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Sparse Signal Reconstruction using Basis Pursuit Algorithm

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ABSTRACT – Compressive Sensing acquire sparse signal significantly at very lower rate than Nyquist sampling rate. For this, a low complexity compressed sensing operation is defined and it is the combination of sampling and compression. The signals formed from compressed sensing operation are compressible signals and a set of random linear measurements accurately reconstructs compressible signals with the use of nonlinear or convex reconstruction algorithms. Basis Pursuit algorithm is one of the convex optimization algorithms to reconstruct the sparse signal. The l_1 minimization theory for linear programming problems is used to formulate the compressive sensing method. Interior point method is used to solve the basis pursuit algorithm for sparse signal reconstruction. In this paper, the methodology of reconstructing sparse signal using basis pursuit algorithm is discussed.

KEYWORDS – compressive sensing, sparse signal, convex optimization, basis pursuit, linear programming, interior point method.

I. INTRODUCTION

Data processing is very important in modern life. The data of interest is generally sparse in specific representation. Hence instead of acquiring all data for further processing, only part of data not thrown away is considered. Such large data having only few amount of desired information are called sparse [1]. For example, a wide spectrum having few channels occupying their frequency bands and all remaining channels not using their frequency bands makes the spectrum sparse in nature. In other words, we can say, spectrum occupancy pattern is sparse for that wide band. A sparse signal uses compressive sensing methodology for reconstruction purpose. In compressive sensing, few samples or measurements less than Nyquist rate are required to recover the sparse signal. Some attractive features of compressive sensing are fault tolerance, noise robustness, almost exact recovery and universality [2].

The representation of signal in certain dictionary is useful while its recovery. This dictionary nature is key information for choosing the reconstruction algorithm for a signal. The details about the same will be discussed later. This sparse signal, in general, implements compressive sensing, by using three main types of algorithms for its recovery. These three types of algorithms are: 1) convex optimization algorithms, 2) combinational algorithms and 3) greedy algorithms [2]. In this paper, we are discussing basis pursuit algorithm for sparse signal recovery which falls under convex optimization algorithm category. Basis pursuit algorithm is basically a technique to recover the original signal. The important advantages of basis pursuit over other algorithms are: 1) prior sparsity knowledge of signal is not necessary, 2) reconstruction problem can be formulated easily as linear programming problem which in turn is easy to solve and 3) the performance of basis pursuit under noisy scenarios is good [3].

This basis pursuit algorithm can be used in many application areas like image processing, communication networks, RADAR etc. The remaining paper is organized as follows: section II overviews sparse signal theory, section III explains compressive sensing fundamentals, section IV describes basis pursuit algorithm, section V discusses the application and section VI concludes the paper.

II. RELATED WORK

The roots of basis pursuit algorithm are mainly around sparse signal theory and compressive sensing fundamentals. This section describes the literature survey about the same.

A. Sparse Signal Theory:

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Signal representation involves choice of dictionary. Dictionary is the set of elementary signals or atoms used to decompose the signal. When the dictionary forms a basis, every signal is uniquely represented as the linear combination of the dictionary atoms [1]. Overcomplete dictionaries were developed over orthogonal and bi-orthogonal dictionaries for signal representation. These overcomplete dictionaries have more atoms than the dimension of the signal and hence signal is represented on wider range. Consider $\underline{D}$ be the dictionary having atoms as $\underline{D} = [\underline{d}_1, \underline{d}_2, \dots \underline{d}_L]$ and $\underline{D} \in R^{N \times L}$ where columns forms the dictionary atoms and $L \geq N$. The linear combination expression to represent a signal $x \in R^N$ is $\delta = \underline{D}^T x$. In the discussed scenario, x is sparse in dictionary $\underline{D}$.

If we are saying x is sparse in canonical basis, then the number of non-zero components in x is far less than its length [4]. To reconstruct such sparse x signal, compressive sensing concept was proposed [5]. According to compressive sensing concept, a set of linear measurement for x is obtained from $\delta = \underline{D}^T x$ where $\underline{D}$ is sensing matrix. The original signal x is then reconstructed from the underdetermined linear system $\underline{D}^T x = \delta$ by using basis pursuit algorithm technique. The fundamentals of compressive sensing concept are discussed in next section.

B. Compressive Sensing Fundamentals:

In [5], it is presented that to accurately reconstruct x from δ, x must be sufficiently sparse relative to the number of measurements and the sensing matrix should possess two important properties: 1) sensing matrix should be designed in such a way that $\delta = \underline{D}^T x$ will contain sufficient information for faithful reconstruction of x and 2) proper algorithm should be chosen for implementing compressive sensing and hence successful recovery of original signal.

There are three important design parameters for sensing matrix $\underline{D}$: 1) spark, 2) mutual coherence property and 3) restricted isometry property. We will look after these properties one by one.

a. Spark:

Spark of a matrix is the smallest number of columns of sensing matrix that are linearly dependent. For proper recovery, the spark of sensing matrix should be greater than or equal to twice the sparsity level of the signal to be recovered.

b. Mutual Coherence Property:

According to this property, the coherence measures the maximum correlation between the columns of δ and $\underline{D}$. The coherence is large when two matrices are closely correlated; otherwise its small. The number of measurement required increases linearly with the sparsity of signal and quadratically with the coherence of the matrices.

c. Restricted Isometry Property:

Using this property, error of signal recovery can be bounded when noise is present. In this, an isometry constant is defined which preserves distance between *k*-sparse signals.

After reviewing the properties, we found that spark and restricted isometry properties are hard to follow. Hence mutual coherence property is taken into consideration while designing sensing matrix [2]. Once the sensing matrix is formed, the recovery of original sparse signal will be possible by implementing efficient algorithm.

We are discussing basis pursuit algorithm for recovery of sparse signal. Basis pursuit algorithm is convex optimization technique. In order to recover x from underdetermined system, consider solutions from below equation

$$\min_{x \in R^L} \{ \|x\|_0 : \underline{D}^T x = \delta \}$$

where $\|x\|_0$ is the number of nonzero elements in x. This l_0 problem is computationally tough and mathematically intractable. Hence the problem is formulated into l_1 minimization problem which can be solved using linear programming method [9]. This l_1 minimization method forms the base for basic pursuit algorithm.

III. BASIS PURSUIT ALGORITHM

Basis pursuit algorithm is a technique to solve l_1 minimization problem which is formulated as linear programming problem for compressive sensing. Let us form a problem statement for communication application to understand the basis pursuit implementation for sparse signal recovery. Consider $x(t)$ be the received the time domain signal which is sampled in time with the Nyquist rate. Let T be the time interval that the sensing system needs to analyze and decide about the occupied channels and assume that T_0 is the sampling period for $x(t)$. Sampling $x(t)$ results in the $R \times 1$ time sequence vector x_t, where $R = T/T_0$ is the number of samples within the available time interval [0, T]. Considering that the frequency representation of the signal x_t is sparse (i.e. assuming that majority of the channels are unoccupied), the frequency representation of the signal x_t is the sequence x_f which is obtained using the Discrete Fourier Transform (DFT) as follows

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$$x_f = F_R x_t$$

where $F\{.\}$ represents the DFT sequence. The sampled vector is compressed into a lower dimension using a random sensing matrix as follows:

$$y = \Phi x_f$$

where Φ is the $N \times R$ random sensing matrix, where $N < R$. Since the signal x_f is sparse, using compressive sensing theory, x_f can be recovered using N measurements by

$$\bar{x}_f = \arg\min_{x_f} \|x_f\|_1$$

$$\text{subject to } y = \Phi F_R^{-1} x_f$$

To solve the linear programming problem, primal dual interior point method is used. The primal and dual vectors are obtained at optimal point. The solution procedure is classical Newton method. At interior point, the system will be linearized and solved. As we progresses through Newton's iterations, relax complementary slackness parameter comes into picture. Due to this, the solution of linearized equations will be biased towards the interior, allowing a smooth, well defined central path from interior point to the solution of boundary [7][8]. Newton's iteration continues until surrogate duality gap decreases below a given value. Surrogate duality gap is an approximation to how close a certain interior point remains optimal.

IV. EXPERIMENTAL RESULTS

We performed an experiment to understand the basic of compressive sensing using basic pursuit algorithm. For this, we have considered 2000 samples of data. Using basis pursuit, at different sparsity level, we tried to reconstruct those 2000 samples and accuracy is calculated against each sparsity value. Fig 1 shows the reconstructed output using BP algorithm. The sparsity value and respective accuracy is tabulated in Table 1. We observed that as the sparsity level increases, accuracy decreases and we get approximately accurate reconstructed signal. More sparsity corresponds to less sparsity level and lesser the accuracy, more accurate reconstructed signal to the original signal.

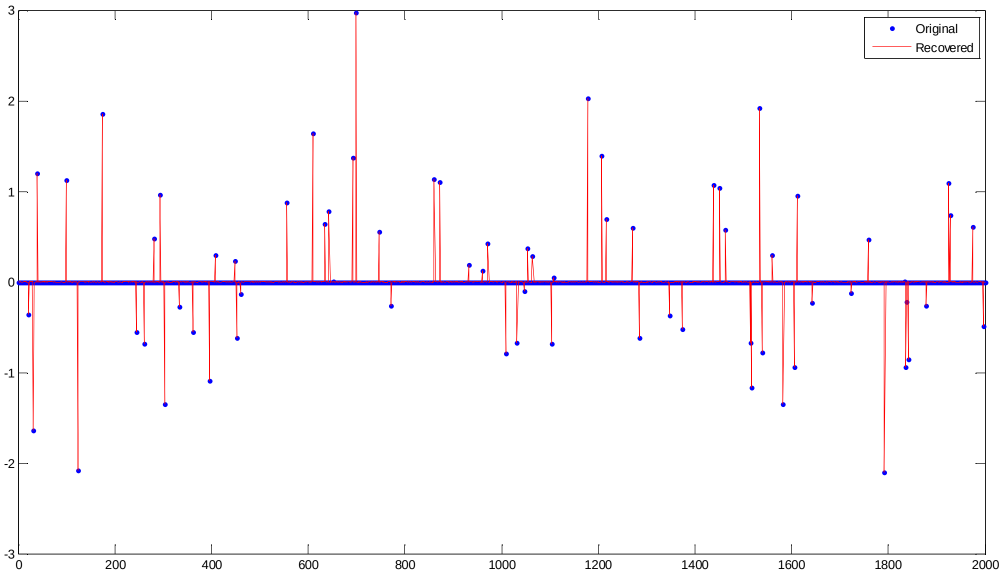

Figure 1: Reconstructed output data using basis pursuit algorithm

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Sr. No.	Sparsity	Accuracy
1	50	$5.7753*10^{-6}$
2	30	0.0979
3	70	$2.2136*10^{-6}$

Table 1: Sparsity level Vs Accuracy for reconstruction using compressive sensing

V. APPLICATIONS

The basis pursuit algorithm finds its applications mainly in image processing for de-noising purpose and in communication area. The applications are discussed below:

a. Cognitive Radio:

In cognitive radio technology, wide band spectrum sensing is the problem of concern. To overcome this, compressive sensing was proposed [6]. It is assumed that received signal spectrum occupancy pattern is sparse and such wide spectrum can be reconstructed using basis pursuit algorithm. Further spectrum estimation techniques will determine which band is exactly vacant so that it can be used by other unlicensed user.

b. Missing data estimation:

Basis pursuit de-noising is used to obtain a sparse set of Fourier coefficients that approximates the noisy signal. De-noising is achieved because the noise-free signal of interest has a sparse representation with respect to the Fourier transform. For other types of the data, it can be more appropriate to use other transforms. A transform should be chosen that admits a sparse representation of the underlying signal of interest. It was also shown that the least square solution achieves no de-noising, only a constant scaling of the noisy data.

c. Signal de-noising:

When the signal to be recovered in its entirety admits a sparse representation with respect to a known transform, then basis pursuit exploits this sparsity so as to fill in the missing data. If the data not only has missing samples but is also noisy, then basis pursuit de-noising should be used in place of basis pursuit. It was shown that the least square solution consists of merely filling in the missing data with zeros.

d. Signal component separation:

When each of the signal components admits a sparse representation with respect to known transforms, and when the transforms are sufficiently distinct, then the signal components can be isolated using sparse representations. If the data is not only a mixture of two components but is also noisy, then basis pursuit de-noising should be used in place of basis pursuit. It was shown that the least square solution achieves no separation; only a constant scaling of the observed mixed signal.

VI. CONCLUSION

In this paper, detailed review of basis pursuit algorithm for sparse signal recovery is discussed. The way this algorithm works is explained using interior point method. Some applications where basis pursuit algorithm can be used are also briefed. The formulation of basis pursuit algorithm in linear programming form reduces its complexity to get the solution. This paper reviews the mathematical analysis for basis pursuit to work. The basic experiment confirms that basis pursuit acquires sparse signal faithfully. The basic knowledge of signal processing, linear algebra and statistics will help to understand the topic in more detail.

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