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Propagation of Plane Waves in Generalized Piezo-thermoelastic Medium: Comparison Of Different Theories

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ABSTRACT: In this paper, a general solution for the propagation of plane waves in generalized piezo-thermoelastic medium for two-dimensional problem under the different thermoelastic theories is investigated. We have included Coupled theory (CT), Lord-Schulman (L-S) and Green-Lindsay (G-L) theories. The normal mode analysis is used to obtain the exact expressions for the considered variables. The results of the physical quantities have been illustrated graphically by comparison between (CT), (L-S) and (G-L) theories.

KEYWORDS: Piezo-thermoelastic, Relaxation time, Normal mode analysis, Generalized thermoelasticity.

I INTRODUCTION

Piezoelectric is considered one of the basic properties of crystals, ceramics, polymers, liquid crystals and some biological tissues (e.g. bone and tendon). Recent interest in the piezoelectric materials stems from their potential applications in intelligent structural systems, and piezoelectric is currently enjoying a greatest resurgence in both fundamental research and technical applications. The theory of thermo-piezoelectric was first proposed by Mindlin [1]. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [2, 3]. Chandrasekharaiah [4] has generalized Mindlin's theory of thermo-piezo-electric to account for the finite speed of propagation of thermal disturbances. Sharma and Kumar [5] have studied plane harmonic waves in piezo-thermoelastic materials. The propagation of Rayleigh waves in generalized piezo-thermoelastic half space has been investigated by Sharma and Walia [6]. Biot [7] has introduced the theory of coupled thermoelasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity, which leads to predict two phenomena not compatible with physical observations. The first one is the equation of heat conduction of this theory which does not contain any elastic terms. The second is the heat equation of a parabolic type, predicting infinite speeds of propagation to heat waves. The governing equations of the Biot theory jointly, eliminate the first paradox of the classical theory. However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic.

Thermoelasticity theories that predict a finite speed for the propagation of thermal signals have aroused much interest in the last three decades. These theories are known as generalized thermoelasticity theories. The first generalization of the thermoelasticity theory is due to Lord and Shulman [8] who introduced the theory of generalized thermoelasticity with one relaxation time through postulating a new law of heat conduction to replace the classical Fourier' law. This law contains the heat flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type which ensuring finite speeds of propagation of heat and elastic waves. The remaining, governing equations for this theory, namely, the equations of motion and the constitutive relations remain the same as those for the coupled and the uncoupled theories. This theory was extended by Dhaliwal and Sherief [9] to general anisotropic media in the presence of heat sources. Othman [10] studied the Lord-

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Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity. Green and Lindsay [11] have obtained another version of the constitutive equations.
Like the (L-S) theory and the (G-L) theory is also a generalization of the coupled (CT) theory, which allows for so-called "second sound" effects. But there exist the following differences between the two theories:
(i) The (L-S) theory modifies only the energy equation of the coupled theory by considering the time needed for the acceleration of heat flow, whereas the (G-L) theory modifies both the constitutive equation and the energy equation. Accordingly, the (L-S) theory involves only one relaxation time of thermoelastic process and the (G-L) theory involves two relaxation times.
(ii) The energy equation of the (L-S) theory depends on both, the strain velocity and strain acceleration whereas the corresponding equation of the (G-L) theory depends only on the strain velocity.
(iii) In the linearized case, accordingly to the (G-L) theory, the heat cannot propagate with a finite speed unless the stresses depend on the temperature velocity. Accordingly, to the (L-S) theory, the heat can propagate with a finite speed even though the stresses are independent of the temperature velocity.
(iv) The two theories are structurally different from one another, and one cannot be obtained as a particular case of the other.
The reflection and refraction problem with the interface of the piezo-thermoelastic materials, under the initial stresses influence in the context of Green and Lindsay theory have been studied by Alshaikh [12]. Othman and Abbas [13] have studied the effect of rotation on plane waves in generalized thermo-microstretch elastic solid by comparing different theories using finite element method. Othman and Lotfy [14] have studied the effect of magnetic field and rotation of the 2-D problem of a fiber-reinforced thermoelastic using different theories with influence of gravity. Abd-Alla and Alsheikh [15] have studied the reflection and refraction of plane quasi-longitudinal waves on the interface of the two piezoelectric media under initial stresses. The reflection phenomena of quasi-vertical transverse and longitudinal waves in the piezo-electric medium under initial stresses have investigated by Abd-Alla et al. [16, 17]; also Abd-Alla et al. [18] have studied the phenomena of reflection and transmission waves in smart nano materials.
The aim of the present work is to study analytically and the numerical results for the field quantities are illustrated graphically by comparison of different theories.

II. BASIC EQUATIONS

The basic, governing field equations of hexagonal piezo-thermoelastic in homogeneous anisotropic solid are [6]:
- Strain-displacement-relation

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

- Stress-strain-temperature

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{ijk}\,E_k - \beta_{ij}\,(1 + t_1\frac{\partial}{\partial t})T\,\delta_{ij}. \tag{2}$$

- Equation of motion

$$\sigma_{ij,j} = \rho\,\ddot{u}_i. \tag{3}$$

- Gauss equation and electric field relation

$$D_{i,i} = 0, \tag{4}$$

$$D_i = e_{ijk}\,\varepsilon_{jk} + \in_{ij}\,E_j + p_i\,(1 + t_1\frac{\partial}{\partial t})T, \tag{5}$$

where $E_i = -\varphi_{,i}$.

- The heat conduction equation

$$K_{ij}T_{,ij} = \rho C_e\,(1 + t_0\frac{\partial}{\partial t})\dot{T} + T_0[\beta_{ij}\,(1 + n_1 t_0\frac{\partial}{\partial t})\dot{u}_{i,j} - p_i\,(1 + n_1 t_0\frac{\partial}{\partial t})\dot{\varphi}_{,i}]. \tag{6}$$

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Where u_i, φ, T are the mechanical displacement, electric potential and an absolute temperature, respectively; ε_{ij}, σ_{ij}, β_{ij} are the components of strain tensor, stress tensor and thermal elastic coupling tensor, respectively; C_{ijkl} is the elastic parameters tensor, $i, j, k, l = 1,2,3$; e_{ijk}, $\in_{ij}$, p_i are the piezo-electric moduli, dielectric moduli and pyro-electric moduli; ρ is the mass density; E_i, D_i are the electric field and the electric displacement; t_0, t_1 are the thermal relaxation time parameters; K_{ij} is the heat conduction tensor; T_0 is the reference temperature; C_e is the specific heat at constant strain.

Equations (2)–(6) are the field equations of the generalized thermoelastic solid, can be defined in terms of the three theories CT, L-S and G-L as follows:

1. The coupled (CT) theory, when $t_1 = t_0 = 0$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{ijk} E_k - \beta_{ij} T\,\delta_{ij}, \tag{7}$$

$$D_i = e_{ijk}\,\varepsilon_{jk} + \in_{ij} E_j + p_i T, \tag{8}$$

$$K_{ij} T_{,ij} = \rho C_e \dot{T} + T_0[\beta_{ij} \dot{u}_{i,j} - p_i \dot{\varphi}_{,i}]. \tag{9}$$

2. Lord-Shulman (L-S) theory, when $n_1 = 1, \quad t_1 = 0, \ t_0 > 0$

Equations (7) and (8) remain unchanged and (6) has the form

$$K_{ij} T_{,ij} = (1 + t_0 \frac{\partial}{\partial t})[\rho C_e \dot{T} + T_0(\beta_{ij} \dot{u}_{i,j} - p_i \dot{\varphi}_{,i})]. \tag{10}$$

3. Green-Lindsay (G-L) theory, when $n_1 = 0, \quad t_1 \ge t_0 > 0$

Equations (2) and (5) remain unchanged and (6) has the form

$$K_{ij} T_{,ij} = \rho C_e (1 + t_0 \frac{\partial}{\partial t})\dot{T} + T_0[\beta_{ij} \dot{u}_{i,j} - p_i \dot{\varphi}_{,i}]. \tag{11}$$

The constitutive relation and electric displacement of the hexagonal (6mm) crystal symmetry are given by

$$\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{13}\varepsilon_{zz} - e_{31}E_z - \beta_1(1 + t_1 \frac{\partial}{\partial t})T, \tag{12}$$

$$\sigma_{zz} = C_{13}\varepsilon_{xx} + C_{33}\varepsilon_{zz} - e_{33}E_z - \beta_3(1 + t_1 \frac{\partial}{\partial t})T, \tag{13}$$

$$\sigma_{zx} = 2C_{44}\varepsilon_{zx} - e_{15}E_x, \tag{14}$$

$$D_x = e_{15}(u_{,z} + w_{,x}) + \in_{11} E_x, \tag{15}$$

$$D_z = e_{31}u_{,x} + e_{33}w_{,z} + \in_{33} E_z + p_3(1 + t_1 \frac{\partial}{\partial t})T. \tag{16}$$

III. FORMULATION OF THE PROBLEM

We consider a homogeneous, anisotropic, piezo-thermoelastic medium of hexagonal type. The basic governing field equations (3), (5), (6) for temperature change $T(x, z, t)$, displacement vector $\boldsymbol{u}(x, z, t) = (u, 0, w)$, and electric potential $\varphi(x, z, t)$, are given by

$$C_{11}u_{,xx} + C_{44}u_{,zz} + (C_{13} + C_{44})w_{,xz} + (e_{31} + e_{15})\,\varphi_{,xz} - \beta_1(1 + t_1 \frac{\partial}{\partial t})T_{,x} = \rho\,\ddot{u}, \tag{17}$$

$$(C_{44} + C_{13})u_{,xz} + C_{44}w_{,xx} + C_{33}w_{,zz} + e_{15}\varphi_{,xx} + e_{33}\varphi_{,zz} - \beta_3(1 + t_1 \frac{\partial}{\partial t})T_{,z} = \rho\,\ddot{w}, \tag{18}$$

$$K_1 T_{,xx} + K_3 T_{,zz} - \rho C_e (1 + t_0 \frac{\partial}{\partial t})\dot{T} = T_0[\beta_1(1 + n_1 t_0 \frac{\partial}{\partial t})\dot{u}_{,x} + \beta_3(1 + n_1 t_0 \frac{\partial}{\partial t})\dot{w}_{,z} - p_3(1 + n_1 t_0 \frac{\partial}{\partial t})\dot{\varphi}_{,z}], \tag{19}$$

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$$(e_{15}+e_{31})u_{,xz}+e_{15}w_{,xx}+e_{33}w_{,zz}-\in_{11}\varphi_{,xx}-\in_{33}\varphi_{,zz}+p_3(1+t_1\frac{\partial}{\partial t})T_{,z}=0\,. \quad (20)$$

For simplification we will use the following non-dimensional variables

$$x'=\frac{\omega^*}{v_p}x,\ z'=\frac{\omega^*}{v_p}z,\quad u'=\frac{\rho\omega^* v_p}{\beta_1 T_0}u,\ w'=\frac{\rho\omega^* v_p}{\beta_1 T_0}w,\ T'=\frac{T}{T_0},\ \sigma'_{ij}=\frac{\sigma_{ij}}{\beta_1 T_0},\ \varphi'=\varepsilon_p\varphi,\ \{t',t_1',t_0'\}=\omega^*\{t,t_1,t_0\},$$

$$D_i'=\frac{D_i}{\beta_1 T_0},\ \omega^*=\frac{c_e c_{11}}{k_{11}},\ \varepsilon_p=\frac{\omega^* e_{33}}{v_1\beta_p T_0},\quad \beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3,\quad \beta_3=2c_{13}\alpha_1+c_{33}\alpha_3. \quad (21)$$

Eqs. (17)–(20) in the non-dimensional forms (after suppressing the primes) reduce to

$$u_{,xx}+\delta_1 u_{,zz}+\delta_2 w_{,xz}+\delta_3\varphi_{,xz}+\delta_4(1+t_1\frac{\partial}{\partial t})T_{,x}=\delta_5\ddot{u}, \quad (22)$$

$$\delta_2 u_{,xz}+\delta_1 w_{,xx}+\delta_6 w_{,zz}+\delta_7\varphi_{,xx}+\delta_5\varphi_{,zz}+\delta_8(1+t_1\frac{\partial}{\partial t})T_{,z}=\delta_5\ddot{w}, \quad (23)$$

$$\delta_9 u_{,xz}+\delta_{10}w_{,xx}+\delta_{11}w_{,zz}+\delta_{12}\varphi_{,xx}+\delta_{13}\varphi_{,zz}+\delta_{14}(1+t_1\frac{\partial}{\partial t})T_{,z}=0, \quad (24)$$

$$\delta_{15}T_{,xx}+\delta_{16}T_{,zz}-(1+t_0\frac{\partial}{\partial t})\dot{T}=(1+n_1 t_0\frac{\partial}{\partial t})[\delta_{17}\dot{u}_{,x}+\delta_{18}\dot{w}_{,z}+\delta_{19}\dot{\varphi}_{,z}]. \quad (25)$$

"The full expressions for δ_j, $j=1-19$ are given in the Supplementary Material as Appendix A".

IV. NORMAL MODE METHOD

The normal mode analysis gives exact solutions without any assumed restrictions on the temperature displacement and stress distributions. The solution of the considered physical variables can be decomposed in terms of normal modes in the following form

$$[u,w,\varphi,T](x,z,t)=[u^*,w^*,\varphi^*,T^*](z)e^{ia(x-ct)}. \quad (26)$$

Where $D=\frac{d}{dz}$, $c=\frac{\omega}{a}$, ω is the complex time constant (frequency), i is the imaginary unit, a is the wave number in the x – direction, and u^*, w^*, φ^*, T^* are the amplitudes of the functions, then

$$(D^2+A_1)u^*+A_2Dw^*+A_3D\varphi^*+A_4T^*=0, \quad (27)$$

$$A_5Du^*+(D^2+A_6)w^*+(A_7+A_8D^2)\varphi^*+A_9DT^*=0, \quad (28)$$

$$A_{10}Du^*+(A_{11}+D^2)w^*+(A_{12}+A_{13}D^2)\varphi^*+A_{14}DT^*=0, \quad (29)$$

$$A_{15}u^*+A_{16}Dw^*+A_{17}D\varphi^*+(D^2+A_{18})T^*=0. \quad (30)$$

"The full expressions for A_k, $k=1-18$ are given in the Supplementary Material as Appendix A".

Eliminating $u^*(z)$, $\varphi^*(z)$ and $T^*(z)$ between equations (27)-(30) we get the following eighth order differential equation for $w^*(z)$

$$(D^8-A\,D^6+B\,D^4-C\,D^2+E)\,w^*(z)=0. \quad (31)$$

In a similar manner we arrive

$$(D^8-A\,D^6+B\,D^4-C\,D^2+E)\,\{u^*(z),\varphi^*(z),T^*(z)\}=0. \quad (32)$$

"The full expressions for A, B, C, E are given in the Supplementary Material as Appendix A".

Equation (32) can be factored as

$$(D^2-k_1^2)(D^2-k_2^2)(D^2-k_3^2)(D^2-k_4^2)w^*(z)=0. \tag{33}$$

The solution of equation (31) which is bound as $z\to\infty$, is given by

$$T^*(z)=\sum_{n=1}^{4} M_n e^{-k_n z}, \tag{34}$$

$$\varphi^*(z)=\sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{35}$$

$$w^*(z)=\sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{36}$$

$$u^*(z)=\sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}. \tag{37}$$

Where $k_n^2,(n=1,2,3,4)$ are the roots of the characteristic equation of equation (33).

Taking dimensionless and normal mode to equations (12)-(16) then by substituting from equations (34)-(37), we obtain

$$\sigma_{xx}^*(z)=\sum_{n=1}^{4} H_{4n} M_n e^{-k_n z}, \tag{38}$$

$$\sigma_{zz}^*(z)=\sum_{n=1}^{4} H_{5n} M_n e^{-k_n z}, \tag{39}$$

$$\sigma_{xz}^*(z)=\sum_{n=1}^{4} H_{6n} M_n e^{-k_n z}, \tag{40}$$

$$D_x^*(z)=\sum_{n=1}^{4} H_{7n} M_n e^{-k_n z}, \tag{41}$$

$$D_z^*(z)=\sum_{n=1}^{4} H_{8n} M_n e^{-k_n z}. \tag{42}$$

"The full expressions for $H_{mn}, m=1-8$ are given in the Supplementary Material as Appendix B".

V. BOUNDARY CONDITIONS

The parameters $M_n\,(n=1,2,3,4)$ have to be chosen such that the boundary conditions on the surface $z=0$ takes the form

$$\sigma_{zz}(x,0,t)=-f_1^*\,e^{ia(x-ct)}, \qquad \sigma_{xz}(x,0,t)=0, \qquad T=0, \qquad \frac{\partial\varphi}{\partial z}=0 \tag{43}$$

Where f_1^* is constant.

Using the expressions of the variables considered into the above boundary conditions (43), we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{4} H_{5n} M_n=-f_1^*, \tag{44}$$

$$\sum_{n=1}^{4} H_{6n} M_n=0, \tag{45}$$

$$\sum_{n=1}^{4} M_n=0, \tag{46}$$

$$\sum_{n=1}^{4} k_n H_{1n} M_n=0. \tag{47}$$

Solving equations (44)-(47) for $M_n\,(n=1,2,3,4)$ by using the inverse of matrix method as follows

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$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} \\ H_{61} & H_{62} & H_{63} & H_{64} \\ 1 & 1 & 1 & 1 \\ k_1 H_{11} & k_2 H_{12} & k_3 H_{13} & k_4 H_{14} \end{pmatrix}^{-1} \begin{pmatrix} -f_1^* \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{48}$$

VI. NUMERICAL CALCULATIONS AND DISCUSSION

The material chosen for the purpose of numerical calculations has been taken as Cadmium Selenide (CdSe) having hexagonal symmetry (6 mm class)

$c_{11} = 7.41\times10^{10}\,Nm^{-2}$, $c_{12} = 4.52\times10^{10}\,Nm^{-2}$, $c_{13} = 3.93\times10^{10}\,Nm^{-2}$, $f_1^* = 10^8$ $c_{33} = 8.36\times10^{10}\,Nm^{-2}$, $c_{44} = 1.32\times10^{10}\,Nm^{-2}$, $T_0 = 298\,K$, $\rho = 5504\,Kgm^{-3}$, $e_{13} = -0.160\,Cm^{-2}$, $e_{33} = 0.347\,Cm^{-2}$, $e_{15} = -0.138\,Cm^{-2}$, $\beta_1 = 0.621\times10^6\,Nk^{-1}.m^{-2}$, $\beta_3 = 0.551\times10^6\,NK^{-1}.m^{-2}$, $p_3 = -2.94\times10^{-6}\,CK^{-1}.m^{-2}$, $K_1 = K_3 = 9\,Wm^{-1}.K^{-1}$, $\in_{11} = 8.26\times10^{-11}\,C^2N^{-1}.m^{-2}$, $\in_{33} = 9.03\times10^{-11}\,C^2N^{-1}.m^{-2}$, $C_e = 260\,J.Kg^{-1}K^{-1}$

Figures 1-6 show the comparison between the displacement components u, w, the normal stress components σ_{zz}, the temperature T, the electric potential φ and the electric displacement D_z. The computations are carried out for the non-dimensional time $t = 0.3$ on the surface plane at $x = 1.5$.

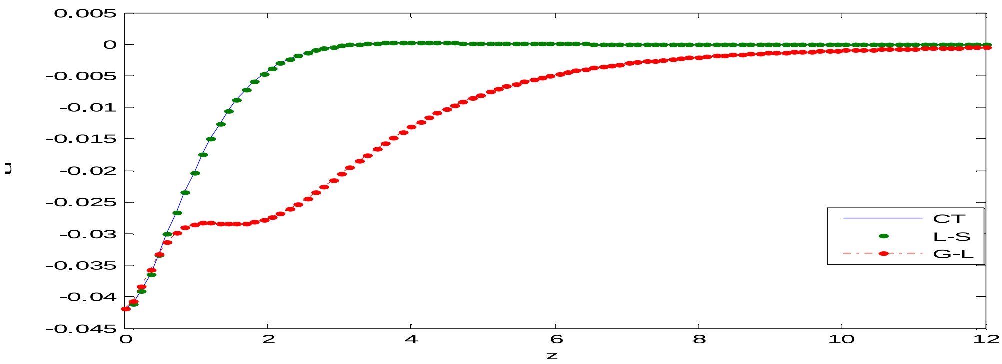

Fig. 1 Distribution of horizontal displacement u versus z-axis.

Figure 1 depicts that the distribution of the horizontal displacement u in the context of the CT, L-S and G-L theory, always begin from negative values, and increases in the range $0 \le z \le 5$. It is observed that the two curves of the CT and L-S theories are coincide. However, u converges to zero with increasing of the distance $z \ge 10$. Figure 2 exhibits that the distribution of the vertical displacement w, in the case of CT, L-S and G-L theory, but decreases and converges to zero for $z > 12$.

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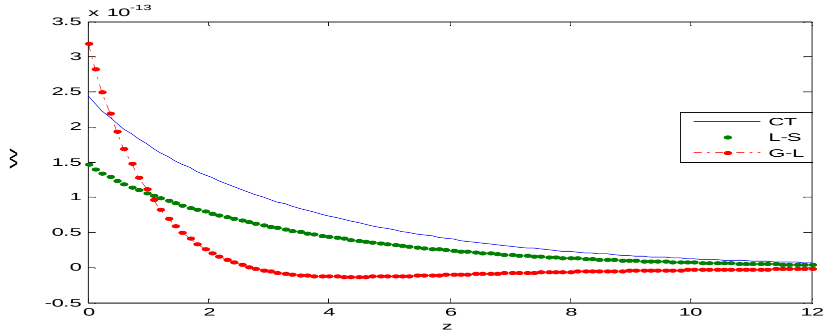

Fig. 2 Distribution of vertical displacement w versus z -axis.

Figure 3 shows that the behavior of three curves of σ_{zz} is increasing in the range $0 \le z \le 5$, while the two curves of the CT and L-S theories are coincide, it is obvious that σ_{zz} converges to zero for $z \ge 10$. Figure 4 demonstrates that for $z \le 2$ the behavior of three curves of the temperature T is increasing, while in the two theories CT and L-S it decreases in the range $2 \le z \le 6$. However the curve of T with G-L theory decreases in the range $2 \le z \le 14$. Then the temperature T converges to zero with decreasing of the distance z at $z \ge 14$. Figure 5 exhibits that the distribution of the curves of φ increases in the range $0 \le z \le 10$. The curve which describes the behavior of φ with the L-S theory is an upper curve while the curve which describes the behavior of φ with the G-L theory, is in the lower, but the curve with CT lies between the two other curves. The values of the electric potential φ, which is based on the three theories CT, L-S and G-L converge to zero with increasing of the distance z for $z \ge 10$.

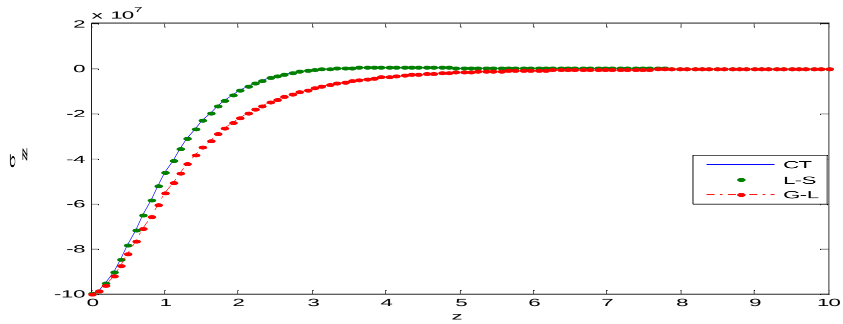

Fig. 3 Distribution of stress component σ_{zz} versus z -axis

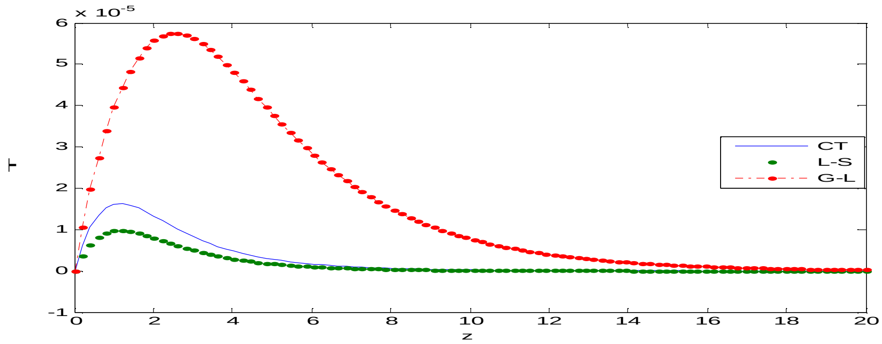

Fig. 4 Distribution of temperature T versus z -axis

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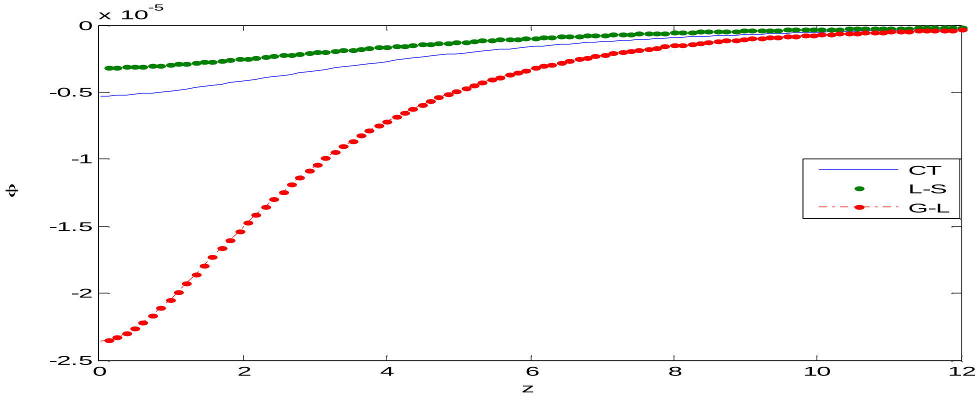

Fig. 5 Distribution of electric potential φ versus z -axis

Figure 6 depicts the distribution of the electric displacement D_z . One can see that the two curves of D_z with the CT and L-S theories are coinciding. The behavior of the three curves of D_z is decreasing in the range $0 \le z \le 5$, but it converges to zero with increasing of the distance $z \ge 7$.

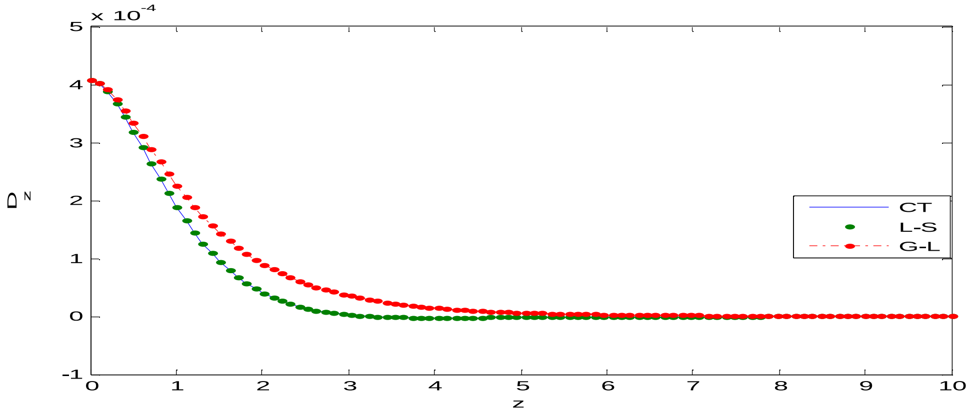

Fig. 6 Distribution of electric displacement D_z versus z -axis

VII. CONCLUSIONS

According to the above results, we can conclude that, analytical solutions based upon normal mode analysis for piezoelectric thermoelastic medium have been developed and utilized. The values of all physical quantities converge to zero with the increasing of the distance z , also all the functions are continuous. All the physical quantities agree with the boundary conditions. The significant effect of thermal time relaxation has been observed in all the various physical quantities of the CdSe material, since all the profiles of considered functions are quite distinguishable. Hence the results for all the considered field parameters show the difference between the three different theories of thermoelasticity namely the CT, L-S and G-L.

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