



# **Optimum and Centralized Detection for Statistical MIMO Radar**

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**ABSTRACT:** Multiple Input Multiple Output (MIMO) radar is a new emerging radar technique developed recently. MIMO radar can exploit the spatial diversity of target scatterers in a variety of ways to extend the radar's performance. This paper introduces the statistical MIMO radar concept and the method of receiving signal processing also is presented. The fundamental difference between statistical MIMO and other radar array systems is that the latter seek to maximize the coherent processing gain, while statistical MIMO radar capitalizes on the diversity of target scattering to improve radar performance. Compared to traditional phased-array radar, statistical MIMO radar can offer better parameter identifiability and higher resolution, better sensitivity to slowly moving targets, enable the direct applicability of adaptive techniques for effective interference and jamming suppression, and allow for much flexibility for transmit beam-pattern design and waveform optimization. Reaping the full benefit of the superior performance enabled by the statistical MIMO radar requires a novel design of its receive filters to minimize the impact of scatterers in nearby range bins on the received signals from the range bin of interest to minimize range compression problem by spacing the antenna elements at the transmitter and at the receiver such that the target angular spread is manifested. In this paper, we focus on the application of the target spatial diversity to improve detection performance. The optimal detector in the Neyman-Pearson sense is developed and analyzed for the statistical MIMO radar in case of stationary target. The performance improvement that can be achieved by the use of angular diversity in statistical MIMO radar is investigated. The results show that at high SNR values statistical MIMO radar provides great improvements of target detection performance over other types of array radars. Whereas, at low SNR values, phased array radar performs better than statistical MIMO radar. Finally, we present a number of numerical examples to demonstrate the effectiveness of the proposed approaches. Therefore, statistical MIMO radar can be applied to enhance radar resolution by allowing the measurement of one scatterer at a time.

**Keywords:** MIMO, radar, statistical, phased array, probability detection, and SNR.

## I. INTRODUCTION

Benefiting from the idea of MIMO communication systems, we explored the potentials of MIMO concept in radar [1]. MIMO radar can be defined simply as a multi antenna radar system which transmits linearly independent or orthogonal waveforms. Our proposed MIMO radar enjoys similar benefits to those enjoyed by MIMO communication systems. Radar targets provide a rich scattering environment. Conventional radars experience target fluctuations of 5-25 dB. Slow RCS fluctuations cause long fades in target RCS, degrading radar performance. The novelty of MIMO radar is that it provides measures to overcome those degradations or even utilizes the RCS fluctuations for new applications. Standard MIMO radar takes the opposite direction of the phased-array radar. The approach is to employ multiple uncorrelated waveforms that are radiated via Omni-directional transmission, in compare to phased-array radar where a single probing waveform is sent via directional transmission. The MIMO systems have increased popularity and involved attention of late for their ability to enhance all areas of system performance. MIMO radar may be

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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configured with its antennas co-located or widely distributed over an area and able to provide independent diversity paths.

In particular, the term “statistical MIMO radar,” refers to the signal model where the signals measured at different antennas are uncorrelated [4]-[5]. If the antennas are separated far enough, the target RCS for different transmitting paths become independent random variables. Thus, each orthogonal waveform carries independent information about the target; spatial diversity about the target is thus created. Exploiting the independence between signals at the array elements, MIMO radar achieves improved detection [6], [7] performance and increased radar sensitivity. Full exploitation of these potentials can result in significant improvement in target detection, parameter estimation; target tracking and recognition performance. This is the motivation for using MIMO radar. Thus, each orthogonal waveform carries independent information about the target; spatial diversity about the target is thus created. Exploiting the independence between signals at the array elements, MIMO radar achieves improved detection performance and increased radar sensitivity.

In this paper, phased array radar and MIMO radar system, namely statistical MIMO radar [2]-[3] are investigated. The similarities and differences of these radar systems from conventional radar systems are explored. The detectors for statistical MIMO radar and phased array radar are provided and their detection performances are compared through numerical simulations. The optimal detector in the Neyman-Pearson sense is developed and analyzed for the statistical MIMO radar. The performance improvements achieved by statistical MIMO radar system better than phased array radar those are summarized.

## II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

### 2.1 Phased array radar

Phased array radar uses antenna arrays for transmitting and receiving signals. These arrays may be linear or planar. In both the linear and planar arrays the separation between the elements is usually uniform. These arrays may be co-located and even transmit and receive functions can be performed by the same array. The two arrays may also be widely separated allowing the radar system to operate in bistatic mode.

#### 2.1.1 System Model

The model of the multistatic phased array radar system is shown in fig. 1 that has  $M$  transmits and  $N$  receives elements. Assume that transmit and receive arrays are uniform linear arrays with inter element spacing of  $d_t$  and  $d_r$  respectively. Since the inter-element spacing of phased array radar antennas is small, the bistatic RCS seen by every transmit-receive pair in a phased array radar system is assumed to be the same.

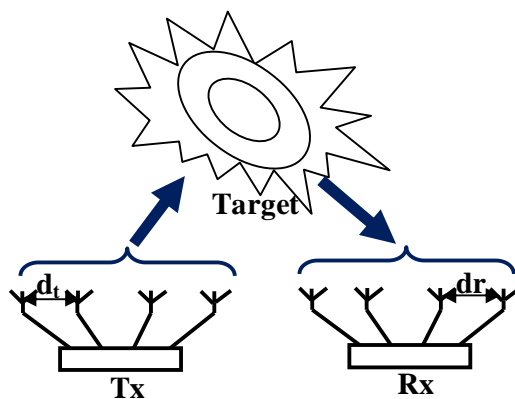


Fig. 1. Phased array radar configuration.



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If the transmit array performs transmit beam-forming in the direction of  $\theta$  the transmitted signal  $\overline{x(t)}$  can be written in the vector form as

$$\overline{x(t)} = a(\theta) \sqrt{\frac{E_t}{M}} x(t) \dots \dots \dots (1)$$

Where,  $\sqrt{E_t/M} x(t)$  denote the discrete time baseband signal transmitted by the transmit antenna elements where  $x(t)$  is the input message signal,  $E_t$  is the total average transmitted energy and where  $a(\theta)$  is the transmitter steering vector. Then the received signal model becomes

$$y = \sqrt{\frac{E_t}{M}} NM\alpha + w \dots \dots \dots (2)$$

Where,  $w$  is a zero mean vector of complex random processes. Note that if  $\alpha$ , where  $\alpha$  is a zero mean complex normal random variable, is small, the amplitude of the received signal will be small despite this processing gain and detection probability will decrease dramatically.

### 2.1.2. Probability Detection

The detection problem in phased array radar can be formulated as binary hypothesis testing problem:

$$\left. \begin{aligned} H_0: & y = w \\ H_1: & y = \sqrt{\frac{E_t}{M}} MN\alpha + \omega \end{aligned} \right\} \dots \dots \dots (3)$$

Where  $H_0$  indicates absence of signal and  $H_1$  indicates presence of signal. Assume that  $\alpha$  is a zero mean complex normal random variable with a variance of  $\sigma_\alpha^2=1$ . It is well known that the optimum solution to this hypothesis testing problem under Neyman-Pearson criterion is the Likelihood Ratio Test (LRT) as,

$$\frac{p(y|H_1, \sigma_w^2, \sigma_\alpha^2)}{p(y|H_0, \sigma_w^2)} \underset{\leq H_0}{\overset{\geq H_1}{\geq}} T \dots \dots \dots (4)$$

In this case the distributions of  $\alpha$  and  $w$  are known, So the likelihood ratio test [4] can be written as,

$$|y|^2 \underset{\leq H_0}{\overset{\geq H_1}{\geq}} T' \dots \dots \dots (5)$$

The false alarm rate ( $P_{fa}$ ) can be defined in the term of threshold  $T'$  as,

$$= \exp\left(\frac{-T'}{N\sigma_w^2}\right) \dots \dots \dots (6)$$

Where,  $T' = -N\sigma_w^2 \ln(P_{fa})$

The probability detection ( $P_d$ ) can be calculated in terms of threshold  $T'$  as,

$$= \exp\left(\frac{-T'}{E_t MN^2 + N\sigma_w^2}\right) \dots \dots \dots (7)$$

Equivalently probability detection ( $P_d$ ) can be written in terms of  $P_{fa}$  and SNR as,

$$P_d = \exp\left(\frac{\ln(P_{fa})}{(SNR)MN + 1}\right) \dots \dots \dots (8)$$

### 2.1.3. Results and Observation

To illustration the probability detection performance of phased array radar, the detector in equation (8) is implemented for which  $P_{fa}$  value is set to  $10^{-2}$ . The resulting  $P_d$  vs. SNR curve is represented in fig. 2.

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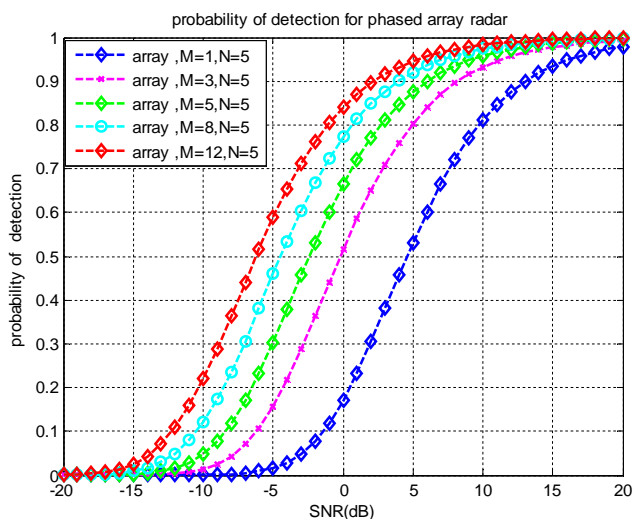


Fig. 2. Probability of detection for phased array radar, changing M.

The detection performance of the phased array radar system enhances as the number of transmit antennas increases although the transmitted power is constant. The gain increases as the number of transmit antenna increases although the noise power in the received signal remains constant.

If the number of transmit elements is held constant at the value of 5 and the number of receive elements is increased, the  $P_d$  vs. SNR curve in fig. 3 is obtained. We can see from the graph that as number of receiving antennas is increased the probability of detection increases, because the total received energy increases.

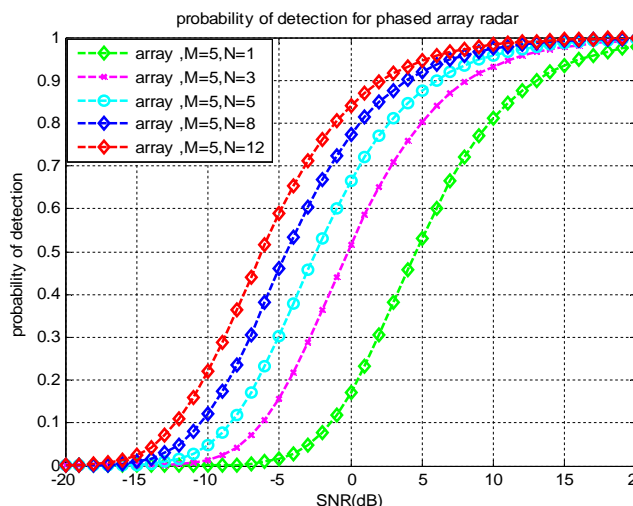


Fig. 3. Probability of detection for phased array radar, changing N.

## 2.2. Statistical MIMO radar

Statistical MIMO radar employs antenna arrays which are widely separated. The inter element spacing in an array is also so large that each transmit-receive pair sees a different aspect of the target and thus sees different RCS due to

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target's complex shape. If the spacing between the antennas elements is wide enough, received signals from each transmit receive pair become independent. This is called spatial or angular diversity. Statistical MIMO radar focuses on this property.

## 2.2.1. System Model

The model of the statistical MIMO radar system is shown in fig. 4 where transmitters and receivers be widely separated.

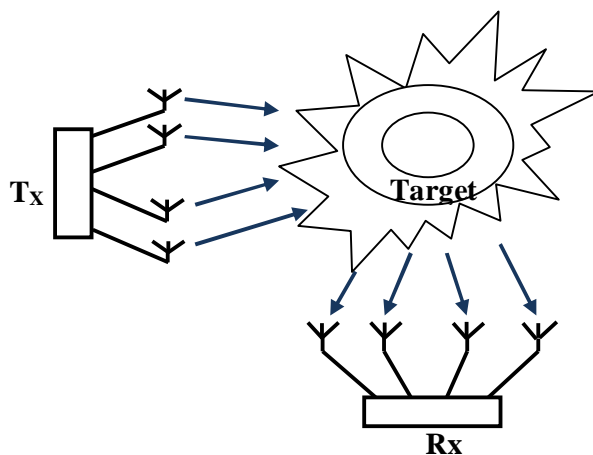


Fig. 4. Statistical MIMO radar configuration.

If the received signal is fed to a bank of matched filters each of which is matched to  $x_m(t)$ , and the corresponding output is sampled at the time instant, then the output of the matched filter bank can be written in the vector form as

$$\bar{y} = \sqrt{\frac{E_t}{M}} \bar{\alpha} + \bar{w} \quad \dots \dots \dots (9)$$

Where  $\bar{y}$  is  $aMN \times 1$  complex vector whose entries correspond to the output of the each matched filter at every receiver,  $\bar{\alpha}$  is a  $MN \times 1$  complex vector that contains all the elements of channel matrix (H) and  $\bar{w}$  is  $aMN \times 1$  complex noise vector.

## 2.2.2. Probability detection

Statistical MIMO Radar can be formulated as binary hypothesis testing problem for  $H_0$  indicates absence of signal and  $H_1$  indicates presence of signal.

$$\left. \begin{aligned} H_0: & \bar{y} = \bar{w} \\ H_1: & \bar{y} = \sqrt{\frac{E_t}{M}} \bar{\alpha} + \bar{w} \end{aligned} \right\} \dots \dots \dots (10)$$

It is well known that the optimum solution to this hypothesis testing problem under Neyman-Pearson criterion is the Likelihood Ratio Test (LRT) as,

$$\frac{p(\bar{y}|H_1, C, A)}{p(\bar{y}|H_0, C)} \underset{H_0}{\overset{H_1}{\geq}} T \dots \dots \dots (11)$$

The  $P_{fa}$  can be calculated as in terms of threshold  $T'$ ,

$$P_{fa} = Prob \left\{ \frac{\sigma_w^2}{2} \chi_{2MN}^2 > T' \right\}$$

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$$\begin{aligned}
 &= Prob\left\{\chi_{2MN}^2 > \frac{2}{\sigma_w^2} T'\right\} \\
 &= 1 - Q_{\chi_{2MN}^2} \exp\left(\frac{2}{\sigma_w^2} T'\right) \dots \dots (12)
 \end{aligned}$$

Where,  $Q_{\chi_{2MN}^2}$  represents the cumulative distribution function of a Chi-squared random. The  $P_d$  can be calculated in terms of  $P_{fa}$  and SNR as,

$$P_d = 1 - Q_{\chi_{2MN}^2} \left( \frac{1}{\left(\frac{SNR}{M} + 1\right)} Q_{\chi_{2MN}^2}^{-1} (1 - P_{fa}) \right) \dots \dots (13)$$

### 2.2.3. Results and Observation

To compare with the detection performance of statistical MIMO radar, the probability detection in equation (13) is implemented for which  $P_{fa}$  value is set to  $10^{-2}$ . If the number of receiving elements is held constant at the value of 5, and the number of transmitting elements is increased, the  $P_d$  vs. SNR curve in fig. 5 is obtained.

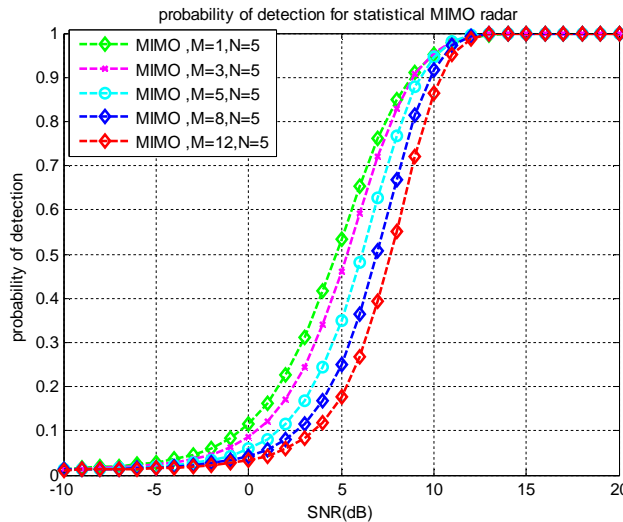


Fig. 5. Probability of detection for statistical MIMO radar, changing M.

It is interesting to see that the detection performance decreases as the number of transmitting antennas increases. Because of this decrease in the detection performance; using more widely separated receiving antennas instead of increasing the number of spatially diverse transmitting antennas seems more reasonable.

If the number of transmitting elements is held constant at the value of 5, and the number of receiving elements is increased, the  $p_d$  vs. SNR curve in fig. 6 is obtained.

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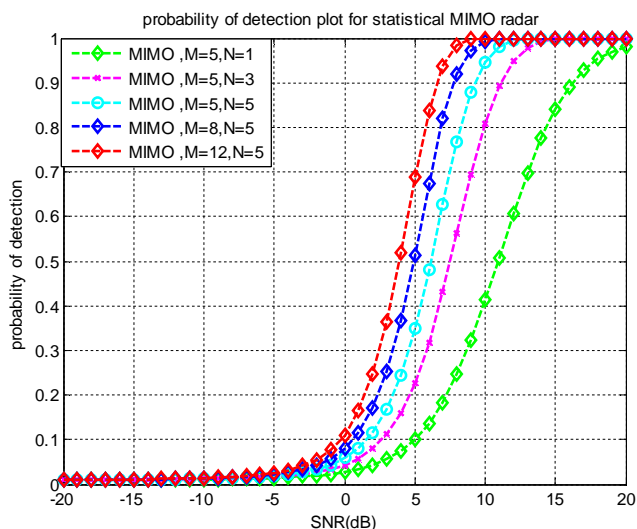


Fig. 6. Probability of detection for statistical MIMO radar, changing N.

The comparison of phased array radar versus statistical MIMO radar is given in fig. 7. This figure is obtained using the analytical expressions given in equations (8) and (13) for  $M = N = 5$ . In the figure the red lines belong to statistical MIMO radar and the blue lines belong to phased array radar. The results in fig. 7 show that at high SNR values and at high detection probabilities, the detection performance of statistical MIMO radar is better than phased array radar. Whereas, at low SNR values, phased array radar performs better than statistical MIMO radar.

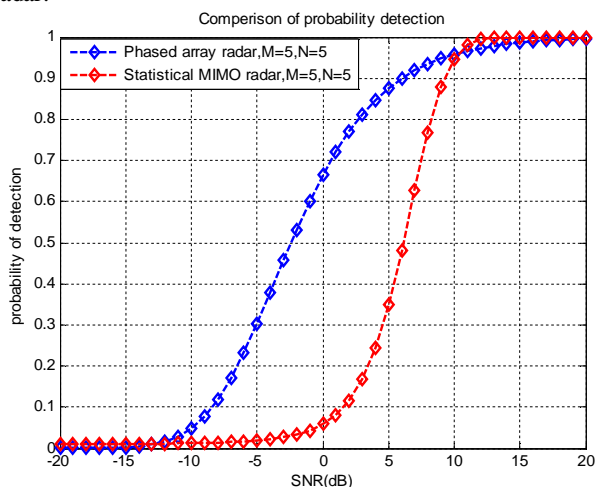


Fig. 7. Comparison of probability detection of statistical MIMO radar vs. phased array radar.

### III. CONCLUSIONS

In this paper, we have presented a wide variety of signal processing algorithms for statistical MIMO radar and phased array radar. We propose novel algorithms for improving the MIMO radar system. This concept differs substantially from current regimes in which closely spaced antenna arrays are used. With closely spaced antenna elements, it is possible to realize a coherent processing gain. However, these systems are prone to severe target fading, and hence they may suffer considerable performance degradation. In contrast, MIMO radar is composed of many independent types of



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radar, each of which sees a different aspect of the target, enabling the exploitation of spatial diversity to overcome target fading. MIMO radar cannot realize any processing gain. Rather, it exploits that target angular spread to combat target fading. We investigated and compared the inherent performance limitations of both conventional phased array radars and the statistical radars. We derived the respective optimal detectors when the target and noise level are either known or unknown. We demonstrated that the MIMO radar outperforms the conventional phased array radar at high detection rates because angular diversity enables RCS fluctuation smoothing.

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## BIOGRAPHY



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