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Some Nontrivial Relationships Between A (N+1) & A(N)

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ABSTRACT: The trivial relationship known between achromatic indices of K_n & K_{n+1} is $A(n+1) \le A(n)+n$. In this paper I have obtain some other nontrivial relationship between the numbers A(n+1) , A(n).

KEYWORDS: Achromatic Index, Colouring of graphs, Edge colouring , Complete Edge colouring, complete graphs.

I. INTRODUCTION

A k-edge colouring of a simple graph G is assigning k colours to the edges of G so that any two adjacent edges receive different colours. If for each pair t_i & t_j of colours there exist adjacent edges with this colours then the colouring is said to be complete. Let G be a simple graph. The achromatic index $\psi'(G)$ of a simple graph G is the maximum number of colours used in the edge colouring of G such that the colouring is complete. The achromatic index of the complete graph K_n is denoted by A(n). Before going to the derivation of the relationships one requires the following definitions & results.

Definition: Colour-Colour adjacency matrix

Consider proper edge colouring C of a simple graph G with k colours.

We define Colour-Colour adjacency matrix C_G with respect to the above colouring as the matrix of order $k \times k$ by $C_G=[c_{ij}]$where c_{ij} = the number of vertices at which colour i is adjacent to colour j in the colouring C of the graph G.

Definition: Prime adjacency:

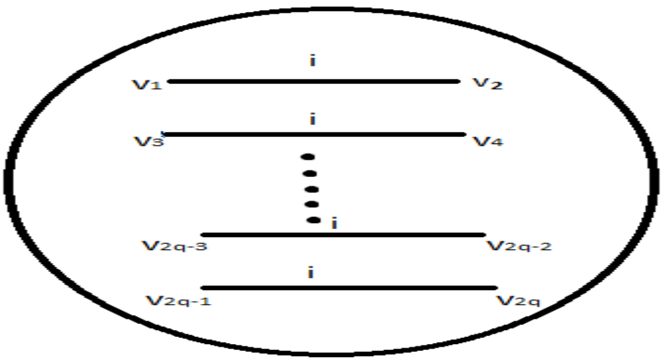
Consider achromatic complete edge colouring of K_n Let C be the colour- colour adjacency matrix with respect to the above colouring. We say that colour i has prime adjacency with the colour j with respect to the above colouring if $c_{ij}=1$.

Definition: spare adjacencies

If colur i is adjacent to colour j in b vertices where b>1 them we say the colour i has b-1 spare adjacencies.

Consider proper edge colouring of the complete graph K_n.

Suppose colour i has q edges in the colouring as shown below.



K_n

Now we will count mutually exclusive spare adjacencies at the colour i.

Spare adjacencies to the colour i due to edges incident at V_1 & V_2 are 2(q-1) each.

Spare adjacencies to the colour i due to edges incident at V_3 & V_4 are 2(q-2) each.

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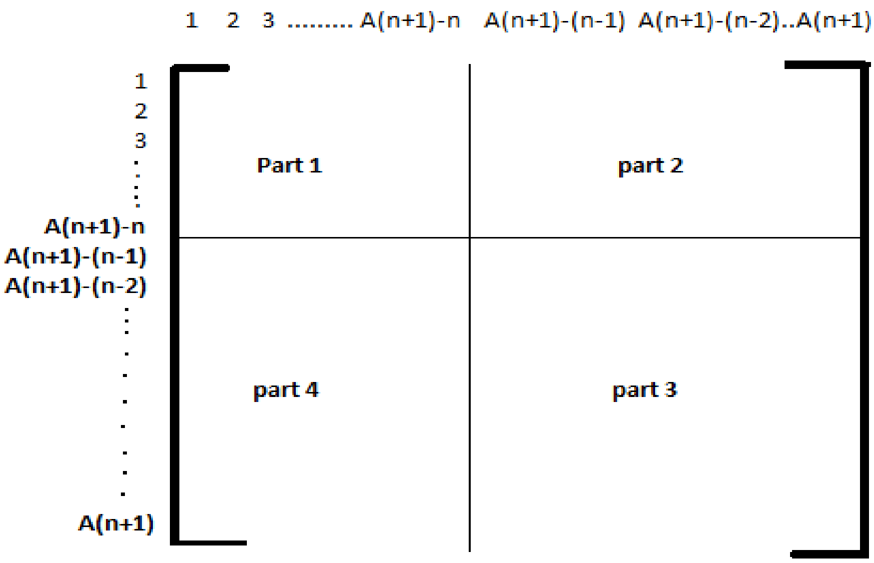
Spare adjacencies to the colour i due to edges incident at V_5 & V_6 are 2(q-3) each
& so counting on in the similar manner we get the total spare adjacencies of the colour i are
4(q-1)+4(q-2)+………………..+4(q-(q-1))
=2q(q-1). …………………(*1)

II. DERIVATION OF THE RELATIONSHIPS

It is known that that A(8)=14.If any colour i in the achromatic complete colouring of K_8 has exactly one edge then the number of distinct colours adjacent to the colour i at the extremities of the edge are 6+6=12.Hence A(8)≤12+1=13 which contradicts to the fact A(8)=14.Hence every colour in the achromatic complete edge colouring of K_8 has at least two edges. Now in the further discussions we always choose $n \geq 9$ As A(8) < A(n), Therefore by arguing similarly as above every colour appearing in the complete achromatic colouring of K_{n+1} has at least two edges in that colouring. Consider the achromatic complete edge colouring of K_{n+1} with A(n+1) colours say 1,2,………A(n+1) colours. Let C be the colour colour adjacency matrix with respect to the above colouring. Let each colour i in the colouring has t_i edges in the colouring. Let k = min{ t_i : i=1to A(n+1)}
Therefore removal of any single vertex V from K_{n+1}, we will remain with proper edge colouring of K_n with A(n+1) colours but the colouring may not be complete& has at the most n colours with exactly k-1 edges of each & at least A(n+1)-n colours with at least k edges of each. Therefore due to (*1), minimum number of spare adjacencies in the proper edge colouring of K_n are 2(k-1)(k-2)n+2k(k-1) (A(n+1)-n)=2(k-1)(n(k-2)+ k (A(n+1)-n))Let C^- be the colour colour adjacency matrix of proper edge colouring of K_{n+1}-{V}. The maximum number of prime adjacencies in C^- are n(n-1)(n-2)- 2(k-1)(n(k-2)+ k (A(n+1)-n))
∴ the lower bound of number of non diagonal zero cells in C^- is
A(n+1)(A(n+1)-1)-[n(n-1)(n-2)- 2(k-1)(n(k-2)+ k (A(n+1)-n))] ………………………(*2)
wlg the colours A(n+1),A(n+1)-1,A(n+1)-2,…………..,A(n+1)-(n-1) be incident at the vertex V.
The C^- matrix is the matrix of order A(n+1) × A(n+1). we will consider four parts of it as shown below.



The numbers written on the side & above the matrix are representing row & column numbers (&also colours) of C^-
Now let's find maximum number of non diagonal zero cells in the C^- matrix. As the colours 1, 2, 3……. A(n+1)-n are not incident at the vertex V, hence removal of the vertex V does not affect mutual adjacencies of the colours involved in the part 1 of the matrix. As the matrix C^- is obtained from C , hence there is no non diagonal zero in the above part 1. After removal of the vertex V, adjacencies broken at extremities of each edge of the colours A(n+1)-(n-1), A(n+1)- (n-2), …….., A(n+1) are at most 2(n-1).
∴ part2 & part 3 together will contain at most 2(n-1)n non diagonal zeros.

Any colour from A(n+1)-(n-1) to A(n+1) can lose adjacencies with at most n-1 colours at the extremity other than the vertex V. Therefore there can be maximum n-1 zeros in each row of part 4.

∴The maximum number of non diagonal zeros in the part 4 are n(n-1)

Hence maximum number of non diagonal zeros in the matrix $C^{'}$ are 2(n-1)n +n(n-1)=3n(n-1) …….(*3)

So from (*2) & (*3) we conclude

$A(n+1)(A(n+1)-1)-[n(n-1)(n-2)- 2(k-1)(n(k-2) + k (A(n+1)-n))] \leq 3n(n-1)$

$A(n+1)(A(n+1)-1)- n(n-1)(n-2)+2(k-1)(n(k-2) + k (A(n+1)-n)) \leq 3n(n-1)$ ………………(*4)

From the description of k as discussed above, there exist a colour i which has exactly k edges in the achromatic complete edge colouring of K_{n+1} .

The maximum number of colours adjacent to the colour i can be 2(n-1)+2(n-3)+……..+2(n-(2k-1))

= 2[nk-(1+2+3+…..+(2k-1))+(2+4+…(2k-2))]

=2[nk-k(2k-1)+k(k-1)]

$=2[nk-k^2]$

$=2nk-2k^2$

$\therefore 2nk-2k^2 \geq A(n+1) -1$

$\therefore 2k^2-2nk+(A(n+1)-1) \leq 0$

$\therefore [k- (2n\pm(4n^2-8(A(n+1)-1))^{1/2})/4] \leq 0$

$\therefore [k- (n\pm(n^2-2(A(n+1)-1))^{1/2})/2] \leq 0$

$\therefore [k- (n+(n^2-2(A(n+1)-1))^{1/2})/2]. [k- (n-(n^2-2(A(n+1)-1))^{1/2})/2] \leq 0$

$\therefore (n-(n^2-2(A(n+1)-1))^{1/2})/2 \leq k \leq (n+(n^2-2(A(n+1)-1))^{1/2})/2$

Right side of the above inequality is trivial as $k \leq n/2$ & $n/2 < (n+(n^2-2(A(n+1)-1))^{1/2})/2$

Neglecting the right side of the above inequality(as itisvery trivial), we remain with

$(n-(n^2-2(A(n+1)-1))^{1/2})/2 \leq k$ ……………..(*5)

Using the monotone property $A(n) \leq A(n+1)$ & the simple algebra we obtain

$(n-(n^2-2(A(n)-1))^{1/2})/2 \leq (n-(n^2-2(A(n+1)-1))^{1/2})/2$

$\therefore (n-(n^2-2(A(n)-1))^{1/2})/2 \leq k$

∴by (*4), $\mathbf{A(n+1)(A(n+1)-1)- n(n-1)(n-2)+2((n-(n^2-2(A(n)-1))^{1/2})/2-1)(n((n-(n^2-2(A(n)-1))^{1/2})/2-2) + (n-(n^2-2(A(n)-1))^{1/2})/2 \; (n-(n^2-2(A(n)-1))^{1/2})/2 \; (A(n+1)-n)) \leq 3n(n-1)}$

The above result is non-trivial relationship between A(n+1) & A(n).We can see that putting n=25 k=3 in the relationship it gives A(26)≤ 119 which is better than the trivial result A(26) ≤ A(25)+25=100+25 =125.

Now we will derive another nontrivial relationship between A(n+1) & A(n).It is obvious that $k.A(n+1) \leq {}^{n+1}c_2$

$\therefore k^{.}A(n+1) \leq n(n+1)/2$

$\therefore k \leq n(n+1)/(2A(n+1))$

$\therefore k \leq n(n+1)/(2A(n))$ …………(*6)

Combining (*5) & (*6) we get $(n-(n^2-2(A(n+1)-1))^{1/2})/2 \leq k \leq n(n+1)/(2A(n))$

hence $(n-(n^2-2(A(n+1)-1))^{1/2})/2 \leq n(n+1)/(2A(n))$.

therefore $\mathbf{(n-(n^2-2(A(n+1)-1))^{1/2}) \leq n(n+1)/A(n).}$

The above result is non-trivial relationship between A(n+1) & A(n).We can see that by putting n=13 & A(n)=39 in the above result we get A(14) ≤ 50,Which is better than the trivial A(14) ≤ A(13)+13=39+13=52

III. CONCLUSION

The above relationships may not improve existing bounds of achromatic indices of complete graphs but they may be helpful to extract some more information about achromatic indices.

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