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OBSERVATIONS ON THE NON-HOMOGENEOUS QUINTIC EQUATION WITH FIVE UNKNOWNS

$$x^4 - y^4 = 2(k^2 + s^2)(z^2 - w^2)P^3$$

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Abstract: **The quintic Diophantine equation with five unknowns $x^4 - y^4 = 2(k^2 + s^2)(z^2 - w^2)p^3$ is analyzed for its infinitely many non-zero distinct integral solutions. A few interesting relations between the solutions and special numbers namely, centered polygonal numbers, centered pyramidal numbers, jacobsthal numbers, Lucas numbers and Keynea numbers are presented.**

Keywords: Quintic equation with five unknowns, Integral solutions, centered polygonal numbers, centered pyramidal numbers.

Mathematics subject classification number: 11D41.

NOTATIONS

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$ - Polygonal number of rank n with size m.

$P_n^m = \left(\frac{n(n+1)}{6}\right)[(m-2)n + (5-m)]$ - Pyramidal number of rank n with size m.

$Pt_n = \frac{n(n+1)(n+2)(n+3)}{24}$ - Pentatope number of rank n

$SO_n = n(2n^2 - 1)$ -Stella octangular number of rank n

$S_n = 6n(n-1) + 1$ -Star number of rank n

$\Pr_n = n(n+1)$ - Pronic number of rank n.

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$J_n = \frac{1}{3}\left(2^n - (-1)^n\right)$-Jacobsthal number of rank n.

$j_n = (2^n + (-1)^n)$ - Jacobsthal lucas number of rank n.

$Ky_n = (2^n + 1)^n - 2$ - Keynea number.

$F_{4,m,3} = \frac{n(n+1)(n+2)(n+3)}{4!}$ -Four dimensional figurative number of rank n whose generating polygon is a triangle.

$F_{5,m,3} = \frac{n(n+1)(n+2)(n+3)(n+4)}{5!}$ -Five dimensional figurative number of rank n whose generating polygon is a triangle.

$CP_n^m = \frac{mn(n-1)}{2} + 1$ - Centered polygonal number of rank n with size m.

I.INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular quintic equations homogeneous or non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1,2,3].For illustration ,one may refer [4-10],for quintic equations with three ,four and five unknowns. This paper concerns with the problem of determining integral solutions of the non-homogeneous quintic equation with five unknowns given by $x^4 - y^4 = 2(k^2 + s^2)(z^2 - w^2)p^3$. A few relations between the solutions and the special numbers are presented..

II.METHOD OF ANALYSIS

The Diophantine equation representing the quintic with five unknowns under consideration is

$$x^4 - y^4 = 2(k^2 + s^2)(z^2 - w^2)p^3 \quad (1)$$

Introducing the transformations

$$x = u + v, y = u - v, z = uv + \alpha^2 + 1, w = uv - \alpha^2 - 1 \quad (2)$$

where α is a distinct positive distinct integer in (1),we get

$$u^2 + v^2 + = (k^2 + s^2)(\alpha^2 + 1)p^3 \quad (3)$$

Assume $p = a^2 + b^2$ (4)

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Substituting (4) in (3) and employing the method of factorization define

$$u+iv=(k+is)(\alpha+i)(a+ib)^3 \qquad (5)$$

equating real and imaginary parts, we get

$$u=\alpha(k-s)(a^3-3ab^2)-(k+s)(3a^2b-b^3)$$

$$v=\alpha(k+s)(a^3-3ab^2)+(k-s)(3a^2b-b^3)$$

Thus, in view of (2), the non-zero distinct integral solutions of (1) are given by

$$x(a,b)=[\alpha(k+s)+(k-s)](a^3-3ab^2)-[\alpha(k-s)-(k+s)](3a^2b-b^3)$$

$$y(a,b)=[\alpha(k-s)-(k+s)](a^3-3ab^2)-[\alpha(k+s)+(k-s)](3a^2b-b^3)$$

$$z(a,b)=[(\alpha^2-1)ks+\alpha(k^2-s^2)][(a^3-3ab^2)^2-(3a^2b-b^3)^2]+$$
$$[\alpha^2-1)(k^2-s^2)-4\alpha ks][(a^3-3ab^2)(3a^2b-b^3]+\alpha^2+1$$

$$w(a,b)=[(\alpha^2-1)ks+\alpha(k^2-s^2)][(a^3-3ab^2)^2-(3a^2b-b^3)^2]+$$
$$[\alpha^2-1)(k^2-s^2)-4\alpha ks][(a^3-3ab^2)(3a^2b-b^3]-\alpha^2-1$$

$$p(a,b)=a^2+b^2$$

.For simplicity and clear understanding we present below the integer solutions and the corresponding properties for $\alpha=0$ and $\alpha=1$.

A. Case:1

Let $\alpha=0$ The non-zero distinct integer solutions of (1) are found to be

$$x(a,b)=(k-s)(a^3-3ab^2)-(k+s)(3a^2b-b^3)$$

$$y(a,b)=-(k-s)(a^3-3ab^2)-(k+s)(3a^2b-b^3)$$

$$z(a,b)=1-ks[(a^3-3ab^2)^2-(3a^2b-b^3)^2]-(k^2-s^2)[(a^3-3ab^2)(3a^2b-b^3)]$$

$$w(a,b)=-1-ks[(a^3-3ab^2)^2-(3a^2b-b^3)^2]-(k^2-s^2)[(a^3-3ab^2)(3a^2b-b^3)$$

$$p(a,b)=a^2+b^2$$

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B.Properties

1) $x(a,1)+y(a,1)+3s(OH_a)+kS_a+7(k-s)[\Pr_a - t_{4,a}] \equiv 0 (\bmod k)$

2) $p(2^n,1)-Ky_{2n}+6J_{2n}=0$

3) $x(1,b)-y(1,b)+2kCp_{6,b}-6sP_b^4-12(k+s)t_{4,b}+2st_{5,b}+6(k+s)t_{6,b}=4k.$

4) $p(2^n,1)-Ky_n \equiv 0(\bmod 2)$

5) $6[x(a,1)-y(a,1)+p(a,1)-kSO_a+sS_a+5(k+s)\Pr_a+(5k+8s)t_{4,a}-2st_{5,a}-3s-1]$ is a nasty number

1) Remark : It is worth to note that when $\alpha = 0$,we have another pattern of solution which is illustrated below.

For this case $\alpha = 0$,(3) reduces to

$$u^2+v^2=(k^2+s^2)p^3 \tag{6}$$

Following the analysis presented above ,the values of u and v are given by

$$u = k(a^3-3ab^2)-s(3a^2b-b^3)$$

$$v = s(a^3-3ab^2)+k(3a^2b-b^3)$$

Hence, the non-zero distinct integral solutions of (1) are given by,

$$x(a,b)=(k+s)(a^3-3ab^2)+(k-s)(3a^2b-b^3)$$

$$y(a,b)=(k-s)(a^3-3ab^2)-(k+s)(3a^2b-b^3)$$

$$z(a,b)=(k^2-s^2)[(a^3-3ab^2)(3a^2b-b^3)]+ks[(a^3-3ab^2)^2-(3a^2b-b^3)^2]+1$$

$$w(a,b)=(k^2-s^2)[(a^3-3ab^2)(3a^2b-b^3)]+ks[(a^3-3ab^2)^2-(3a^2b-b^3)^2]-1$$

$$p(a,b)=a^2+b^2$$

The above values of x, y, z and w are different from that of in case (1) presented above.

C.Properties:

1) $x(a,2)+y(a,2)-2k[3P_a^4-t_{3,a}]+2s[CP_{12,a}-S_a)-s \equiv 0(\bmod 2).$

2) $x(a, a+1) + 12(k+s)P_a^3 - (k-s)SO_a + (k-3s)(2t_{4,a} - t_{6,a}) = 0..$

3) $6[\,p(a^2-1, a^2) - 4P^5_{a^2-1} + So_{a^2-1}]$ is a nasty number.

4) $y(b-1, b) - (k-s)[3OH_b + 8t_{3,b}] - 3(k+s)[2P_b^4 - 3t_{4,b}] \equiv 0(\mathrm{mod}\,2).$

5) $p(a^2, a^2+1) - 48Pt_a + 36P_a^4 - S_a + 6t_{4,a} = 0$

D.Case:2

Let $\alpha = 1$ .After performing a few calculations as in case (1) the non-zero distinct integer solutions are obtained as

$$x(a,b) = 2k(a^3 - 3ab^2) - 2s(3a^2b - b^3)$$

$$y(a,b) = -2s(a^3 - 3ab^2) - 2k(3a^2b - b^3)$$

$$z(a,b) = (k^2 - s^2)[(a^3 - 3ab^2)^2 - (3a^2b - b^3)^2] - 4ks[(a^3 - 3ab^2)(3a^2b - b^3) + 2$$

$$w(a,b) = (k^2 - s^2)[(a^3 - 3ab^2)^2 - (3a^2b - b^3)^2] - 4ks[(a^3 - 3ab^2)(3a^2b - b^3) - 2$$

$$p(a,b) = a^2 + b^2$$

E.Properties:

1) $x(a,1) + y(a,1) - 2(k-s)[P_a^5 - \mathrm{Pr}_a + 4t_{3,a-1}] + 2(k+s)[CP_{6,a} - 6t_{3,a} - 2] - 2(s-5k)t_{4,a} = 0.$

2) $x(a,1) + y(a,1) - 12kP_a^3 + 2kS_a + sSO_a + 2st_{8,a} \equiv 0(\mathrm{mod}\,2)$

3) $p(2^n, 2^n+1) - Ky_n - j_{2n} = 1$

III.CONCLUSION

In addition to the above patterns of solutions, there are other forms of integer solutions to (1). For illustration, when $\alpha = 0$, the equation (6) is written as

$$u^2 + v^2 = (k^2 + s^2)p^3 * 1 \qquad (7)$$

Write 1 as,

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$$1 = \frac{[(m^2 - n^2) + i2mn][(m^2 - n^2) - i2mn]}{(m^2 + n^2)^2} \quad (8)$$

or

$$1 = \frac{[2mn + i(m^2 - n^2)][2mn - i(m^2 - n^2)]}{(m^2 + n^2)^2} \quad (9)$$

Using (4) and (8) in (7) and employing the method of factorization, define,

$$u + iv = (k + is)(a + ib)^3 \left[\frac{(m^2 - n^2) + i2mn}{m^2 + n^2} \right]$$

Equating the real and imaginary parts, the values of u and v are obtained.Substuting these values of u and v in (2) and choosing a and b suitably, many different integer solutions to (1) are obtained. Similar process is carried out by considering (4) and (9).

To conclude one may search for other choices of solutions to (1) along with the

corresponding properties.

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