

# Rigid Dynamics Solution Methodology for 3-PSU Parallel Kinematic Manipulators

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**ABSTRACT:** In the world of robotics, the parallel kinematic manipulators have proved their unique potentials like rigid structural capacities, better dynamic behaviours, and excellent spatial positional accuracy for many applications. One of the prime focus areas of the paper is to test and to establish a simplified novel approach for the Inverse Kinematics problem solution for a 3-PSU Parallel Kinematic Manipulator platform. Additionally, the methodology for simulation of Inverse Kinematics has been checked and verified. The obtained solution is further used for positional simulation of virtual prototype of 3-PSU PKM using Pro/Mechanism module. Further, these derived data is used for simulation of the Rigid Dynamics Solution for PKM (considering inertial forces acting on actuator) is also simulated in case of circular contouring.

**KEYWORDS:** Parallel Kinematic Manipulator, PKM, Parallel Manipulator, Inverse Kinematics, Rigid Dynamics, Simulation.

## I. INTRODUCTION

The Parallel Kinematic Manipulator has gained a lot of attention to researchers and academicians from research, academics and industrial organizations just because of the reason behind this, that is their high dynamic characteristics, rigid structures, non-additive error propagation and high positional accuracies of the parallel kinematic manipulator (PKM) [1]. The closed loop connections of linkages establish a rigid structure of PKM. The payload to self weight ratio is also enhanced because of multiple linkages support the moving link, which further results into good dynamic characteristics. The PKMs generally have different theoretically possible configurations of joints and actuators [2], notations, different kinematic and dynamic characteristics, control schemes and workspaces [3].

The first aim of this paper focuses on to study and to verify a novel, simplified geometrical approach to solve inverse kinematic solution of 3-PSU PKM configurations using advanced CAD software and Simulation software like Pro/E mechanism and MATLAB.

The second aim is to study a simulation methodology for Rigid Dynamics solution for same setup for a specific tool-path (i.e. circular contouring) at lower speeds in order to check if considerable amount of forces act on actuators while motion due to inertial forces developed at joints or not. The obtained results will be checked with reference to the odel and result profile as suggested by Saha and Rao [4].

As the wide variations in PKMs are possible as far as the configuration of the setup is concerned, firstly the notation scheme and Degree of Freedom must be discussed to decide the methodology for solution. These discussions are carried out in next sections.

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## II. METHODS, NOTATIONS SCHEME AND DEGREE OF FREEDOM

In literature, uniform notation scheme of mechanical link arrangement of PKM has not been defined adequately. Because of lack in standard scheme, no sophisticated software has been designed with proper modular library facility. Although, the configuration (also known as 3-PSU configuration) of Parallel Manipulator can be elaborated as following; ‘3’ denotes three Degree of Freedom (DoF), ‘P’ stands for actuated prismatic joint, ‘S’ refers to ball & socket (spherical) joint & ‘U’ means Universal joint. Three serial P-S-U links are connected in parallel manner in spatial space. [5].

These three translatory motions are considered along mutually orthogonal axes, and three rotary motions around these axes. Degree of Freedom (DoF) does not answer the question if the PKM is Planar or Spatial. The total numbers of variable required for actuator parameter (either prismatic or rotary) can be related with total DoF.

For example, DoF of manipulator of Stewart platform is six, means all three rotational and all three translatory motions are available with this PKM. The PKM has been suggested with 3-DoF (actuated by three stepper motors) though, it is an example of planar robot manipulator [6].

Depending upon the DoF the range of applications of application ranging from spatial applications like pick and place, material handling, MEMS, medical application, wire robots, CMM, astronomy, vibrations, sun-tracking, biomechanics, simulators, machine tools [7] to Planar applications [8].

If,  $n$  = number of rigid bodies  
 $j$  = number of joints  
 $f_i$  = DoF of the  $i^{\text{th}}$  joint

Then,

$$\text{DoF} = 6(n - j - 1) + \sum_{i=1}^g f_i$$

$$\text{DoF} = 6(14 - 18 - 1) + [(3 \times 1) + (6 \times 3) + (6 \times 2)]$$

$$\text{DoF} = 3$$

Now, it can be established that 3 sets of actuators are necessary and sufficient to control 3 numbers of coordinates in the space, based on this derived Degree of Freedom. Further, Inverse Kinematics model will be developed with consideration of 3 DoF for this configuration of the PKM.

## III. INVERSE KINEMATICS

### A. Mathematical Modelling

3-PSU configuration of PKM can be modeled as per figure,

Where,

- $A_i B_i$  =  $i^{\text{th}}$  rail on which spherical joint moves (prismatic joint)
- $A_i P_i = d_i$  = Distance of actuator from  $A_i$
- $O_{xyz}$  = Co-ordinate of fixed frame
- $M_{xyz}$  = Co-ordinate of moving frame
- $T_{xyz}$  = Co-ordinate of Machine Tool
- $P_i Q_i = l_i$  = Length of  $i^{\text{th}}$  link

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With the condition that two consecutive pairs are always kept parallel, resulting into  $1 \parallel 2, 3 \parallel 4, 5 \parallel 6$ . Which, mathematically means:  $d_1 = d_2; d_3 = d_4; d_5 = d_6$ .

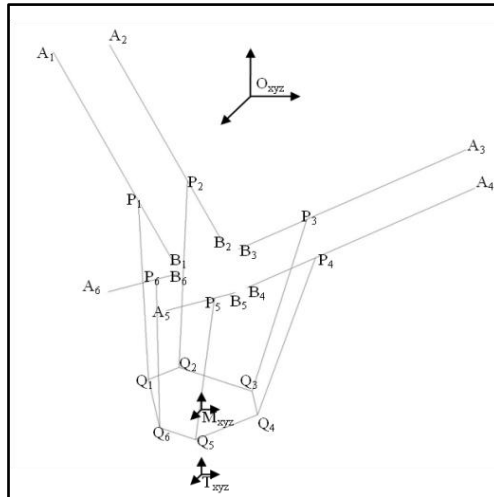


Fig 1 General model for 3-PSU PKM

### B. Inverse Kinematic Solution

Inverse Kinematic solution for 3-PSU PKM focuses to establish the relation between  $(x,y,z)$  variables of vector  $T_{xyz}$  and actuator lengths  $(d_1,d_3,d_5)$ . A single link shown in figure 2 is to be considered for analysis of inverse kinematic solution. A transformation of vector from  $T_{xyz}$  to  $M_{xyz}$  is found out first. Because of symmetric arrangement of linkages parallelepiped construction is made and no rotational component exists for  $M_{xyz}$ . Further, from  $M_{xyz}$ , different coordinates for  $Q_i$  ( $i=1$  to 6) is obtained. Due to Spherical Joint(S),  $Q_i$  ( $x_q, y_q, z_q$ ) will be a centre of sphere and radius will be equal to  $P_iQ_i$  ( $=l_i$ ).  $P_i$  ( $x_p, y_p, z_p$ ) is a point on sphere as well as on line  $A_iB_i$  both, so it satisfies the equations as following.

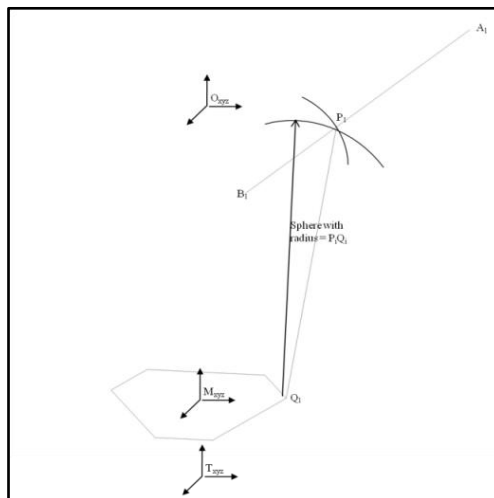


Fig 2 Single link of 3-PSU PKM

Line  $A_iB_i$  is represented by, for  $i^{th}$  link and  $m \in [0, 1]$ .

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$$\begin{aligned} x_p &= x_a + m(x_b - x_a) \\ y_p &= y_a + m(y_b - y_a) \\ z_p &= z_a + m(z_b - z_a), \end{aligned}$$

For sphere,

$$(x_p - x_q)^2 + (y_p - y_q)^2 + (z_p - z_q)^2 = l_i^2$$

$$(x_a + m(x_b - x_a) - x_q)^2 + (y_a + m(y_b - y_a) - y_q)^2 + (z_a + m(z_b - z_a) - z_q)^2 = l_i^2$$

Which on expansion forms a quadratic equation of m, by solving it two values of m will be obtained, among them only one value would belong to [0,1]. This value of valid m is used to solve the P(x<sub>p</sub>,y<sub>p</sub>,z<sub>p</sub>) from equations 1,2,3.

From  $d^2 = (x_p - x_a)^2 + (y_p - y_a)^2 + (z_p - z_a)^2$ , a set of d<sub>i</sub> is achieved among them d<sub>1</sub> = d<sub>2</sub>; d<sub>3</sub> = d<sub>4</sub>; d<sub>5</sub> = d<sub>6</sub> only three values (d<sub>1</sub>,d<sub>3</sub>,d<sub>5</sub>) is used to operate prismatic actuator.

### C. Simulation

The Inverse Kinematic solution of 3-PSU PKM as per the geometric approach discussed above is solved with the use of software Matlab. The obtained data of (d<sub>1</sub>, d<sub>3</sub>, d<sub>5</sub>) are sent to Pro/E mechanism for preliminary CAD model developed in Pro/E (see figure 3).

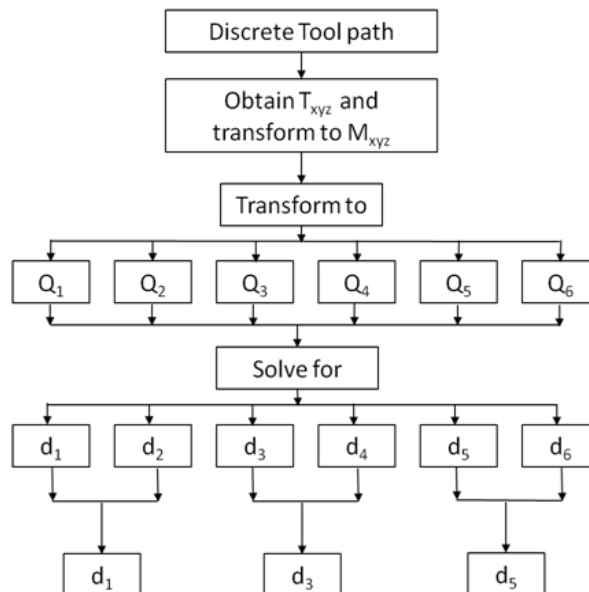


Fig 3 The process of Inverse Kinematics

### D. Examples

Case I:

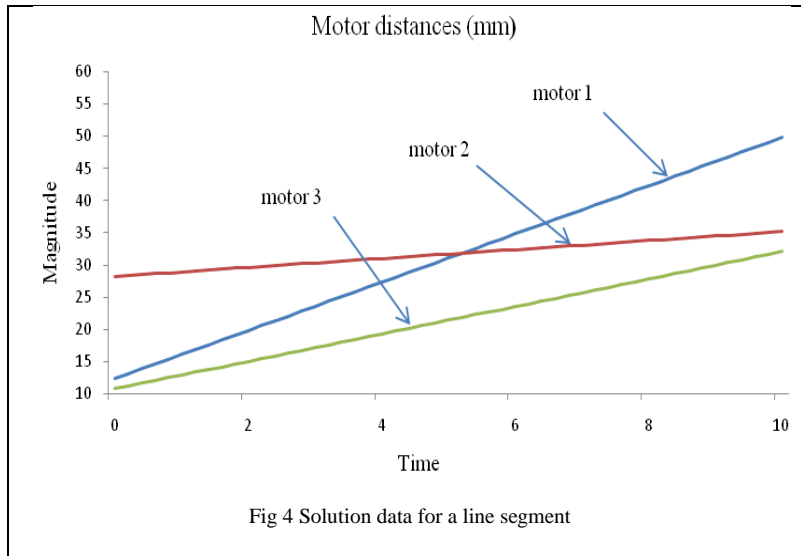
For the following line segment with start and end points given in 3D space three set of motor data was found for simulation (refer to figure 4).

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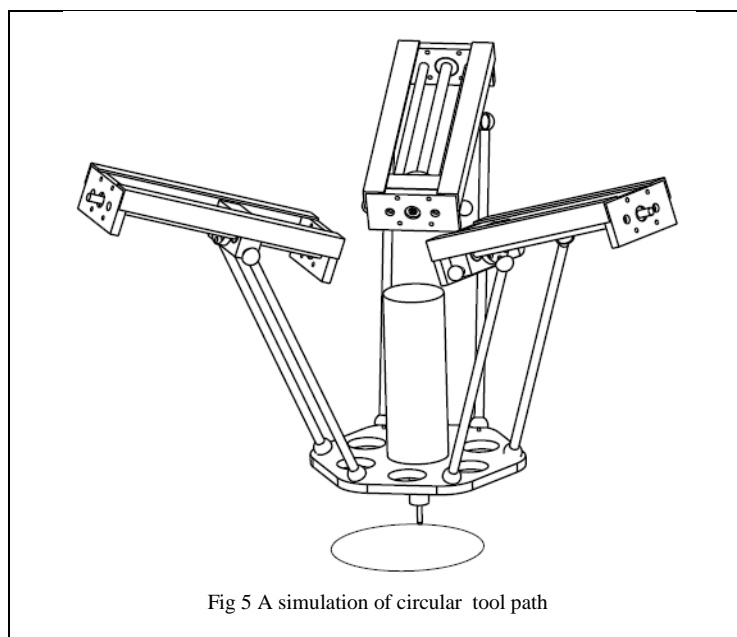
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Start point: [27.788946, -136.043956, 11.930000]  
 End point: [04.511992, -122.605000, 22.140000]



Case II:

A circle in offset plane with known parameter is simulated for Inverse Kinematic solutions. A typical setup for the PKM (shown as per figure 5) is developed in Pro/E before solving an Inverse Kinematics Problem for a circular contouring [9].

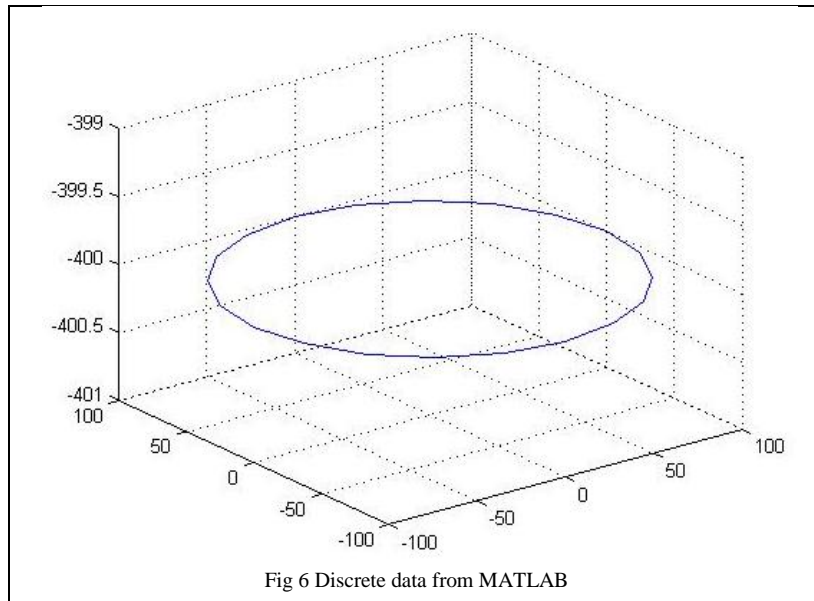


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To create the circle on plane with circle radius = 100 mm, circle centre 400 mm below the machine co-ordinate and completing one revolution in 20 second (low speed of feed), a discrete data set was created in MATLAB as shown in figure 6.



To the joint space data of a circle on plane PKM inverse kinematic problem was solved in MATLAB and solution data was plotted for three of the motor distances with respect to time this plot is as per figure 7 below. The plot clearly shows a harmonic behavior of the motors which further encourages better interpolation strategy to develop with control unit.

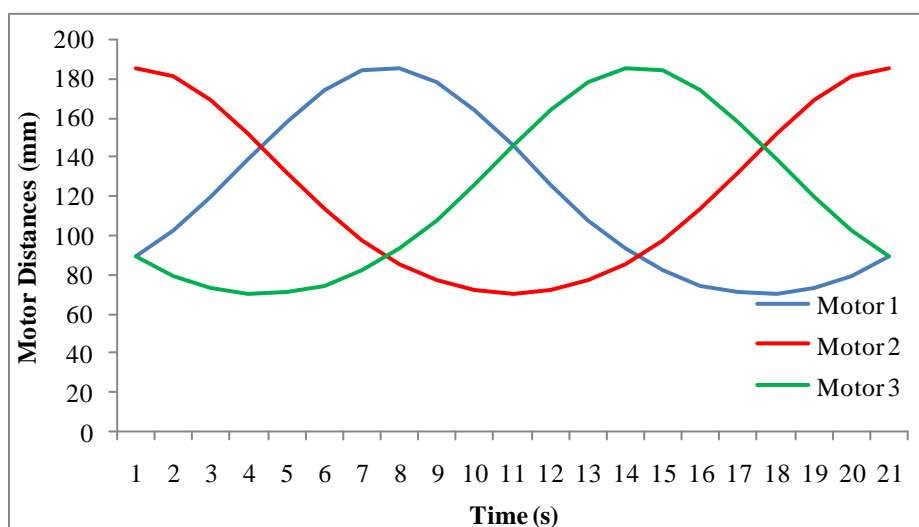


Fig 7 Inverse Kinematic Solution for circular contouring

**IV. RESULT**

Dynamic Analysis of the 3-PSU PKM, with the model setup discussed in previous section measures for the reaction forces acting on three different actuated moving parts. The force, F-d3 shown in figure 5 measures the reaction force on moving actuator due to the inertia of all moving parts. Similarly all the three forces along with its motion direction are measured separately during the single revolution of tool-platform on circular contouring of 100 mm radius. As the result of the Rigid Dynamics analysis of the current case, the forces were found as shown in figure 6.

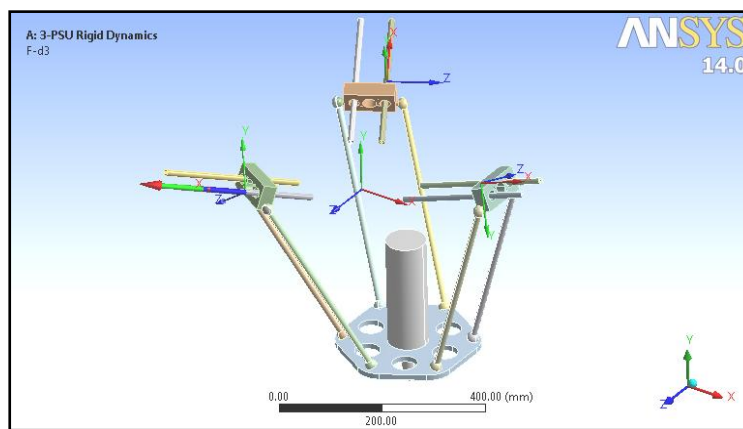


Fig 5 Actuator force on slider along with motion direction

*Result of Rigid Dynamics Analysis*

Here, forces change in specific manner, creating a regular wave form with some phase difference among them. The results are very equivalent to the results gained by Saha and Rao, (refer to the figure 7). It is to be noted that figure 7 shows six different actuator forces while in 3-PSU PKM, alternate forces should be considered for three actuator forces.

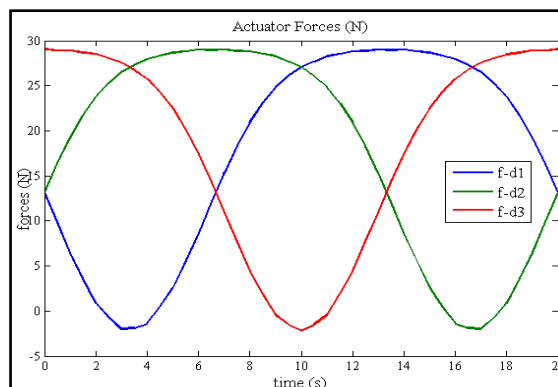


Fig 6 Results obtained by Simulation

## V. CONCLUSION

For a line segment and a circular path inside workspace, first Inverse Kinematic solution has been done with the use of MATLAB and further the solution has been verified with Pro/E mechanisms. Thus, a simulation methodology has been checked for a sample virtual prototype.

Further, dynamic analysis of the structure for a sample case has been done with the use of Rigid Dynamics (Ansys). The nature of the results is analyzed and compared with the results obtained by Saha and Rao who have worked on DeNOC methodology.

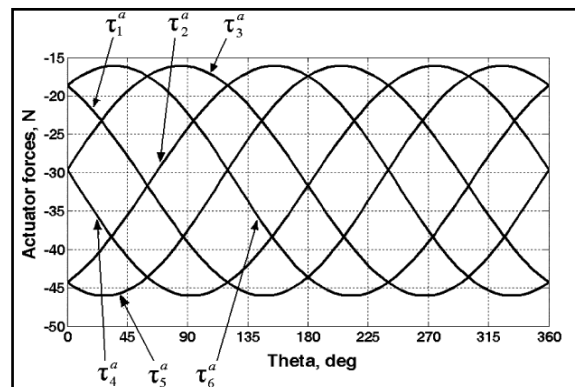


Fig 7 Results from DeNOC method derived by Saha and Rao [4]

It is to be concluded that, inertial forces produced by the motion of moving components of 3-PSU PKM at lower speeds are needed to be considered. The concept significantly suggests that these forces cannot be neglected while designing actuator components and while designing control scheme for actuation.

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