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Remarks on $\hat{\Omega}$-multifunctions via filters

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Abstract: Aim of this paper is to obtain a new class of separation known as $\hat{\Omega}$-compact Spaces. Their properties are investigated in terms of nets, filterbase and $\hat{\Omega}$-complete accumulation point. Also I-lower (resp.upper) $\hat{\Omega}$-continuous and $\hat{\Omega}$-multifunctions have been introduced to study $\hat{\Omega}$-compact spaces.

Key words and Phrases: $\hat{\Omega}$-closed sets., $\hat{\Omega}$-complete accumulation point., $\hat{\Omega}$-compactness., $\hat{\Omega}$-multifunctions.

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I INTRODUCTION

Compactness is one of the most useful and fundamental notions of not only general topology but also for other advanced branches of Mathematics. Lellis Thivagar et.al [5] recently introduce the class of $\hat{\Omega}$-closed sets which form a topology and properly lies between the class of δ-closed sets and that of ω-closed sets. The aim of this paper is to investigate some characterizations of $\hat{\Omega}$-compact Spaces in terms of nets and filterbase. By introducing the notion of $\hat{\Omega}$-complete accumulation points, we investigate some characterizations of $\hat{\Omega}$-compact Spaces. This paper is to introduce concepts such as I-lower (resp.upper) $\hat{\Omega}$-continuous and $\hat{\Omega}$-multifunctions by which $\hat{\Omega}$-compactness is studied. Also some characterizations of $\hat{\Omega}$-multifunctions is obtained.

II PRELIMINARIES

Throughout this paper (X,τ) (or briefly X) represent a topological space with no separation axioms assumed unless otherwise explicitly stated. For a subset A of (X,τ), we denote the closure of A, the interior of A and the complement of A as $cl(A), int(A)$ and A^c respectively. Some of the following notations which are used in this paper. The family of all open (resp. δ-open, $\hat{\Omega}$-open, $\hat{\Omega}$-closed) sets on X are denoted by $O(X)$ (resp. $\delta O(X), \hat{\Omega}O(X)$, $\hat{\Omega}C(X)$). Also $O(X,x)=\{U\in X: x\in U\in O(X)\}$; $\delta O(X,x)=\{U\in X: x\in U\in \delta O(X)\}$; $\hat{\Omega}O(X,x)=\{U\in X: x\in U\in \hat{\Omega}O(X)\}$;

Let us sketch some existing definitions, which are useful in the sequel as follows.

Definition 2.1 *[12] A subset A of X is called δ-closed in a topological space (X,τ) if $A=\delta cl(A)$, where $\delta cl(A)=\{x\in X: int(cl(U))\cap A\neq\varnothing, U\in O(X,x)\}$. The complement of δ-closed set in (X,τ) is called δ-open set in (X,τ). From [3] Lemma 3, $\delta cl(A)=\cap\{F\in\delta C(X): A\subseteq F\}$ and from Corollary 4, $\delta cl(A)$ is a δ-closed for a subset A in a topological space (X,τ).*

Definition 2.2 *A subset A of a topological space (X,τ) is called*

1) semiopen set in (X,τ) [7] if $A\subseteq cl(int(A))$.

2) $\hat{\Omega}$-closed set [5] if $\delta cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi open in (X,τ).

The complement of $\hat{\Omega}$-closed (resp. semi open) is said to be $\hat{\Omega}$-open (resp. semi closed).

Definition 2.3 *By a multivalued function [2], F on a set X into a set Y, denoted by $F: X\to Y$, we mean a relation from X into Y. That is $F\subseteq X\times Y$. Let $F: X\to Y$ be a multifunction. The upper and lower inverse of a subset V of Y*

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are denoted by $F^{+}(V)$ *and* $F^{-}(V)$ *respectively. They are defined as* $F^{+}(V)=\{x\in X:F(x)\subseteq V\}$ *and* $F^{-}(V)=\{x\in X:F(x)\cap V\neq\varnothing\}$.

Definition 2.4 *A multifunction* $F:X\to Y$ *is said to be*

1)upper continuous (or upper semi-continuous) [2](resp.lower continuous (or lower semi continuous)) if $F^{+}(V)$ (resp. $F^{-}(V)$) is open in X for every open set V in Y.

2) lower (resp.upper) α-continuous at a point $x\in X$ [8] if for each open set V in Y containing $F(x)$, there exists a α-open set in X containing x such that $F(x)\cap V\neq\varnothing$ (resp. $F(U)\subseteq V$).

3)lower (resp.upper) pre-continuous at a point $x\in X$ [9] if for each open set V in Y containing $F(x)$,there exists a pre open set in X containing x such that $F(x)\cap V\neq\varnothing$ (resp. $F(U)\subseteq V$).

4)lower (resp.upper) β-continuous at a point $x\in X$ [10][11] if for each open set V in Y containing $F(x)$, there exists a β-open set in X containing x such that $F(x)\cap V\neq\varnothing$ (resp. $F(U)\subseteq V$).

Definition 2.5 [6] A space (X,τ) is called ${}_{\omega}T_{\hat{\Omega}}$-space if every ω-closed set in X is $\hat{\Omega}$-closed in X.

Theorem 2.6[6] **(Theorem 3.14)** *A space* (X,τ) *is* ${}_{\omega}T_{\hat{\Omega}}$ *-space if and only if every closed set is* $\hat{\Omega}$ *-closed in* (X,τ).

III $\hat{\Omega}$-COMPACT SPACES VIA FILTERS

Definition 3.1 *Let* Λ *be a directed set. A net* $\lambda=\{x_{\alpha}:\alpha\in\Lambda\}$ $\hat{\Omega}$ *-accumulate at a point* $x\in X$ *if the net is frequently in every* $U\in\hat{\Omega}O(X,x)$. *That is, for each* $U\in\hat{\Omega}O(X,x)$ *and for each* $\alpha_0\in\Lambda$, *there exists* $\alpha\square\alpha_0$ *such that* $x_{\alpha}\in U$.

Definition 3.2 *A filter base* $B=\{B_{\alpha}:\alpha\in J\}$ $\hat{\Omega}$ *converge to a point* $x\in X$ *if for each* $U\in\hat{\Omega}O(X,x)$, *there exists an* B *in* B *such that* $B\subseteq U$.

Definition 3.3 *A point* $x\in X$ *is said to be* $\hat{\Omega}$ *-adherent point of a filter base* B *on a space* X *if* $x\in\hat{\Omega}cl(B)$ *for every* $B\in B$.

Definition 3.4 *A point* $x\in X$ *is said to be* $\hat{\Omega}$ *-complete accumulation point of a subset* S *of a space* X *if* $|S\cap U|=|S|$ *for each* $U\in\hat{\Omega}O(X,x)$. *Where* $|S|$ *denotes the cardinality of* S.

Example 3.5 *Let* $X=\{a,b,c\},\tau=\{\varnothing,\{a,b\},X\}$. *Then* $\hat{\Omega}O(X)=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$. If $S=\{b\}$, then the points b and c are $\hat{\Omega}$-complete accumulation points whereas a is not.

Definition 3.6 *A family* $\{U_{\alpha}:U_{\alpha}\in\hat{\Omega}O(X),\alpha\in J\}$ *(where J is an indexed set) is said to be* $\hat{\Omega}$ *-open cover of a subset A of X if* $A\subseteq\bigcup_{\alpha\in J}U_{\alpha}$. *If there exists a finite set* J_0 *of J such that* $A\subseteq\bigcup_{\alpha\in J_o}U_{\alpha}$ *,then it is known that* $\hat{\Omega}$ *-open cover of a subset* A *has a finite sub cover.*

Definition 3.7 *A space* X *is a* $\hat{\Omega}$ *-compact if every* $\hat{\Omega}$ *-open cover of* X *has a finite sub cover.*

Characterization of $\hat{\Omega}$-compact spaces.

Theorem 3.8 *A space* X *is said to be a* $\hat{\Omega}$ *-compact if and only if each infinite subset of* X *has a* $\hat{\Omega}$ *-complete accumulation point.*

Proof. **Necessity-** Suppose that X is $\hat{\Omega}$-compact and S is any infinite subset of X. If F is the set of all points $x\in X$ which are not $\hat{\Omega}$-complete accumulation points of S. Then, for each point $x\in F$, there exists $U_x\in\hat{\Omega}O(X,x)$ such that $|S\cap U_x|\neq|S|$. If $F=X$, then the collection $U=\{U_x:x\in X\}$ is a $\hat{\Omega}$-open cover of X. By hypothesis, there exists finite number of points $x_1,x_2,x_3,...,x_n$ in X such that $X\subseteq\bigcup_{i=1}^{i=n}U_{x_i}$. Then, $S\subseteq\bigcup_{i=1}^{i=n}(U_{x_i}\cap S)$. Therefore, $|S|=\max\{|U_{x_i}\cap S|:i=1,2,...n.\}$ a contradiction to $|S\cap U_x|\neq|S|$ for any $x\in F$. Therefore, there exists $x\in X\setminus F$ which is $\hat{\Omega}$-complete accumulation points of S.

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Vol. 2, Issue 11, November 2013

Sufficiency- Assume that every infinite subset of X has an $\hat{\Omega}$-complete accumulation point in X and Suppose that X is not $\hat{\Omega}$-compact. Then there exists a $\hat{\Omega}$-open cover $\mathcal{U}$ of X which has no finite sub cover. If we define, $\delta = \min\{|\Phi| : \Phi \subseteq \mathcal{U}$, where Φ is an $\hat{\Omega}$-open cover of $X\}$, then we have a $\hat{\Omega}$-open cover Γ of X such that $\Gamma \subseteq \mathcal{U}$ and $\delta = |\Gamma|$.

By hypothesis, $\delta \square |N|$, where N is the set of all natural numbers. By well ordering of Γ, by some minimal well ordering say : suppose that U is any member of Γ. By minimal well ordering : , $|\{G : G \in \Gamma, G : U\}| \lhd |\{G : G \in \Gamma\}|$. Since Γ can not have any sub cover with cardinality less than δ, we have for each $U \in \Gamma, X \neq \bigcup\{G : G \in \Gamma, G : U\}$. It is always possible that for each $U \in \Gamma$, choose $x(U) \in X \setminus \bigcup\{G \cup x(G) : G \in \Gamma, G : U\}$, if not,one can choose a cover of smaller cardinality from Γ. If $H = \{x(U) : U \in \Gamma\}$, then it is enough to show that H has no $\hat{\Omega}$-complete accumulation point in X. Let $z \in X$. Since Γ is a cover of X, there exist $W \in \Gamma$ such that $z \in W$. Since $U : W, x(U) \in W$. It follows that $T = \{U : U \in \Gamma$ and $x(U) \in W\} \subseteq \{G : G \in \Gamma, G : W\}$. But $|T| < \delta$. Therefore, $|H \cap W| < \delta$. But $|H| = \delta \square |N|$, since for any two distinct members U, V in $\Gamma, x(U) \neq x(W)$. This means that H has no $\hat{\Omega}$-complete accumulation point in X, a contradiction to our assumption.Therefore, X is $\hat{\Omega}$-compact.

Theorem 3.9 *A space X is $\hat{\Omega}$-compact if and only if every net in X with a well ordered directed set as it's domain $\hat{\Omega}$-accumulates to some point of X.*

Proof. **Necessity-** Suppose that X is $\hat{\Omega}$-compact and $\lambda = \{x_\alpha : \alpha \in \Lambda\}$ is a net with well ordered directed set Λ as domain. Assume that λ never $\hat{\Omega}$-accumulates at any point of X. By the definition of a net $\hat{\Omega}$-accumulation,for each $x \in X$, there exists $U_x \in \hat{\Omega}O(X, x)$ and $\alpha_x \in \Lambda$ such that $U_x \cap \{x_\alpha : \alpha \square \alpha_x\} = \varnothing$. Therefore, $\{x_\alpha : \alpha \square \alpha_x\} \subseteq X \setminus U_x$. Now the collection $C = \{U_x : x \in X\}$ is a $\hat{\Omega}$-open cover of X. By hypothesis, there exists a finite set of points $x_1, x_2, x_2, \ldots, x_n$ in X such that $X = \bigcup_{i=1}^{i=n} U_{x_i}$. Since Λ is a well ordered set,every finite set has the largest element. Therefore, denote the largest element of the subset $\{\alpha_{x_i} : i = 1, 2, \ldots, n\}$ of Λ as $\alpha_{x_j}, 1 \square j \square n$. Then for $\gamma \square \alpha_{x_j}$, we have $\{x_\delta : \delta \square \gamma\} \subseteq \bigcap_{i=1}^{i=n} (X \setminus U_{x_i}) = X \setminus \bigcup_{i=1}^{i-n} U_{x_i} = \varnothing$ which is not possible. Therefore, λ has at least one $\hat{\Omega}$-accumulation point in X.

Sufficiency- Suppose that S is any infinite subset X. By Zorn's Lemma, S is a well ordered subset of X. Then we can assume that S to be a net with a domain which is well ordered index set. By hypothesis, it has an $\hat{\Omega}$-accumulation point say z in X. Therefore, z is an $\hat{\Omega}$-complete accumulation point of S. By Theorem 3.9, X is $\hat{\Omega}$-compact.

Theorem 3.10 *A space X is $\hat{\Omega}$-compact if and only if for every collection of $\hat{\Omega}$-closed sets in X with the finite intersection property has a non empty intersection.*

Proof. **Necessity-** Suppose that $Y = \{F : F \in \hat{\Omega}C(X)\}$ be a collection which satisfies finite intersection property and suppose that $\cap\{F : F \in Y\} = \varnothing$. By De-Morgan's Law, $X = \cup\{X \setminus F : X \setminus F \in \hat{\Omega}O(X)\}$. By hypothesis, there exists a finite number of $\hat{\Omega}$-closed sets $F_1, F_2, F_3, \ldots, F_n$, such that $X = \bigcup_{i=1}^{i=n}\{X \setminus F_i : X \setminus F_i \in \hat{\Omega}O(X)\}$. Again by De-Morgan's Law, $\bigcap_{i=1}^{i=n} F_i = \varnothing$, a contradiction.

Sufficiency- If a space X is not compact then there exists a $\hat{\Omega}$-open cover Γ of X which has no finite sub cover.It follows that for any n, $X \neq \bigcup_{i=1}^{i=n}\{G_i : G_i \in Y\}$ and hence $\varnothing \neq \bigcap_{i=1}^{i=n}\{X \setminus G_i : X \setminus G_i \in \hat{\Omega}C(X)\}$. Therefore, there exists a family

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Vol. 2, Issue 11, November 2013

$B=\{X\setminus G : G\in\hat{\Omega}O(X)\}$ of $\hat{\Omega}$-closed sets in X with finite intersection property. By hypothesis, $\cap\{(X\setminus G) : G\in B\}\neq\varnothing$ and hence $X\neq\cup\{G : G\in\hat{\Omega}O(X)\}$, a contradiction.

Theorem 3.11 *A space X is $\hat{\Omega}$-compact if and only if each filter base in X has at least one $\hat{\Omega}$-adherent point in X.*

Proof. **Necessity-** Suppose that X is $\hat{\Omega}$-compact and $\Phi=\{F_\alpha : \alpha\in J\}$ is a filter base in it. Since in a filter Φ, every finite intersection of sets of Φ belongs to Φ and the empty set is not in Φ, Φ has finite intersection property and hence $\{\hat{\Omega}cl(F_\alpha) : \alpha\in J\}$ has finite intersection property. By Theorem 3.10, $\bigcap_{\alpha\in J}\hat{\Omega}cl(F_\alpha)\neq\varnothing$. Therefore, Φ has at least one $\hat{\Omega}$-adherent point in X.

Sufficiency- Suppose that Φ is any family of $\hat{\Omega}$-closed sets and each finite intersection is non-empty. The sets F_α with their finite intersection provide a filter base B. By hypothesis, B has at least one $\hat{\Omega}$-adherent point x in X. Then, $x\in\bigcap_{\alpha\in J}\hat{\Omega}cl(F_\alpha)=\bigcap_{\alpha\in J}F_\alpha$. Therefore, $\bigcap_{\alpha\in J}F_\alpha\neq\varnothing$ and by Theorem 3.10, X is $\hat{\Omega}$-compact.

Theorem 3.12 *A space X is $\hat{\Omega}$-compact if and only if each filter base Φ on X with at most one $\hat{\Omega}$-adherent point in X is $\hat{\Omega}$-convergent.*

Proof. **Necessity-** Suppose that X is $\hat{\Omega}$-compact and $x\in X$ is any point and Φ is any filter base on X. The $\hat{\Omega}$-adherence of Φ is a subset of $\{x\}$. By Theorem 3.11, $\hat{\Omega}$-adherence of Φ is equal to $\{x\}$ and hence $\bigcap_{F\in\Phi}\hat{\Omega}cl(F)=\{x\}$.

Assume the contrary that, there exists $U\in\hat{\Omega}O(X,x)$ such that for all $F\in\Phi, F\cap(X\setminus U)\neq\varnothing$. Then $X=\{F\setminus U : F\in\Phi\}$ is a filter base on X. By Theorem 3.11, $\hat{\Omega}$-adherence of X is non-empty. However, $\bigcap_{F\in X}\hat{\Omega}cl(F\setminus U)\subseteq\bigcap_{F\in X}\hat{\Omega}cl(F)\cap(X\setminus U)=\{x\}\cap(X\setminus U)=\varnothing$, a contradiction. Therefore, assumption is wrong and hence for each $U\in\hat{\Omega}O(X,x)$, there exist $F\in\Phi$ with $F\subseteq V$. Therefore, Φ is $\hat{\Omega}$-convergent to a point x.

Sufficiency- It is enough to show that each filter base in X has at least one $\hat{\Omega}$-accumulation point. Suppose that there exist a filter base Φ on X with no $\hat{\Omega}$-adherent point. By hypothesis, Φ $\hat{\Omega}$-converges to a point $x\in X$.Let F_α be an arbitrary element of Φ. Then, for each $U\in\hat{\Omega}O(X,x)$, $F_\beta\in\Phi$ such that $F_\beta\subseteq U$. Since Φ is a filter base, there exists a γ such that $\varnothing\neq F_\gamma\subseteq F_\alpha\cap F_\beta\subseteq F_\alpha\cap U$. Therefore, $F_\alpha\cap U\neq\varnothing$ for any $U\in\hat{\Omega}O(X,x)$ and hence $x\in\hat{\Omega}cl(F_\alpha)$ for each α and hence $x\in\bigcap_{\alpha}\hat{\Omega}cl(F_\alpha)$. Therefore, x is a $\hat{\Omega}$-adherent point of Φ, a contradiction. Therefore, our assumption is wrong and hence X is $\hat{\Omega}$-compact.

Definition 3.13 *A mapping $f:(X,\tau)\to P$ is said to be 1-lower (resp. 1-upper) $\hat{\Omega}$-continuous at the point $x\in X$ if for each real number $r>0$, there exists a $\hat{\Omega}$-open set $U\in\hat{\Omega}O(X,x)$, $f(u)>f(x)-r$ (resp. $f(u)>f(x)+r$)for every $u\in U$. The function f is 1-lower (resp. 1-upper) $\hat{\Omega}$-continuous in X if f is 1-lower (resp. 1-upper) $\hat{\Omega}$-continuous at every point of X.*

Theorem 3.14 *A function $f:(X,\tau)\to P$ is 1-lower $\hat{\Omega}$-continuous in X if and only if for each $r\in P$, $\{x\in X : f(x)\square r\}$ is $\hat{\Omega}$-closed.*

Proof. The collection $\sigma=\{(r,\infty) : r\in P\}\cup P$ is a topology on P. Then, function $f:(X,\tau)\to P$ is 1-lower $\hat{\Omega}$-continuous in X if and only if $f:(X,\tau)\to(P,\sigma)$ is $\hat{\Omega}$-continuous. Then, inverse image of every closed set in (P,σ) is $\hat{\Omega}$-closed in X. Since for each $r\in P,(-\infty,r]$ is closed in (P,σ), $f^{-1}((-\infty,r])=\{x\in X : f(x)\square\, r\}$ is $\hat{\Omega}$-closed in X.

Corollary 3.15 *A subset A of a space X is compact if and only if the characteristic function χ_A is 1-lower $\hat{\Omega}$-continuous map.*

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Vol. 2, Issue 11, November 2013

Theorem 3.16 *A function* $f:(X,\tau)\to P$ *is* 1*-upper* $\hat{\Omega}$*-continuous in* X *if and only if for each* $r\in P$, $\{x\in X: f(x)\square r\}$ *is* $\hat{\Omega}$*-closed.*

Proof. It is similar to that of Theorem 3.15.

Corollary 3.17 *A subset* A *of a space* X *is compact if and only if the characteristic function* χ_A *is* 1*-upper* $\hat{\Omega}$*-continuous map.*

Theorem 3.18 *If* $F(x)=\sup_{i\in J}\{f_i(x)\}$ *exists where each* f_i : $(X,\tau)\to P$ *is a* 1*-lower* $\hat{\Omega}$*-continuous, then* $F(x)$ *is* 1*-lower* $\hat{\Omega}$*-continuous.*

Proof. Suppose that $r\in P$ be arbitrary and suppose that $F(x)<r$. Then for each $i\in J$, $f_i(x)<r$. Then $\{x\in X: F(x)\square r\}=\bigcap_{i\in J}\{x\in X: f_i(x)\square r\}$. Since each f_i is a 1-lower $\hat{\Omega}$-continuous, $r\in P$,$\{x\in X: f_i(x)\square r\}$ is $\hat{\Omega}$-closed. By [5] Theorem 4.16, $\bigcap_{i\in J}\{x\in X: f_i(x)\square r\}$ is $\hat{\Omega}$-closed. Thus, $F(x)$ is 1-lower $\hat{\Omega}$-continuous.

Theorem 3.19 *If a function* $f:(X,\tau)\to P$ *is* 1*-lower* $\hat{\Omega}$*-continuous in a* $\hat{\Omega}$*-compact space* X, *then* f *assumes the value* $p=\inf_{x\in X} f(x)$.

Proof. Suppose that $f:(X,\tau)\to P$ is 1-upper $\hat{\Omega}$-continuous in a $\hat{\Omega}$-compact space X and suppose $r>p$.By the infimum property, $A_r=\{x\in X: f(x)\square r\}$ is a non-empty $\hat{\Omega}$-closed set in X. Then the collection $\{A_r : r>p\}$ is a family of non-empty $\hat{\Omega}$-closed sets in X with finite intersection property. By Theorem 3.11, there exist $x\in X$ such that $x\in\bigcap_{r>p}\{A_r\}$. Therefore, $p=f(x)$ and hence the result.

Theorem 3.20 *If* $G(x)=\inf_{i\in J}\{f_i(x)\}$ *exists where each* $f_i:(X,\tau)\to P$ *is a* 1*-upper* $\hat{\Omega}$*-continuous, then* $G(x)$ *is* 1*-upper* $\hat{\Omega}$*-continuous.*

Proof. Similar to that of Theorem 3.18.

Theorem 3.21 *If a function* $f:(X,\tau)\to P$ *is* 1*-upper* $\hat{\Omega}$*-continuous in a* $\hat{\Omega}$*-compact space* X, *then* f *assumes the value* $q=\sup_{x\in X} f(x)$.

Proof. Similar to that of 3.19.

Remark 3.22 *If a function* f *on a space* X *satisfy the conditions of Theorems 3.18 and 3.20, then* f *is bounded and attains its bounds.*

IV $\hat{\Omega}$-MULTIFUNCTIONS

Definition 4.1 *A multifunction* $F: X\to Y$ *is said to be upper* $\hat{\Omega}$*-continuous at* $x\in X$ *if for each open subset* V *of* Y *with* $F(x)\subseteq V$, *there exists a* $\hat{\Omega}$*-open set* U *in* X *containing* x *such that* $F(U)\subseteq V$.

Definition 4.2 *A multifunction* $F: X\to Y$ *is said to be lower* $\hat{\Omega}$*-continuous at* $x\in X$ *if for each open subset* V *of* Y *with* $F(x)\cap V\neq\varnothing$, *there exists a* $\hat{\Omega}$*-open set* U *in* X *containing* x *such that* $F(u)\cap V\neq\varnothing$ *for each* $u\in U$.

Example 4.3 *Let* $X=Y=\{a,b.c.d\}$, $\tau=\{\varnothing,\{b,c\},\{a,b,c\},\{b,c,d\},X\}$ *and* $\sigma=\{\varnothing,\{b,c\},Y\}$. Define the multifunction $F: X\to Y$ by $F(a)=\{a,d\}$, $F(b)=\{b,c\}$, $F(c)=\{c,d\}$ and $F(d)=d$. Then F is upper $\hat{\Omega}$-continuous and lower $\hat{\Omega}$-continuous mappings.

Theorem 4.4 *In a topological space* (X,τ) *,every upper (resp.lower)* $\hat{\Omega}$*-continuous is* upper (resp.lower) pre-continuous upper (resp.lower) β-continuous.

Proof. It follows from the fact [5] every $\hat{\Omega}$-open set is pre-open as well as β-open.

Remark 4.5 *From the following example,the converse of Theorem 4.4 is not always true.*

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Vol. 2, Issue 11, November 2013

Example 4.6 *Let* $X = Y = \{a,b.c.d\}$, $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},X\}$ *and* $\sigma = \{\emptyset,\{a,b,c\},Y\}$. *Define the multifunction* $F:X \to Y$ *by* $F(a) = \{a,c\}$, $F(b) = \{b,c\}$, $F(c) = \{a,b\}$ and $F(d) = \{a,d\}$. Then F is upper pre-continuous and upper β-continuous but not a F is upper $\hat{\Omega}$-continuous.

Remark 4.7 *From Examples 4.3 and 4.6 it is known that the notion of upper* $\hat{\Omega}$ *-continuous and upper* α *-continuous, upper* $\hat{\Omega}$ *-continuous and upper continuity are independent.*

Theorem 4.8 *In a topological space* (X,τ), *every upper (resp.lower) super continuous function is upper (resp.lower)* $\hat{\Omega}$ *-continuous.*

Proof. It follows from the fact [5] every δ-open set is $\hat{\Omega}$-open set and [1], Theorem 1 and Theorem 2.

The following theorem states some characterizations of upper $\hat{\Omega}$-continuous mappings.

Theorem 4.9 *The following statements are equivalent for a multifunction* $F:X \to Y$.

i) F is upper $\hat{\Omega}$-continuous.

ii) $F^{+}(V) \in \hat{\Omega}O(X)$ for any open set V of Y.

iii) $F^{-}(C) \in \hat{\Omega}C(X)$ for any closed set C of Y.

iv) For any subset B of Y, $\hat{\Omega}cl(F^{-1}(B)) \subseteq F^{-1}(cl(B))$.

Proof. $(i) \Rightarrow (ii)$ Suppose that V is any open set in Y and $x \in F^{+}(V)$. Then $F(x) \subseteq V$. By hypothesis, there exists a a $\hat{\Omega}$-open set U in X containing x such that $F(U) \subseteq V$. Then, $F^{+}(V) = \cup\{U : U \in \hat{\Omega}O(X,x), F(U) \subseteq V\}$. By [5] Theorem 4.16, $F^{+}(V)$ is $\hat{\Omega}$-open in X.

$(ii) \Rightarrow (iii)$ Since $F^{+}(Y \setminus B) = X \setminus F^{-}(B)$ for any subset B of Y and since complement of $\hat{\Omega}$-open set is $\hat{\Omega}$-closed, (ii) holds.

$(iii) \Rightarrow (iv)$ Suppose that B is any subset of Y .Since $cl(B)$ is a closed set in Y and by hypothesis, $F^{-}(B)$ is $\hat{\Omega}$-closed set in X. Also $B \subseteq cl(B)$ and so $F^{-}(B) \subseteq F^{-}(cl(B))$.By [5] Remark 5.2, $\hat{\Omega}cl(F^{-}(B))$ is the smallest $\hat{\Omega}$-closed set containing $F^{-}(B)$. Therefore, $\hat{\Omega}cl(F^{-}(B)) \subseteq F^{-}(cl(B))$.

$(iv) \Rightarrow (iii)$ Suppose that C is any closed subset of Y. By hypothesis, $\hat{\Omega}cl(F^{-}(C)) \subseteq F^{-}(cl(C)) = F^{-}(C)$. Therefore, $F^{-}(C)$ is $\hat{\Omega}$-closed subset of X.

$(iii) \Rightarrow (ii)$ Suppose that V is any open subset of Y. Then $Y \setminus V$ is a closed subset of Y. By hypothesis, $F^{-}(Y \setminus V)$ is $\hat{\Omega}$-closed subset of X. That is, $X \setminus F^{+}(V)$ is $\hat{\Omega}$-closed subset of X and hence $F^{+}(V)$ is $\hat{\Omega}$-open subset of X.

$(ii) \Rightarrow (i)$ Suppose that $x \in X$ and V be any open subset of Y such that $F(x) \subseteq V$. Then, $x \in F^{+}(V)$ and by hypothesis, $F^{+}(V)$ is $\hat{\Omega}$-open subset of X. Therefore, there exists $U \in \hat{\Omega}O(X,x)$ such that $x \in U \subseteq F^{+}(V)$. Thus, $F(U) \subseteq V$.

Characterizations of lower $\hat{\Omega}$-continuous mappings.

Theorem 4.10 *The following statements are equivalent for a multifunction* $F:X \to Y$.

1) F is lower $\hat{\Omega}$-continuous.

2) $F^{-}(V) \in \hat{\Omega}O(X)$ for any open set V of Y.

3) $F^{+}(C) \in \hat{\Omega}C(X)$ for any closed set C of Y.

4)For any subset B of Y, $\hat{\Omega}cl(F^{+}(B)) \subseteq F^{+}(cl(B))$.

5)For any subset A of X, $F(\hat{\Omega}cl(A)) \subseteq cl(F(A))$.

proof. It is similar to that of Theorem 4.3.

Theorem 4.11 *The following statements are equivalent for a multifunction* $F:X \to Y$.

i) F is lower $\hat{\Omega}$-continuous.

ii)If U is any open subset of Y and $x \in F^{-}(U)$, then there exists $V \in \hat{\Omega}O(X,x)$ such that $V \subseteq F^{-}(U)$.

iii) If D is any closed subset of Y and $x \notin F^{+}(D)$, then there exists $\hat{\Omega}$-closed set C in X such that $x \notin C$ $F^{+}(D) \subseteq C$.

IJIRSET

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

Proof. (i)$\Rightarrow$(ii) Suppose that U is any open subset of Y and $x \in F^-(U)$. By Theorem 4.10 (ii), $F^-(U)$ is a $\hat{\Omega}$ -open subset of X. By letting $V = F^-(U)$, $V \in \hat{\Omega}O(X,x)$ such that $V \subseteq F^-(U)$.

(ii)$\Rightarrow$(iii) Suppose that D is any closed subset of Y and $x \notin F^+(D)$. Then, $x \in X \setminus F^+(D) = F^-(Y \setminus D)$. Therefore, $Y \setminus D$ is open in Y and $x \in F^-(Y \setminus D)$. By hypothesis, there exists $V \in \hat{\Omega}O(X,x)$ such that $V \subseteq F^-(Y \setminus D)$. Then, $F^+(D) = X \setminus F^-(Y \setminus D) \subseteq X \setminus V$. By letting $C = X \setminus V$, it is shown that there exists $\hat{\Omega}$ -closed set C in X such that $x \notin C$ and $F^+(D) \subseteq C$.

(iii)$\Rightarrow$(i) Suppose that D is any closed subset of Y and $x \notin F^+(D)$. By hypothesis, there exists $\hat{\Omega}$ -closed set C in X such that $x \notin C$ and $F^+(D) \subseteq C$. Then, $\hat{\Omega}clF^+(D) \subseteq \hat{\Omega}cl(C) = C$ and so $x \notin \hat{\Omega}cl(F^+(D))$. Thus, $\hat{\Omega}cl(F^+(D)) = F^+(D)$ and hence $F^+(D)$ is $\hat{\Omega}$ -closed subset of X .By Theorem 4.10 (ii), F is lower $\hat{\Omega}$ -continuous.

Definition 4.12 *The graph* $G_F : X \to Y$ *of a multifunction* $F : X \to Y$ *is given by* $G_F(x) = \{x\} \times F(x)$ *for each point* $x \in X$.

Lemma 4.13 *[?] The following statements are true for any multifunction* $F : X \to Y$ for any subsets A of X and B of Y 1) $G_{F^+}(A \times B) = A \cap F^+(B)$ .2) $G_{F^-}(A \times B) = A \cap F^-(B)$.

Theorem 4.14 *Let* $F : X \to Y$ *is a multifunction such that* $F(x)$ *is compact for each point* $x \in X$. *Then* F *is upper* $\hat{\Omega}$ *-continuous if and only if* $G_F : X \to X \times Y$ *is upper* $\hat{\Omega}$ *-continuous.*

Proof. **Necessity-** Suppose that $F : X \to Y$ is a upper $\hat{\Omega}$ -continuous and $x \in X$, W is any open subset of $X \times Y$ containing $G_F(x)$. Then for each $y \in F(x)$, there exists open subsets U_y in X and V_y in Y respectively such that $(x,y) \in U_y \times V_y \subseteq W$. The collection $\{V_y : y \in F(x)\}$ is an open cover of $F(x)$. Since $F(x)$ is compact, there exists a finite number of points $y_1, y_2, ... y_n$ in $F(x)$ such that $F(x) \subseteq \bigcup_{i=1}^{i=n} V_{y_i}$ .Put $U = \bigcap_{i=1}^{i=n} U_{y_i}$ and $V = \bigcup_{i=1}^{i=n} V_{y_i}$. Then U and V are open subsets of X and Y respectively such that $(x,y) \in U \times V \subseteq W$ .Since F is upper $\hat{\Omega}$ -continuous, there exists $G \in \hat{\Omega}O(X,x)$ such that $F(G) \subseteq V$. By Lemma 4.6, $U \cup G \subseteq U \cap F^+(V) = G_{F^+}(U \times V) \subseteq G_{F^+}(W)$. Therefore, $U \cap G \in \hat{\Omega}O(X,x)$ and $G_F(U \cap G) \subseteq W$ and hence G_F is upper $\hat{\Omega}$ -continuous.

Sufficiency- Suppose that $G_F : X \to X \times Y$ is upper $\hat{\Omega}$ -continuous, $x \in X$ and V is any open subset of Y containing $F(x)$. Since $X \times V$ is an open set in $X \times Y$ and $G_F(x) \subseteq X \times V$, there exists $U \in \hat{\Omega}O(X,x)$ such that $G_F(U) \subseteq X \times V$. By Lemma 4.6, $U \subseteq G_{F^+}(X \times V) = F^+(V)$ and $F(U) \subseteq V$. Thus, F is upper $\hat{\Omega}$ -continuous.

Theorem 4.15 *A multifunction* $F : X \to Y$ *is lower* $\hat{\Omega}$ *-continuous if and only if* $G_F : X \to X \times Y$ *is lower* $\hat{\Omega}$ *-continuous.*

Proof. **Necessity-** Suppose that $F : X \to Y$ is a lower $\hat{\Omega}$ -continuous and $x \in X$, W is any open subset of $X \times Y$ such that $x \in G_{F^-}(W)$ .Then $W \cap (\{x\} \times F(x)) \neq \varnothing$ .Therefore,there exists $y \in F(x)$ such that $(x,y) \in W$ and hence $(x,y) \subseteq U \times V \subseteq W$ for some open subsets U and V of X and Y respectively.Since $F(x) \cap V \neq \varnothing$, there exists $G \in \hat{\Omega}O(X,x)$ such that $G \subseteq F^-(V)$. Then , $U \cap G \subseteq U \cap F^-(V) = G_{F^-}(U \times V) \subseteq G_{F^-}(W)$.

Thus, $U \cap G \in \hat{\Omega}O(X,x)$ and hence G_F is lower $\hat{\Omega}$ -continuous.

Sufficiency- Assume $G_F : X \to X \times Y$ is lower $\hat{\Omega}$ -continuous, $x \in X$, V is any open subset of Y such that $x \in F^-(V)$ .Now $X \times V$ is open in $X \times Y$, and $G_F(x) \cap (X \times V) = (\{x\} \times F(x)) \cap X \times V = \{x\} \times (F(x) \cap V) \neq \varnothing$.

.Since F is lower $\hat{\Omega}$ -continuous,there exists $U \in \hat{\Omega}O(X,x)$ such that $U \subseteq G_{F^-}(X \times V)$. By Lemma 4.6, $U \subseteq F^-(V)$ and hence F is lower $\hat{\Omega}$ -continuous.

Definition 4.16 *A subset* A *of a topological space* X *is said to be*

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

1) α - para compact [13], if every cover of A by open sets of X is refined by a cover of A which consists of open sets of X and is locally finite in X.
2) α -regular [4] if for each $a \in A$ and each open set U of X containing a, there exists an open set G of X such that $a \in G \subseteq \delta cl(G) \subseteq U$.

Lemma 4.17 *[4], If A is α-para compact and α-regular in a topological space X and U is an open neighbourhood of A, then there exists an open set G of X such that $A \subseteq G \subseteq cl(G) \subseteq U$.*

Definition 4.18 *Let $F: X \to Y$ is a multifunction. $(\hat{\Omega}clF): X \to Y$ is defined as $(\hat{\Omega}clF)(x) = \hat{\Omega}cl(F(x))$ for each $x \in X$.*

Lemma 4.19 *If a multifunction $F: X \to Y$ is such that $F(x)$ is α-para compact and α-regular at each $x \in X$ and Y is ${}_{\omega}T_{\hat{\Omega}}$, then $(\hat{\Omega}clF)^{+}(V) = F^{+}(V)$ for every open set V in Y.*

Proof. Let V be any open set in Y and $x \in (\hat{\Omega}clF)^{+}(V)$. Then $(\hat{\Omega}clF)(x) \subseteq V$ and hence $F(x) \subseteq \hat{\Omega}cl(F(x)) \subseteq V$. Thus, $x \in F^{+}(V)$. Therefore, $(\hat{\Omega}clF)^{+}(V) \subseteq F^{+}(V)$. On the other hand, if $x \in F^{+}(V)$, then $F(x) \subseteq V$. By lemma 4.10, there exists an open set G of Y such that $F(x) \subseteq G \subseteq cl(G) \subseteq V$. Since in a space ${}_{\omega}T_{\hat{\Omega}}$, every closed set is $\hat{\Omega}$ closed set, $\hat{\Omega}cl(F(x)) \subseteq \hat{\Omega}cl(G) \subseteq cl(G) \subseteq V$. Thus, $x \in (\hat{\Omega}clF)^{+}(V)$. Therefore, $(\hat{\Omega}clF)^{+}(V) = F^{+}(V)$

Theorem 4.20 *Suppose that $F: X \to Y$ is a multifunction such that $F(x)$ is α-para compact and α-regular at each $x \in X$ and Y is ${}_{\omega}T_{\hat{\Omega}}$. If F is upper $\hat{\Omega}$-continuous if and only if $(\hat{\Omega}clF): X \to Y$ is upper $\hat{\Omega}$-continuous.*

Proof. **Necessity-** Suppose that F is upper $\hat{\Omega}$-continuous and $x \in X$, V is any open subset of Y containing $\hat{\Omega}cl(F(x))$. Then $x \in (\hat{\Omega}clF)^{+}(V)$. By Lemma 4.13, $x \in (\hat{\Omega}clF)^{+}(V) = F^{+}(V)$ and hence $F(x) \subseteq V$. Since F is upper $\hat{\Omega}$-continuous, there exists $U \in \hat{\Omega}O(X, x)$ such that $F(U) \subseteq V$. Then there exists an open set W in Y such that $F(u) \subseteq W \subseteq cl(W) \subseteq V$ and hence $\hat{\Omega}cl(F(u)) \subseteq \hat{\Omega}cl(W) \subseteq cl(W) \subseteq V$ for each $u \in U$. Therefore, $(\hat{\Omega}clF)(U) \subseteq V$ for each $u \in U$ and hence $(\hat{\Omega}clF)$ is upper $\hat{\Omega}$-continuous.

Sufficiency- Suppose that $(\hat{\Omega}clF)$ is upper $\hat{\Omega}$-continuous $x \in X$, V is any open subset of Y containing $F(x)$. Then $x \in F^{+}(V)$. By Lemma 4.13, $x \in (\hat{\Omega}clF)^{+}(V)$ and hence $\hat{\Omega}cl(F(x)) = (\hat{\Omega}clF)(x) \subseteq V$. Since $(\hat{\Omega}clF)$ is upper $\hat{\Omega}$-continuous, there exists $U \in \hat{\Omega}O(X, x)$ such that $\hat{\Omega}cl(F(U)) = (\hat{\Omega}clF)(U) \subseteq V$. Therefore, $U \subseteq (\hat{\Omega}clF)^{+}(V) = F^{+}(V)$ and hence $F(U) \subseteq V$. Thus, F is upper $\hat{\Omega}$-continuous.

Lemma 4.21 *If a multifunction $F: X \to Y$ is such that $F(x)$ is α-para compact and α-regular at each $x \in X$, then $(\hat{\Omega}clF)^{-}(V) = F^{-}(V)$ for every open set V in Y.*

Proof. suppose that V is any open subset of Y and $x \in (\hat{\Omega}clF)^{-}(V)$. Then $(\hat{\Omega}clF)(x) \cap V \neq \varnothing$. Then $\hat{\Omega}cl(F(x)) \cap V \neq \varnothing$. Since V is a open set in Y, $F(x) \cap V \neq \varnothing$ and so $x \in F^{-}(V)$. Then, $(\hat{\Omega}clF)^{-}(V) \subseteq F^{-}(V)$. On the other hand, if $x \in F^{-}(V)$, then $F(x) \cap V \neq \varnothing$ and hence $\hat{\Omega}cl(F(x)) \cap V \neq \varnothing$. That is $(\hat{\Omega}clF)(x) \cap V \neq \varnothing$. Thus, $F^{-}(V) \subseteq (\hat{\Omega}clF)^{-}(V)$. Therefore, $F^{-}(V) = (\hat{\Omega}clF)^{-}(V)$.

Theorem 4.22 *A multifunction $F: X \to Y$ is lower $\hat{\Omega}$-continuous if and only if $(\hat{\Omega}clF): X \to Y$ is lower $\hat{\Omega}$-continuous.*

Proof. Similar to that of previous Theorem.

Theorem 4.23 *Let $\hat{\Omega}$ be a both open and preclosed subset of X. If $F: X \to Y$ is upper (resp.lower) $\hat{\Omega}$-continuous, then $F|_A: A \to Y$ is upper (resp.lower) $\hat{\Omega}$-continuous.*

Proof. Suppose that F is upper $\hat{\Omega}$-continuous, $x \in A$ and V is any open subset of Y such that $(F|_A)(x) \subseteq V$. Therefore, for each $x \in A, F(x) = (F|_A)(x) \subseteq V$. Since F is upper $\hat{\Omega}$-continuous, there exists $U_1 \in \hat{\Omega}O(X, x)$ such that $F(U_1) \subseteq V$. By [5] Theorem 6.10, $U_1 \cap A$ is $\hat{\Omega}$-open in $(A, \tau|_A)$ containing x. If $U = U_1 \cap A$, then U is $\hat{\Omega}$-open in $(A, \tau|_A)$ containing x such that $(F|_A)(U) = F(U) \subseteq V$. Thus, $F|_A$ is upper $\hat{\Omega}$-continuous.

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

Theorem 4.24 *Let* $\{A_\lambda : \lambda \in \Lambda\}$ *be a cover of* X *by sets both* δ*-open and preclosed in* X. *If the multifunction* $F|_{A_\lambda}$ *is upper* $\hat{\Omega}$*-continuous for each* $\lambda \in \Lambda$, *then* $F : X \to Y$ *is upper* $\hat{\Omega}$*-continuous.*

Proof. Suppose that $F|_{A_\lambda}$ is upper $\hat{\Omega}$-continuous for each $\lambda \in \Lambda$, $x \in X$ and V is any open subset of Y such that $F(x) \subseteq V$. Then $x \in A_\lambda$ for some $\lambda \in \Lambda$. Then, $F(x) = F|_{A_\lambda}(x) \subseteq V$. By hypothesis, there exists a $\hat{\Omega}$-open set U in the subspace $(A, \tau|_\lambda)$ containing x such that $F|_{A_\lambda}(U) \subseteq V$. By [5] Theorem 6.9, $U \in \hat{\Omega}O(X, x)$ and $F(U) = F|_{A_\lambda}(U) \subseteq V$. Thus, F is upper $\hat{\Omega}$-continuous.

Characterization of $\hat{\Omega}$-compactness.

Theorem 4.25 *A space* X *is* $\hat{\Omega}$*-compact space if and only if every lower* $\hat{\Omega}$*-continuous multifunction from* X *into the closed sets of a space assumes a minimal value with respect to set inclusion relation.*

Proof. **Necessity-** Suppose that X is a $\hat{\Omega}$-compact space and F is a lower $\hat{\Omega}$-continuous multifunction from X into the closed sets of space Y. Let the poset of all closed subsets of space Y with the set inclusion $\subseteq$ be labeled by γ. It is enough to show that $F : X \to \gamma$ is lower $\hat{\Omega}$-continuous function. That is, to show that for every closed set D in X, $F^-(\{S \subseteq Y : S \in \gamma, S \subseteq D\})$ is $\hat{\Omega}$-closed in X. Put $C = F^-(\{S \subseteq Y : S \in \gamma, S \subseteq D\})$ and take $x \in X$ such that $x \notin C$. Claim that $x \notin \hat{\Omega}cl(C)$. Since $x \notin C$, $F(x) \cap \{S \subseteq Y : S \in \gamma, S \subseteq D\} = \varnothing$ and so $F(x) \neq S$ for any closed set S of Y. Moreover, $Y \setminus D$ is a open subset of Y such that $x \in F^-(Y \setminus D)$. By theorem 4.11 (ii), there exists $W \in \hat{\Omega}O(X, x)$ such that $W \subseteq F^-(Y \setminus D)$. Therefore, $F(w) \cap Y \setminus D \neq \varnothing$ for each $w \in W$. Therefore, for each $w \in W$, $F(w) \setminus D \neq \varnothing$. Hence, $F(w) \setminus S \neq \varnothing$ for any closed subset S of Y such that $S \subseteq D$. Therefore, $W \cap C = \varnothing$. It is shown that there exists $W \in \hat{\Omega}O(X, x)$ such that $W \cap C = \varnothing$. By [5] Theorem 5.11, $x \notin \hat{\Omega}cl(C)$. Therefore, $C = \hat{\Omega}cl(C)$ and so C is $\hat{\Omega}$-closed set in X. It is known that F assumes a minimal value.

Sufficiency- Suppose that X is not a $\hat{\Omega}$-compact space. By Theorem 3.9, for a well ordered set Λ, $\{x_i \in X : i \in \Lambda\}$ is a net with no $\hat{\Omega}$-accumulation point. We give Λ the order topology. Let $M_j = \hat{\Omega}cl(\{x_i : i \square j\})$ for every $j \in \Lambda$. Define a multifunction $F : X \to \Lambda$ by $F(x) = \{i \in \Lambda : i \square j_x\}$, where j_x is the first element of all those j's for which $x \notin M_j$. Since Λ has the order topology, $F(x)$ is closed. By the fact that $\{j_x : x \in X\}$ has no greatest element in Λ, F does not assume any minimal value with respect to set inclusion. Next claim that for every open set U in Λ, $F^-(U) \in \hat{\Omega}O(X)$. If $U = \Lambda$, then $F^-(U) = X$ which is $\hat{\Omega}$-open in X. Suppose that $U \subseteq \Lambda$ and $z \in F^-(U)$. Then $F(z) \cap U \neq \varnothing$. Choose $j \in F(z) \cap U$. That is, $j \in U$ and $j \in F(z) = \{i \in \Lambda : i \square j_x\}$. Therefore, $M_j \square M_{j_x}$. Since $z \notin M_{jx}$, $z \notin M_j$. There exists $W \in \hat{\Omega}O(X)$ such that $W \cap \{x_i : i \in \Lambda\} = \varnothing$. Then, $W \cap M_j = \varnothing$, and hence $w \notin M_j$ and since j_w is the first element for which $w \notin M_j$, then $j_w \square j$. Therefore, $j \in \{i \in \Lambda : i \square j_w\} = F(w)$. Since $j \in U$ and $j \in F(w)$, $F(w) \cap U \neq \varnothing$. Then, $w \in F^-(U)$ and hence $z \in W \subseteq F^-(U)$. It is shown that there exists $W \in \hat{\Omega}O(X)$ such that $z \in W \subseteq F^-(U)$. Therefore, $F^-(U)$ is $\hat{\Omega}$-open set in X and hence the multifunction F is lower $\hat{\Omega}$-continuous, a contradiction to hypothesis. Thus, X is $\hat{\Omega}$-compact.

Theorem 4.26 *A space* X *is* $\hat{\Omega}$*-compact space if and only if every upper* $\hat{\Omega}$*-continuous multifunction from* X *into the subsets of a* T_1 *space* Y *attains a maximal value with respect to set inclusion relation.*

Proof. It is similar to that of Theorem 4.25.

The next theorem concerns the existence of a fixed point for multifunction on $\hat{\Omega}$-compact.

Theorem 4.27 *Let* $F : X \to Y$ *be a multifunction from a* $\hat{\Omega}$*-compact space* X *into itself and* $F(S)$ *be* $\hat{\Omega}$*-closed set in* X *for being a* $\hat{\Omega}$*-closed set* S *in* X. *If* $F(x) \neq \varnothing$ *for every point* $x \in X$, *then there exists a non-empty* $\hat{\Omega}$*-closed subset* C *of* X *such that* $F(C) = C$.

Proof. Suppose that $\Lambda = \{S \subseteq X : S \neq \varnothing, S \in \hat{\Omega}C(X), F(S) \subseteq S\}$ .Since $X \in \Lambda$, Λ is non-empty and also a partially ordered set by set inclusion.Suppose that $\{S_\gamma\}$ is a chain in Λ ,then $F(S_\gamma) \subseteq S_\gamma$ for each γ. Since domain space is $\hat{\Omega}$ -compact, $S = \bigcap_\gamma S_\gamma \neq \varnothing$. By [5] Theorem 4.16, arbitrary intersection of $\hat{\Omega}$ -closed set is $\hat{\Omega}$ -closed and so $S \in \hat{\Omega}C(X)$. It follows that $F(S) \subseteq F(S_\gamma) \subseteq S_\gamma$ for each γ and hence $S \in \Lambda$ and $S = \inf\{S_\gamma\}$. By Zorn's Lemma, Λ has a minimal element C. Therefore, $C \in \hat{\Omega}C(X)$ and $F(C) \subseteq C$. Since C is the minimal element of Λ, $F(C) = C$.

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