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Performance Analysis of Time Moments, Markov's Parameters and Eigen Spectrum Using Matching Moments

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ABSTRACT: Time moments (TM) & Markov's parameters (PM) are combined with Eigen spectrum analysis using approximate reduced order system. Original higher order systems are compared to study the performance. The proposed method guarantees stability of the reduced model if the original higher-order system is stable and is comparable in quality with the order of other existing methods.

Three reduction methods are studied and analysed. The methods are illustrated using 3rd and 4th order numerical examples.

KEYWORDS: High order systems, Model reduction, Time moments, Markov's parameters, Eigen spectrum.

I. Introduction

The problem of reducing a higher order system to a lower order system is the need of the hour. This has been the subject of much research because higher order systems cost high and take extremely large computation time, thereby making reduction techniques more than necessary. The reduction technique is to define a model of reduced dimension, retaining the main physical aspects of the initial system and also ensuring stability. In our work, we study a technique for order reduction of large systems that combines time moments, Markov's parameters and Eigen spectrum analysis. Therefore, by comparing this approach with the well-known existing matching moments, interesting results are obtained, as it is shown by Step and Impulse response in MATLAB 9.1.

II. Related work

- **MATHEMATICAL CALCULATION FOR TIME MOMENTS AND MARKOV'S PARAMETERS:**

- **For 4th order system:** matching two time moments & two Markov's parameters and one Markov's and three time moments to reduce the system $G_n(s)$

$$G_n(s) = \frac{a_1 s^3 + a_2 s^2 + a_3 s + a_4}{s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4} \;\ldots\ldots\ldots\ldots (1)$$

- **For higher order system:**

Time Moments-　　　　Markov's Parameters-

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$$t_1 = \frac{a_4}{b_4}$$

$$t_2 = \frac{a_2 + b_2 - t_1 b_3}{b_4} \qquad M_1 = a_1$$

$$M_2 = b_3 - M_1 b_2$$

$$t_3 = \frac{a_1 - t_1 b_2 - t_2 b_3}{b_4}$$

- **For reduce order system:**

$$G_r(s) = \frac{a_1^{'} s + a_2}{s^2 + b_1^{'} s + b_2^{'}} \quad \text{........................} \ (2)$$

Time Moments- Markov Parameters-

$$t_1^{'} = \frac{a_2^{'}}{b_2^{'}}$$

$$t_2^{'} = \frac{a_1^{'} - t_1^{'} b_1^{'}}{b_2^{'}} \qquad M_1^{'} = a_1^{'}$$

$$M_2^{'} = a_2^{'} - M_1^{'} b_1^{'}$$

$$t_3^{'} = \frac{-t_1^{'} - b_1^{'} t_2^{'}}{b_2^{'}}$$

- **For 3rd order system:** Matching two time moments & two Markov's parameters and one Markov's and three time moments to reduce the system $G_n(s)$

$$G_n(s) = \frac{a_1 s^2 + a_2 s + a_3}{s^3 + b_1 s^2 + b_2 s + b_3} \quad \text{...................} \ (3)$$

- **For higher order system:**

Time Moments- Markov Parameters-

$$t_1 = \frac{a_3}{b_3} \qquad M_1 = a_1$$

$$t_2 = \frac{a_2 - t_1 b_2}{b_3} \qquad M_2 = a_2 - M_1 b_1$$

$$t_3 = \frac{a_1 - t_1 b_1 - t_2 b_2}{b_3}$$

- **For reduce order system:**

$$G_n(s) = \frac{a_1^{'} s + a_2}{s^2 + b_1^{'} s + b_2^{'}} \quad \text{...........} \ (4)$$

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Time Moments-

$$t_1^{'} = \frac{a_2^{'}}{b_2^{'}}$$

$$t_2^{'} = \frac{a_1^{'} - t_1^{'} b_1^{'}}{b_2^{'}}$$

$$t_3^{'} = \frac{-t_1^{'} - b_1^{'} t_2^{'}}{b_2^{'}}$$

Markov Parameters-

$$M_1^{'} = a_1^{'}$$

$$M_2^{'} = a_2^{'} - b_1^{'} M_1^{'}$$

- **EIGEN SPECTRUM ANALYSIS USING TIME MOMENTS AND MARKOV'S PARAMETERS:**

- **Eigen Spectrum Analysis:**
  Let the transfer function of the high-order system (HOS) of order 'n' is

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_o + b_1 s + b_2 s^2 + \ldots\ldots\ldots + b_{n-1} s^{n-1}}{(s+\lambda_1)(s+\lambda_2)\ldots\ldots\ldots(s+\lambda_n)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where $-\lambda_1 < -\lambda_2 < -\lambda_3 \ldots\ldots < -\lambda_{n.}$ are poles of HOS and that of low-order system (LOS) of order 'r' is:

$$G_r(s) = \frac{d_0 + d_1 s + d_2 s^2 + \ldots\ldots + d_{n-2} s^{n-2}}{(s+\lambda_1^{'})(s+\lambda_2^{'})\ldots\ldots(s+\lambda_r^{'})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where, $-\lambda_1^{'} < -\lambda_2^{'} < \ldots\ldots < -\lambda_r^{'}$ are poles of LOS.

III. PROPOSED ALGORITHM

(a) Draw the Eigen spectrum zone (ESZ) of the HOS as shown in Fig. 1

(b) If poles - λ_i (i = 1,n) are located at – (Re λ_i ± Imλ_i) (i = 1,p) within the ESZ, then the two lines passing through the nearest (Re λ_1) and farthest (Re λ_n) real poles when cut by two lines passing through the farthest imaginary pole pairs (±Im(max)) form the ESZ.

(c) To obtained pole centroid and stiffness of higher order system, pole centroid is defined as the mean of real parts of the poles and is expressed as,

$$\lambda_m = \frac{\sum_{i=1}^{p} \mathrm{Re}\,\lambda_i}{p} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

System stiffness is defined as the ratio of the nearest to the farthest pole of a system in terms of real parts only and is written as

$$\lambda_s = \frac{\mathrm{Re}\,\lambda_1}{\mathrm{Re}\,\lambda_p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

(d) Determination of Eigen spectral points of LOS: If λ_m and λ_s are pole centroid and system stiffness of LOS such λ'_m that and $\lambda_s^{'}$, then following situation arises

$$\lambda_m^{'} = \frac{\mathrm{Re}\,\lambda_1 + \mathrm{Re}\,\lambda_2 + \ldots\ldots + \mathrm{Re}\,\lambda_p}{p} = \lambda_m \ldots\ldots\ldots\ldots\ldots (9)$$

$$\lambda_s^{'} = \frac{\mathrm{Re}\,\lambda_1^{'}}{\mathrm{Re}\,\lambda_p^{'}}$$

Where λ'_i (i=1,r) are the poles of LOS located at $-(\mathrm{Re}\,\lambda_i^{'} \pm \mathrm{Im}\,\lambda_i^{'})i = 1, p^{'}$

Now if,

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$$\frac{\mathrm{Re}\,\lambda_p^{'} - \mathrm{Re}\,\lambda_1^{'}}{p-1} = M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

i.e. Re λ'_1+M= Reλ'_2, Re λ'_2 + M = Re λ'_3 and so on till

Reλ_{p-1}+ M = Re$\lambda_p$then Eq. (5) can be put as

$$\lambda_m = \frac{\mathrm{Re}\,\lambda_1 + \mathrm{Re}\,\lambda_p + (\mathrm{Re}\,\lambda_1 + M) + (\mathrm{Re}\,\lambda_2 + M) + \ldots\ldots + (\mathrm{Re}\,\lambda_{P-1} + m)}{p}$$

$$or$$

$$N = \mathrm{Re}\,\mathrm{Re}\,\lambda_1(p-1) + \mathrm{Re}\,\lambda_p + QM \quad \ldots\ldots\ldots (11)$$

$$where N = \lambda_{mp}, and QM = M + 2M + \ldots\ldots (p^{'} - 2)M.$$

By putting $\mathrm{Re}\lambda_1^{'} = \lambda_s \mathrm{Re}\lambda_p^{'}$ Equation (8) and (9) will be as under:

$$\begin{aligned} &\mathrm{Re}\,\lambda_p^{'} - \lambda_s\,\mathrm{Re}\,\lambda_p^{'} = M(p^1 - 1) \\ &\lambda_s\,\mathrm{Re}\,\lambda_p^{'}(p-1) + \mathrm{Re}\,\lambda_p^{'} + QM = N \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Equation (10) and (11) can be put as,

$$\begin{aligned} &\mathrm{Re}\,\lambda_p(1-\lambda_s) + M(1-p) = 0 \\ &\mathrm{Re}\,\lambda_p[\lambda_s(p-1)+1] + MQ = Q \end{aligned}$$

$$\begin{matrix} \lambda_s(p-1)+1 & Q \\ (1-\lambda_s) & (1-p) \end{matrix} \begin{bmatrix} \mathrm{Re}\,\lambda_P \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots (13)$$

Equation (12) can be solved for $\mathrm{Re}\,\lambda_p^{'}$ and M enabling thereby to locate the Eigen spectral points (ESP) as shown in Fig 1.and denominator equation can be written as:

$$\bar{D}(s) = e_0 + e_1 s + e_2 s^2 + \ldots\ldots\ldots \ldots. + e_{r-1} s^{r-1} + e_r s^r$$

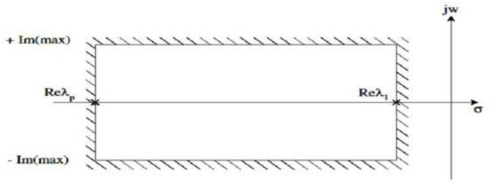

Fig.1 Eigen spectrum zone (ESZ) of HOS

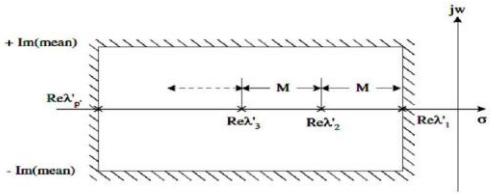

Fig.2 Eigen spectrum zone (ESZ) of LOS

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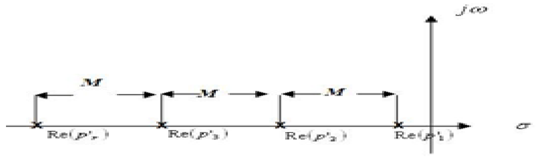

Fig.3 Eigen spectral zone (ESZ) of LOS

- **Time Moments Matching Method:**

  Order original system given in equation (5) is equated to the r^{th} order reduced model represented by the equation (6)

$$\frac{a_o + a_1 s + a_2 s^2 + \ldots\ldots + a_{n-1} s^{n-1}}{b_o + b_1 s + b_2 s^2 + \ldots\ldots\ldots + b_{n-1} s^{n-1} + b_n s^n} = \frac{d_o + d_1 s + d_2 s^2 + \ldots\ldots\ldots + d_{n-1} s^{n-1}}{e_o + e_1 s + e_2 s^2 + \ldots\ldots\ldots + e_{n-1} s^{n-1} e_n s^n} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

  On cross multiplying and rearranging the equation (14)

$$a_o e_o + (a_o e_1 + a_1 e_o)s + (a_o e_2 + a_1 e_1 + a_2 e_o)s^2 + \ldots\ldots + a_{n-1} e_k s^{n-1+k} = b_o d_o + (b_o d_1 + b_1 d_o)s + \ldots\ldots + b_n d_{k-1} s^{n-1+k} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

Equating the coefficients of the same power of s on both sides in the equation (15), the following relations are obtained:

$$\begin{aligned} & a_o e_o = b_o d_o \\ & a_o e_1 + a_1 e_o = b_o d_1 + b_1 d_o \\ & \ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots (16) \\ & a_o e_{r-1} + a_1 e_{r-2} + \ldots\ldots = b_o d_{r-1} + b_1 d_{r-2} + \ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots \\ & a_{n-1} e_r = b_n d_{r-1} \end{aligned}$$

By solving above equations, the unknown values of reduced order transfer function in form (2) are calculated with the help of constant term e_o.

IV. Pseudo Code

- **NUMERICAL APPLICATION**

There are two numerical examples of 4th order and 3rd order. Performance analysis of higher order and reduced order systems by Time moments, Markov's parameters and Eigen spectrum.

Example 1. Consider the 4th order system by Avadh Pati et al.

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

- **For two markov's parameters and two time moments reduced system –** For better matching we have toconsider: $M_1 = M_1^{'}, M_2 = M_2^{'}$ and $t_1 = t_1^{'}, t_2 = t_2^{'}$.

  Then, reduce order system will be-

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$$G_2(s) = \frac{s - 192.77}{s^2 - 207.77s - 192.77} \text{.........}(17.1)$$

- **For one Markov's parameters and three time moments reduced system –** For better matching we have to consider: $M_1 = M_1^{'}$ and ${}_1 \quad {}_1^{'} \; {}_2 \quad {}_2^{'} \; {}_3 \quad {}_3^{'}$.

  Then, reduce order system will be-

$$G_2(s) = \frac{s + 0.249}{s^2 + 1.269s + 0.249} \text{.............}(17.2)$$

- **For time matching moments reduced system –** For better matching we have to consider: $\lambda_m = \lambda_m^{'}$ and $\lambda_s = \lambda_s^{'}$.Then, reduced order system will be-

$$G_2(s) = \frac{0.666s + 4}{s^2 + 5s + 4} \text{..............}(17.3)$$

Example 2. Consider the 3rd order system by Katsuhiko Ogata.

$$G_3(s) = \frac{2s^3 + 5s^2 + 3s + 6}{s^3 + 6s^2 + 11s + 6} \text{.....................} (18)$$

- **For two markov's parameters and two time moments reduced system –** For better matchingwe have toconsider: $M_1 = M_1^{'}, M_2 = M_2^{'}$ and $t_1 = t_1^{'}, t_2 = t_2^{'}$.

  Then, reduce order system will be-

$$G_2(s) = \frac{2s + 3}{s^2 + 5s + 3} \text{.........}(18.1)$$

- **For one markov's parameters and three time moments reduced system –** For better matching we have to consider: $M_1 = M_1^{'}$ and $t_1 = t_1^{'}, t_2 = t_2^{'}, t_3 = t_3^{'}$

  Then, reduce order system will be-

$$G_2(s) = \frac{2s + 1.515}{s^2 + 3.51s + 1.515} \text{..........}(18.2)$$

- **For time matching moments reduced system –** For better matching we have to consider: $\lambda_m = \lambda_m^{'}$ and $\lambda_s = \lambda_s^{'}$

  Then, reduce order system will be

$$G_2(s) = \frac{0.03s + 2.97}{s^2 + 3.99s + 2.97} \text{............}(18.3)$$

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V. SIMULATION RESULTS

In this present work, two higher order systems have been taken into consideration. In order to make the higher order system into a lower order system, Time moments, Markov‟s parameters and Eigen spectrum analysis have been employed to reduce the complex system. A comparative study shown by fig.(4), fig.(5), fig.(6) and fig. (7) Using MATLAB 9.1 shows the results to be almost same and improved.

A comparison between 4th order original system and reduced order systems of step responses is shown Fig.4.

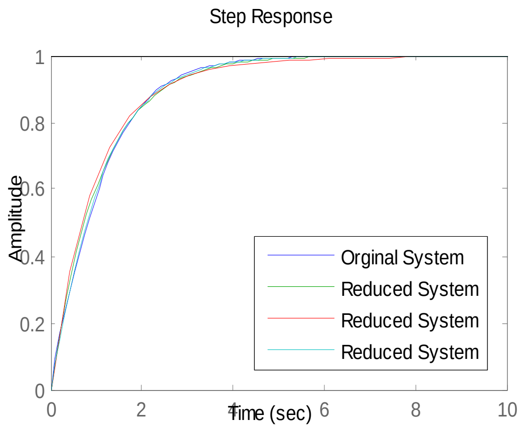

Figure 4 Step response of original system and reduced order system

A comparison between 4th order original system and reduced ordersystemsof impulse responses is shown Fig.5.

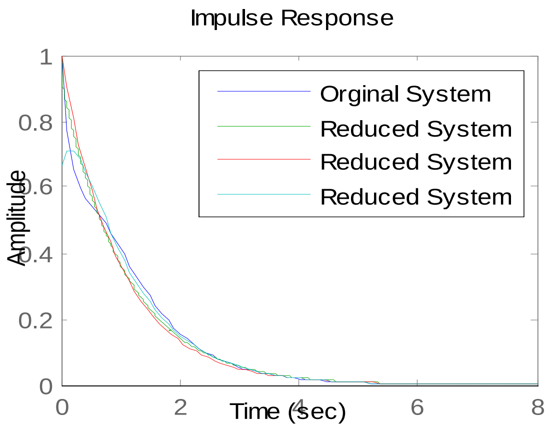

Figure 5 Impulse response of original system and reduced order system

A comparison between 3rd order original system and reduced order systems' of step responses is shown in Fig.6

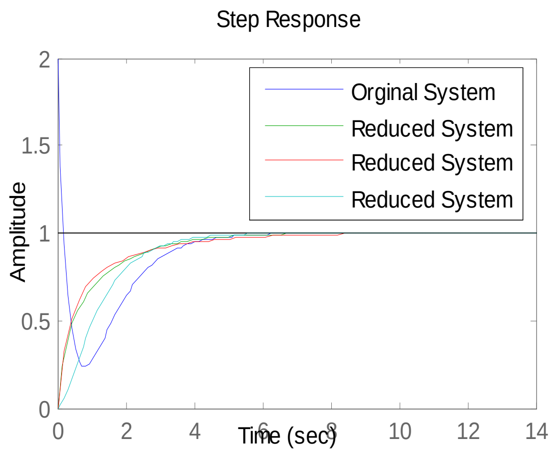

Figure.6 Step response of original system and reduced order system

A comparison between 3rd order original system and reduced order systems'of impulse responses is shown in Fig.7

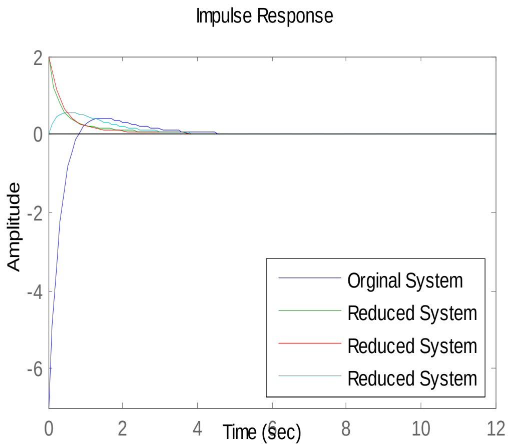

Figure.7 Impulse response of original system and reduced order system

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VI. Conclusion and Future Work

A mixed method for order reduction based on Time moments, Markov's parameters and Eigen spectrum analysis is studied. This method produces stable reduced models from stable higher order system. In the proposed method used the time matching technique allows matching of the step and impulse response is quite good. The poles are synthesized by eigen spectrum analysis and zeros are determined by time matching. The method is simple, robust and takes little computational time. The comparison between the proposed and other well-known existing order reduction technique is shown. The study establishes proposed method comparable in quality with other existing techniques of model order reduction.

The model order reduction techniques (MORT) based on the mathematical calculation, can be further improved by considering different cases of matching moments. Reduced model order parameters make the system stable and developed. Lack of time and difficulties in industrial areas, made it difficult to apply it to higher order system. Using different MORTs in near future, there is possibility to develop and analyse system performance.

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BIOGRAPHY

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