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RESEARCH PAPER

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Role of Degree Sequence in Determination of Maximal Clique of a Graph

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Abstract: The paper high lights the relation between Discrete Mathematics and Computer Science and Engineering. Since, Graphs and Graph-Algorithms are core topic in combinatorial Algorithms[6]. Our attempt is religious. This is a practice even adopted by D.E. Knuth[6] and shows that we are in a era where visual representation of graphs is a tertiary affair.

Keywords: Graphic sequence, Clique Number, Isomorphism, drop-m, p-laying off.

INTRODUCTION

Please take it with a grain of salt. The reason is the solution is limited to isomorphism constraints. In this paper $G(V,E)$ denoted as simple, symmetric and connected graph. In this paper we have concentrated on the problem of determination of maximum clique number for a graph with given degree sequence. The same problem was solved by A. Ramachandra Rao and others. It is clear that different graphs can have same degree sequence [1-4]. Then at least one graph exists with the determined maximum clique number. They have proposed a Havel-Hakimi type procedure for determination of maximum clique number of a graph with given degree sequence [5]. We have seen that there is a problem that it can not determine the required clique number with first attempt. Here we proposed an algorithm that can uniquely determine the maximum clique number and it takes much less time than previously proposed algorithm.

PRELIMINARIES

In this section we are going to define some terms related to our paper and also some required theorems that are the basis of the proposed algorithm.

Definition 1: A sequence $\xi = d_1, d_2, d_3, \ldots, d_n$ of nonnegative integers is said to be *graphic sequence* if there exists a graph G whose vertices have degree d_i and G is called *realization* of ξ [1].

Definition 2: A sequence ξ has property A_p iff there exists a graph with degree sequence ξ in which the first p vertices form a complete subgraph [5].

Definition 3: If $p < n$ and $(p-1) \leq d_1 \leq (n-1)$ then *p-laying off* d_1 from ξ and reducing $d_2, d_3, d_4, \ldots, d_p$ and (d_1-p+1) largest terms among $d_{p+1}, d_{p+2}, \ldots, d_n$ by unity [5].

Definition 4: ξ has property A_p iff the sequence obtained by *p-laying off* d_1 from ξ has property A_{p-1} [5].

Definition 5: If ξ is the degree sequence of a graph G and G contains a maximum clique number k, then

$$m = \Sigma\,(d_i - k+1)$$

drop-m implies reducing largest terms among $d_{k+1}, d_{k+2}, \ldots, d_n$ by unity.

Theorem 1: Let, a sequence $\xi = (d_1, d_2, d_3, \ldots, d_n)$ and let Let, a sequence $\xi^{p,k}=(d_1^{p,k}, d_2^{p,k}, \ldots, d_{n-k+1}^{p,k})$ be the sequence obtained from $\xi^{p,k-1}$ by $(p-k+2)$-laying off $d_1^{p,k-1}$ for $k = 2, 3, 4, \ldots, p$, where $\xi^{p,1} = \xi$. Then ξ has the property A_p if and only if $\xi^{p,p}$ is graphic [5].

***Proof** [5]:*
This theorem gives a procedure to determine whether ξ has property A_p since it is easy to find out whether $\xi^{p,p}$ is graphic or not. This can be used to construct a graph on p vertices (when it exists) starting from a graph with degree sequence ξ can also be determined easily.

Illustration of the procedure with an example:
Consider a sequence,

$\xi^{4,1} = 6, 5, 4, 4, 4, 4, 4, 1$

With $p = 4$, $d_1=6$
and since $p-1=3 \leq d_1 \leq (n-1)$

$\Rightarrow 3 \leq 6 \leq 7$
$\therefore \xi^{4,2} = 4, 3, 3, 3, 3, 3, 1$

Now, $d_1=4$ since, $p-1=3 \leq d_1 \leq (n-1)$
$\Rightarrow 3 \leq 4 \leq 7$
$\therefore \xi^{4,3} = 2, 2, 3, 2, 2, 1$

Now, $\xi^{4,4} = 1, 2, 2, 2, 1$

Now, $\xi^{4,4}$ is graphic. Working backward, it is easy to construct a graph with degree sequence ξ in which 4 vertices form a complete subgraph.

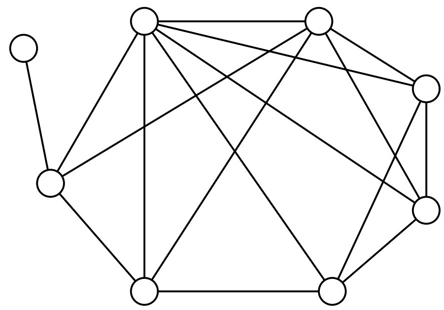

Figure 1

$\xi^{5,1}$ = 6, 5, 4, 4, 4, 4 ,4, 1

With $p = 5$, d_1=6
and since $p-1=4 \le d_1 \le (n-1)$

$\Rightarrow 4 \le 6 \le 7$
$\therefore \xi^{5,2}$ = 4, 3, 3, 3, 3, 3, 1

Now, d_1=4 since, $p-1=4 \le d_1 \le (n-1)$

$\Rightarrow 4 \le 4 \le 7$
$\therefore \xi^{5,3}$ = 2, 2, 2, 3, 2, 1

Now, $\xi^{5,4}$ = 1, 1, 3, 2, 1

Now, $\xi^{5,5}$ = 0, 3, 2, 1

Since, $\xi^{5,5}$ is not graphic we conclude that maximum clique number can not be 5 it must be 4 for the given degree sequence ξ.

FOUNDATION OF THE PROPOSED ALGORITHM

The problem is to find out the maximum clique number of a graph *G*, with a degree sequence ξ. Since maximum clique number is nothing but the maximum number of vertices forming complete subgraph of *G*. If *k* be the maximum clique number of *G* then in ξ there must be at least *k* number of integers in ξ is $\ge k-1$.

So, we have taken 1st *k* integers with value greater than equal to $k-1$ fromξ. And this represents a complete subgraph of *k* vertices. Deleting this subgraph from *G* we get *G′* with total edges $E'=E-\{k(k-1)/2+m\}$ where *E* is total number of edges of *G*, $m=\sum(di-k+1)$. Therefore we get ξ′ from ξ after *drop-m*. If ξ′ represents a graphic sequence then *k* is maximum clique number.

The proposed theorem which is the basis of the Algorithm is now given below.

Theorem 2: Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of a graph *G*. The graph G contains a maximum clique number k iff $d_1 \ge d_2 \ge d_3 \ldots\ldots d_k \ge k-1$, $k \le n$ and the sequence after droop-m $(d_{k+1}, d_{k+2}, \ldots, d_n)$ is graphic.

Proof:

Let us considerξ represents a graphic degree sequence of graph *G* and after *drop-m* we have ξ′ from ξ, which is not graphic sequence. Since, *drop-m* implies deletion of a complete subgraph of *k* vertices from *G* and reducing next (d*k*+1, d*k*+2, …., d*n*) maximum *m* terms by unity. Then after *drop-m* we have ξ′ from ξ. Since ξ is graphic, after *drop-m* ξ′ must represent a graphic sequence. That contradicts our assumption. Hence the theorem is proved. ♦

In the next section we are going to propose an algorithm for

PROPOSED ALGORITHM

The proposed algorithm MAX_CLQ is given in this section.

ALGORITHM MAX_CLQ

Input: Sequence of nonnegative integers.
Output: *k* maximum clique number.

Step 1:
$\xi = d_1 \ge d_2 \ge d_3 \ge \ldots \ge d_n$. $k = d_1$;
Step 2:
If $(d_1 \ge d_2 \ge d_3 \ge \ldots \ge d_k \ge (k-1))$ Then
Continue;
Else
Jump to Step 4.
End If
Step 3:
If $((d_{k+1}, d_{k+2}, \ldots, d_n)$ is graphic after *drop-m*) Then
Jump to Step 5.
Else
Continue.
End If
Step 4:
$k = k-1$
Step 5:
Print "The maximum clique number of the sequence ξ is *k*"
Step 6:
Stop.

EXPLANATION OF THE ALGORITHM MAX_CLQ WITH AN EXAMPLE

Let us consider a degree sequence ξ = 6, 6, 4, 4, 4, 4, 2, 2 and have to find out the maximum click number for the sequence, representing a graph G.
Now,
Since d_1=6 therefore $k = d_1 = 6$.
Now, $d_2 \ge 5$, but $d_3 < 5$.
$\therefore k = k-1 = 5$

Now, $d_2 \ge d_3 \ge \ldots \ge d_5 \ge 4$
$\therefore m = \Sigma (d_i - k+1) \ \forall \ i$=1,2,..5.
$\therefore m = 4$

∴ *drop-m* from (d_6, d_7, d_8) we have
$\xi' = (2, 1, 1)$
which is graphic.

∴The sequence ξ contains a maximum clique number 5. Figure 2 shows the corresponding graph.

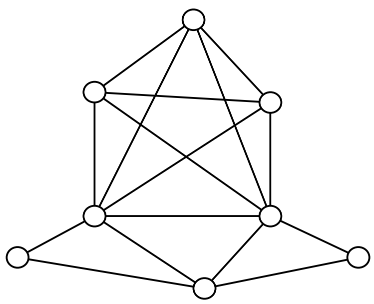

Figure 2

EXPERIMENTAL RESULTS

The experiment is carried out on real data. Each sequence is generated randomly and then checked whether it is a valid Tree Sequence or not. The program was run in a Pentium IV(1.33GHz) PC, Windows XP platform and in Borland C compiler.

Table 1

No. of Vertices	Present	Previous[5]
100	0	6.758
200	0	6.758
300	0.11	6.7581
400	0.17	6.7581
500	0.33	7.1043
600	0.6	7.1046
700	0.94	7.4519
800	1.59	8.1445
900	2.2	8.8382
1000	3.46	9.8784
1100	4.01	11.2646
1200	5.54	13.3435
1300	6.76	15.7687
1400	8.62	18.5406
1500	10.71	22.3527
1600	12.31	26.8563
1700	15.27	32.0532
1800	17.85	55.2185
1900	18.4015	57.0873
2000	19.3884	61.9856

In the Table 1, the first column (No. of Vertices) indicates number of vertices for which random sequences are generated and the 2^{nd} and the 3^{rd} columns indicates time required for finding the clique number for present(2^{nd}) and previous(3^{rd}) algorithms.

CONCLUSION

The decision problem, whether a graph *G* contains a clique of size *K* or not is NP-Complete [7]. It is polynomially reducible from Satisfiability problem, to colorability problem to clique detection problem. In fact the problem is complementary of *K*-Chromatic decision problem. We have abstracted the problem to finite integers. This is a zooming concept as the integers may be represented as chromosomes which forecasts that soft computing may be allowed to be applied in this case. The application area is intersystem-intersystem communication problem specially in the area of congestion of Mobile Computing.

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