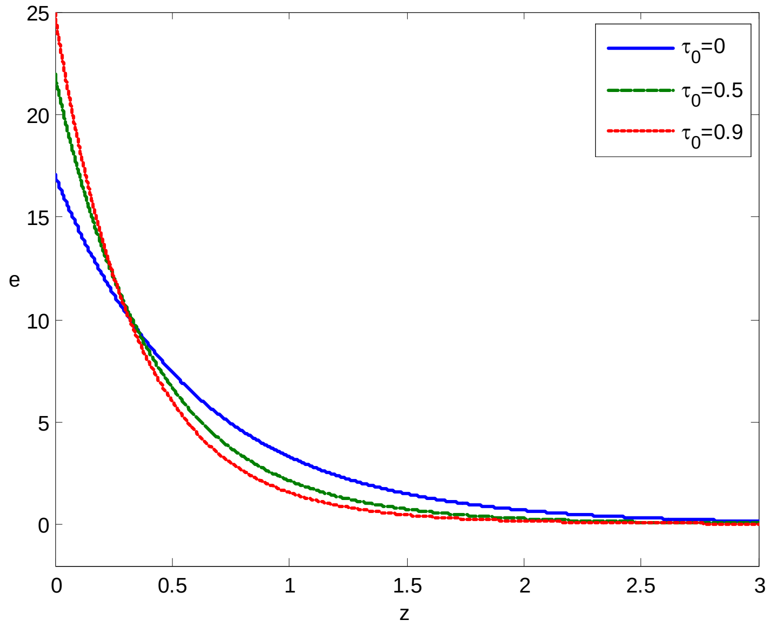

Fig. 8. Strain distribution at $x = y = 0.1$, $t = 0.1$ and $H_0 = 10^9$

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time τ_0 has an increasing effect on the stress σ and strain e in the range $(0 \le z \le 0.33)$ approximately, while it has a decreasing effect in the range $(0.33 \le z \le 3)$ approximately. Figure 9 displays the variations of the temperature T with distance z and shows that the temperature T increases with the increase of the distance z for all values of τ_0 and finally all curves terminate at the zero value for $z > 3$. It is clearly observed that the relaxation time τ_0 has a decreasing effect on the temperature distribution in the range $(0 \le z \le 0.55)$ while it has an increasing effect in the range $(0.55 \le z \le 3)$ approximately.

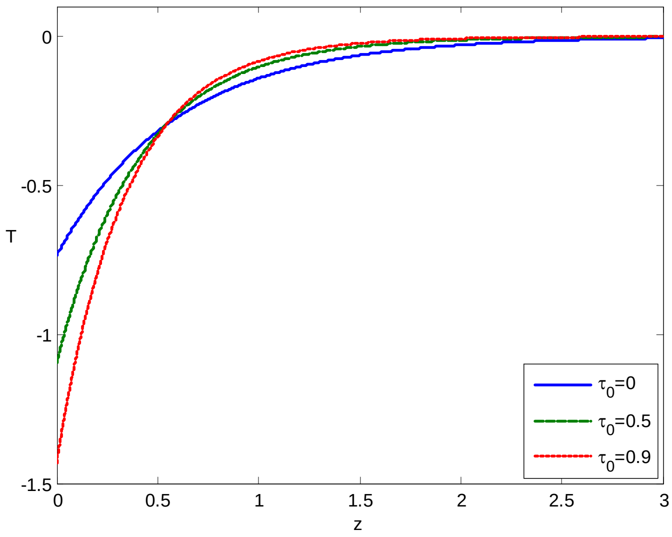

Fig. 9. Temperature distribution at $x = y = 0.1,\quad t = 0.1$ and $H_0 = 10^9$

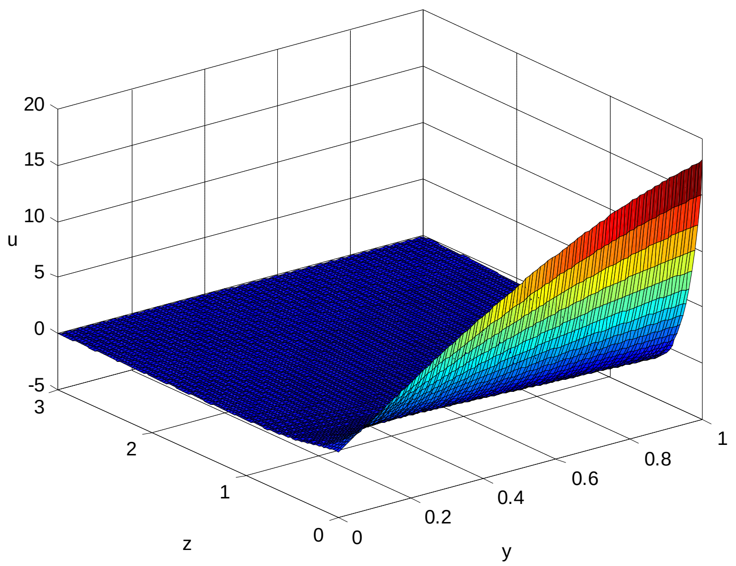

Fig. 10. Displacement distribution at $x = 0.1,\quad t = 0.1$ and $H_0 = 10^9$

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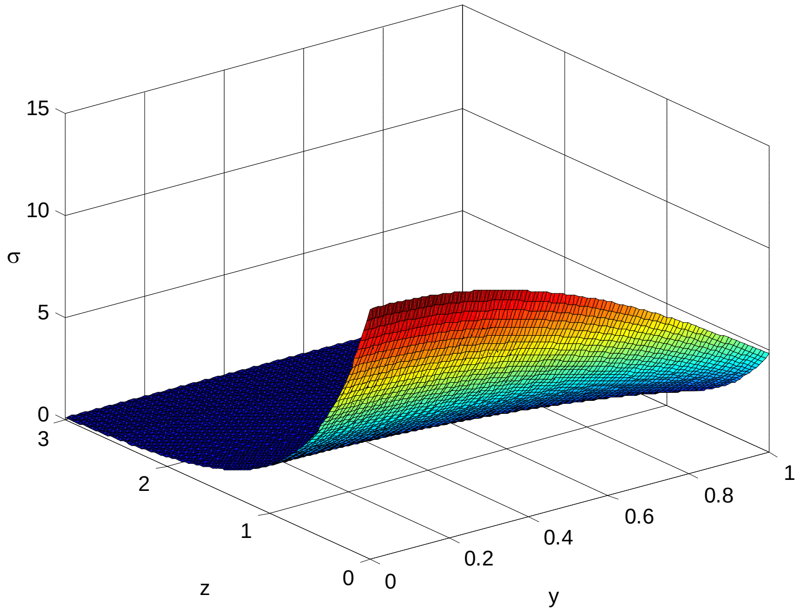

Fig. 11. Stress distribution at $x = 0.1$, $t = 0.1$ and $H_0 = 10^9$

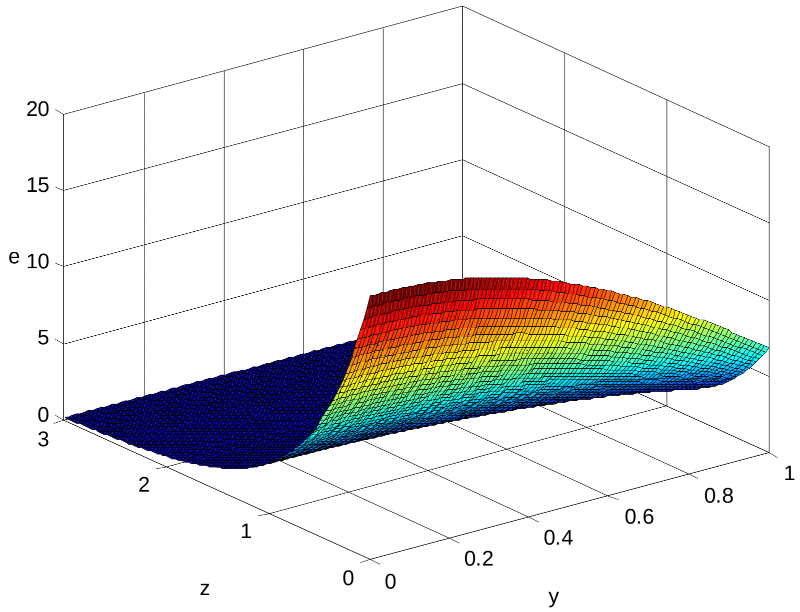

Fig. 12. Strain distribution at $x = 0.1$, $t = 0.1$ and $H_0 = 10^9$

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Figures 10-17 present, in three dimensions, the variations of the displacement, stress, strain and temperature at $H_0 = 10^9$. In these figures, the effects of the changing of the distances y, z and the time t on all the studied fields are pronounced and also we can see that some quantities increase on the negative direction of the distance, while some on positive direction.

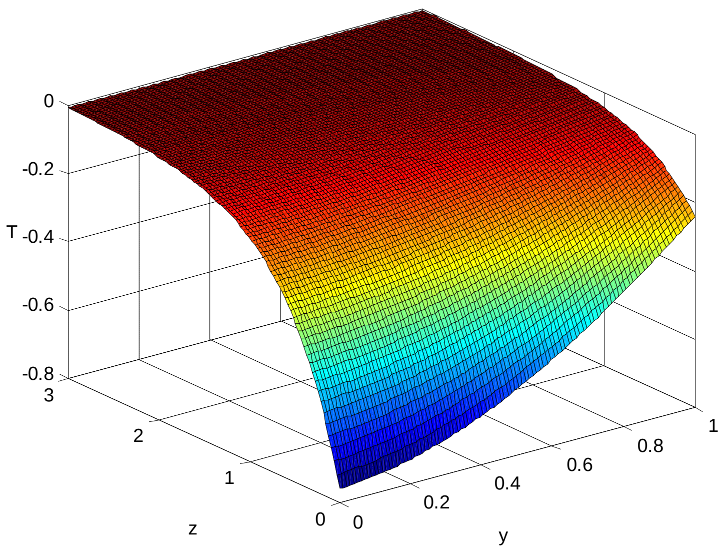

Fig. 13. Temperature distribution at $x = 0.1, \quad t = 0.1$ and $H_0 = 10^9$

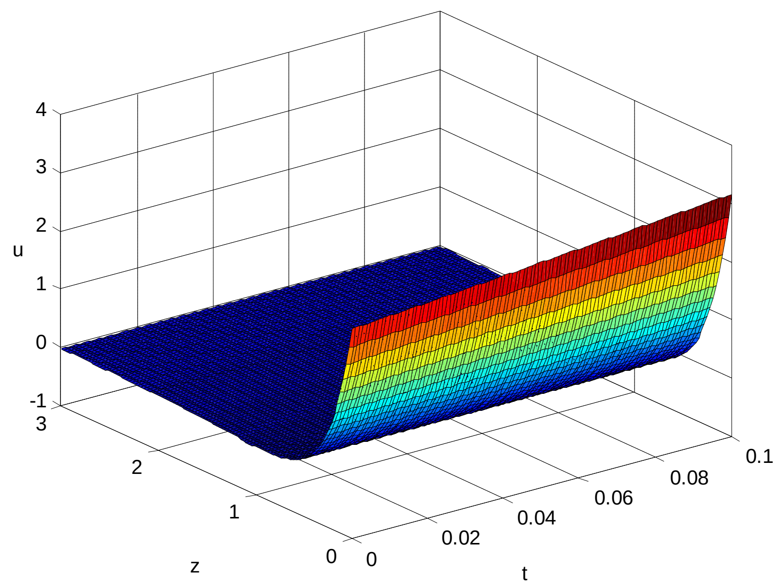

Fig. 14. Displacement distribution at $x = y = 0.1,$ and $H_0 = 10^9$

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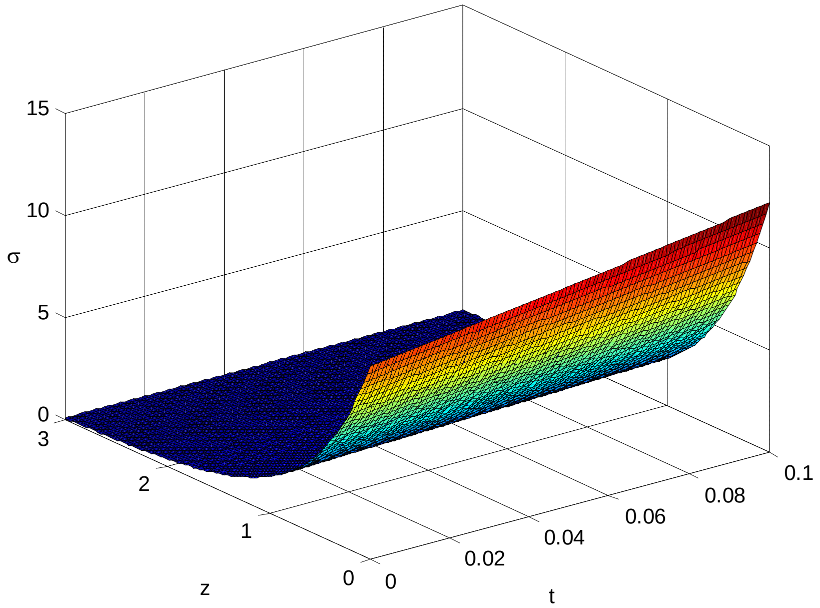

Fig. 15. Stress distribution at $x = y = 0.1,$ and $H_0 = 10^9$

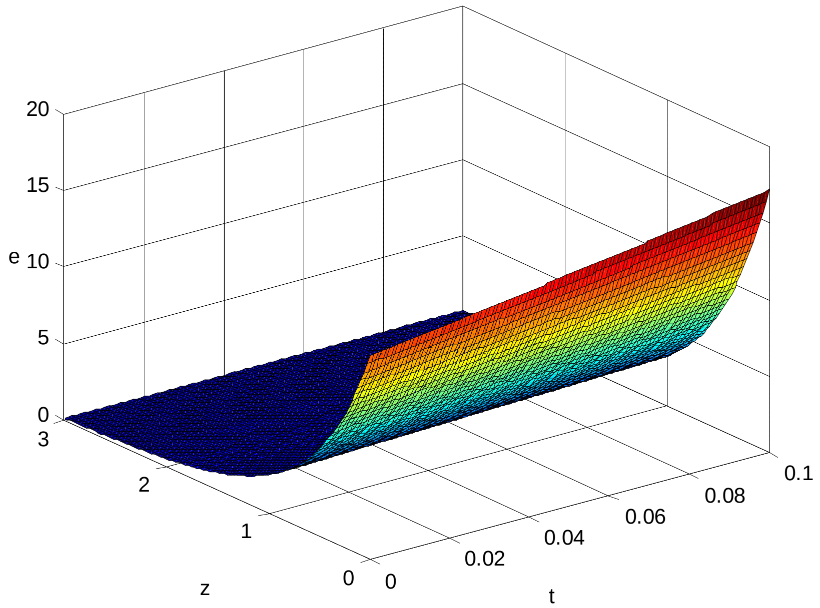

Fig. 16. Strain distribution at $x = y = 0.1,$ and $H_0 = 10^9$

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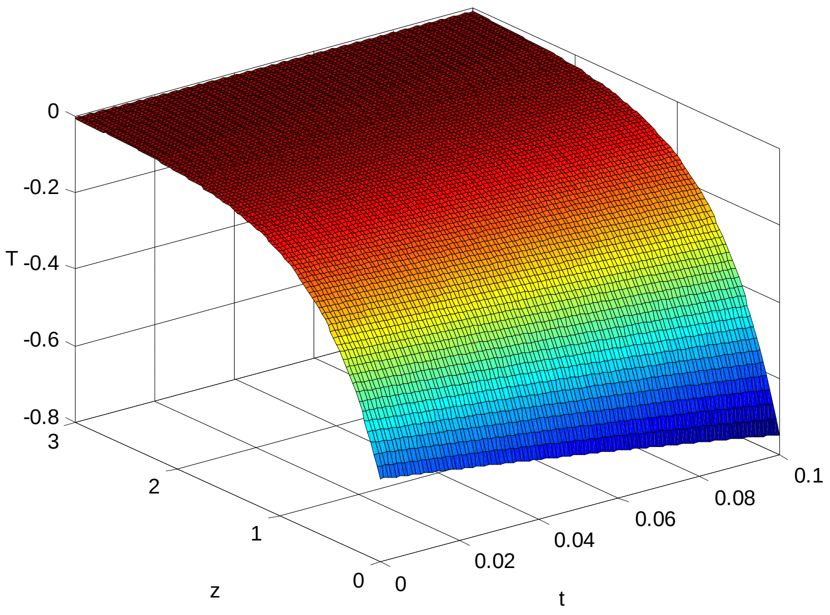

Fig. 17. Temperature distribution at $x = y = 0.1$, and $H_0 = 10^9$

VI. CONCLUDING REMARKS

A three-dimensional model of the generalized thermoelasticity under the influence of thermal relaxation time and magnetic field was established and according to the results the following conclusions can be obtained:

1. It was observed that the magnetic field has a great effect on the components of the displacement, stress, strain and temperature distributions.
2. The effect of relaxation time (τ_0) is clearly observed on the distributions of all physical quantities.
3. The normal mode analysis, used in this article to solve the problem, is applicable to a wide range of problems in thermodynamics and thermoelasticity. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions.
4. The values of the distributions of all physical quantities converge to zero with increasing distance *z*. Using these results; it possible to investigate the disturbance caused by more general sources for practical applications.
5. Physical applications are found in the mechanical engineering, geophysical, and industrial sectors.

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