ISSN (Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

An ISO 3297: 2007 Certified Organization, Volume 2, Special Issue 1, December 2013

Proceedings of International Conference on Energy and Environment-2013 (ICEE 2013)

On 12th to 14th December Organized by

Department of Civil Engineering and Mechanical Engineering of Rajiv Gandhi Institute of Technology, Kottayam, Kerala, India

NUMERICAL STUDIES ON HYDRAULIC TRANSIENTS DURING PUMP STARTUP AND COAST-DOWN IN AN ADIABATIC CLOSED LOOP

Seik Mansoor Ali, Ananya Mohanty, Krishna Chandran. R, V. Balasubramaniyan
Safety Research Institute, Atomic Energy Regulatory Board, Kalpakkam – 603 102, India
Safety Research Institute, Atomic Energy Regulatory Board, Kalpakkam – 603 102, India
Safety Research Institute, Atomic Energy Regulatory Board, Kalpakkam – 603 102, India
Safety Research Institute, Atomic Energy Regulatory Board, Kalpakkam – 603 102, India

ABSTRACT
The focus of this paper is on investigating the flow development and decay in a single closed loop, under pump start up as well as trip conditions. The relevant governing differential equations are derived from first principles. A numerical solution technique is described for integration of these equations that uses data from the complete pump characteristic curves. A step by step solution algorithm is provided for the same. Benchmarking and validation studies against available numerical and experimental data are presented. Thereafter, the solution technique is demonstrated using actual performance curves of several pumps. Finally, parametric studies are presented to highlight the influence of specific speed on the flow development and decay in the closed loop.

Keywords: Transient, coastdown, startup, closed loop, pump characteristic curves

NOMENCLATURE
A Area of cross section of the pipe (m^2)
c Constant
D Pipe diameter or equivalent diameter (m)
f Darcy friction factor
F Force (N)
g Gravitational constant (m/s^2)
h Dimensionless head
J Polar moment of inertia of rotating parts ($kg.m.s^2$)
L Pipe or channel length (m)

m Dimensionless impeller torque
M Pump impeller torque (N. m), mass of the fluid
n Number of control volumes
N Pump speed (rpm), number of loops
P Pressure (Pa)
q Pump discharge at any time (m^3/s)
Q Dimensionless flow rate
t Time (s)
T Dimensionless time

Greek symbols

α System half-time
Δ Increment
ρ Density of fluid(kg/m^3)
Ω Dimensionless pump speed
ω Rotational speed of the pump (rad/s)
τ Pump half-time(s)

Subscripts

o Initial or Steady state conditions
P Pump

1. INTRODUCTION

Transient variation of pressure and flow are produced in a closed loop by starting or stopping a pump, by opening or closing of valves or by the regulation of pump speed. Such transient flow problems are encountered in many engineering applications. In the safety analysis of nuclear power reactors, the precise evaluation of rapid flow transients is an important factor in reactor design. Flow coast down studies are often undertaken to predict of rate of decrease of coolant flow through the reactor core and its influence on the decay heat removal capability of the circulating system following the pump trip. Similarly, during the starting of a centrifugal pump, before the final steady state conditions are reached, certain transient conditions can produce high heads and consequently require torques much higher than design values. Therefore, knowledge of startup transients is also very important. In view of above, investigations into transient flow behavior in closed loops systems are essential.

The mathematical formulation of the problem that describes the interaction between the system fluid and pump impeller involves the equations of motion of fluid in the loop and that of the pump rotating parts. These equations need to be integrated simultaneously in order to obtain the system behavior. An elegant mathematical model for flow coast down in a single closed loop centrifugal pump system was presented by Burgreen[1]. The governing equation for flow was derived assuming the fluid in the loop to be rigid, neglecting the elastic effects. Burgreen proposed analytical solution to this problem that did not require the use of complete pump characteristic curves. Although this method was approximate, involving a number of assumptions, yet it was shown to yield results that were in fair agreement with those obtained by the use of pump characteristic curves. Later, he extended this model for investigating pump start-up transients also. Analytical as well as experimental studies on flow coast down in centrifugal pump systems were conducted by Yokomura [2]. The focus of his investigation was on the flow coast down behavior of centrifugal pumps in water cooled reactor systems. The experimental test loop was a scaled down model of the first Japanese nuclear ship reactor. The test loop consisted of ½ scaled reactor core structural model, a full scale fuel assembly, steam generator plenum model and circulating water pumps operating at near ambient conditions. He proposed and discussed an analytical model that did not require any information on pump characteristics. The results of the analytical predictions were compared with those from the experimental setup and shown to be satisfactory.

More recently, Farhadi et al. [3] presented a numerical model for analyzing pump start-up transients in the piping system of nuclear reactors. A case study was presented wherein the results of numerical predictions were compared with the experimental data from the Tehran Research Reactor (an open pool MTR-type reactor). Later Farhadi [4-5] presented mathematical models for flow coast down transients. These models could account for the mechanical and electrical losses in TRR three-phase induction motor and thus were an improvement over existing models. Several case studies with application to the Tehran Research Reactor (TRR) primary piping system were presented and discussed. These works also elucidated the influence of key variables on the system transient behavior, through parametric studies. Further, the results of the model predictions were compared with those obtained from other similar mathematical models. These predictions were shown to be in good agreement with available experimental data and numerical predictions.

At the outset, this paper presents a mathematical model for investigating flow development and decay in a single closed adiabatic loop, under pump start up as well as trip conditions. The numerical solution technique for integration of these equations uses data from the complete pump characteristic curves. A step by step solution algorithm is developed for this purpose and explained. The model is first benchmarked and validated using available numerical and experimental data. Thereafter, the solution technique is demonstrated using actual performance curves of several types of pumps. The influence of type of pump and specific speed on the transients is elucidated using parametric studies. The objective of this work is to make available a tool for safety studies on reactor system flow transients. Subsequently, it is proposed to enhance its capability by incorporating appropriate energy balance equations, so that the temperature transients in the nuclear fuel elements, clad and coolant can also be obtained. The model can also be extended in future for investigating multiple loop transients. The availability of such a tool will be useful for conducting quick reactor safety assessment studies.

2. MATHEMATICAL MODEL

a. Equations of Motion for Fluid in the Loop

The basic assumptions used in the analysis are: (a) the loop is vertically oriented, (b) fluid in the loop is incompressible and (c) Pipe walls are rigid. A typical loop of the reactor system is shown in Figure 1. It is subdivided into a number of control regions. Consider a single control volume in the loop. Applying the conservation of momentum principle to a control volume containing the pump, we get:

$$\sum_{i=1}^{n}\left(\frac{L}{A}\right)_i \frac{dq}{dt} + \left[\frac{q}{2}\sum_{i=1}^{n}\left(\frac{fL}{DA^2}\right)_i\right] = \frac{\Delta P_p}{\rho_i} \qquad (1)$$

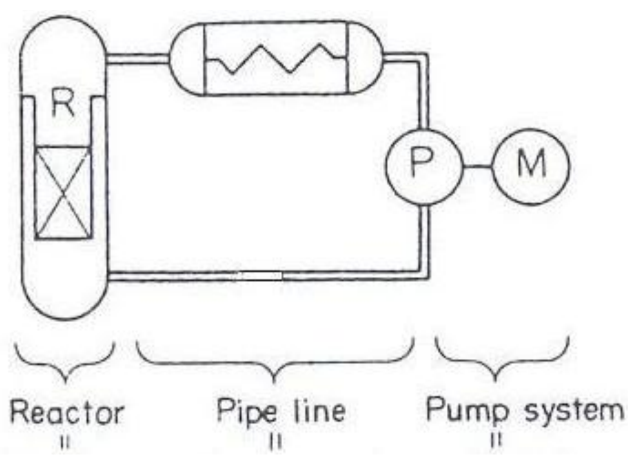

FIGURE 1. ARRANGEMENT OF SINGLE LOOP SYSTEM

The above equation can be further simplified to get the following equation:

$$\sum_{i=1}^{n}\left(\frac{L}{A}\right)_i \frac{dq}{dt}+\frac{q^2}{2}\left[\sum_{i=1}^{n}\left(\frac{fL}{DA^2}\right)_i\right]=gh_p \qquad (2)$$

Under steady state conditions,

$$\frac{q_o^2}{2}\left[\sum_{i=1}^{n}\left(\frac{fL}{DA^2}\right)_i\right]=gh_{po} \qquad (3)$$

Equation (3) can be substituted in Eqn. (2). If coast-down occurs in a loop that does not contain a pump impeller to influence the flow, then the right side of Eqn. (2) becomes zero, thereby giving

$$\sum_{i=1}^{n}\left(\frac{L}{A}\right)_i \frac{dq}{dt}+gh_{po}\left(\frac{q}{q_o}\right)^2=0 \qquad (4)$$

Now, the time in which the flow in the loop is reduced to half of its initial value can be obtained as follows.

The loop half time $t_{1/2}$ is given by:

$$t_{1/2}=\frac{Q_o\sum_{i=1}^{n}\left(\frac{L}{A}\right)_i}{gh_{po}} \qquad (5)$$

Equation (4) can be normalized using the following dimensionless quantities.

$$T=\left(\frac{t}{t_{1/2}}\right),\ Q=\frac{q}{q_o},\ h=\frac{h_P}{h_{po}},\ \Omega=\frac{\omega}{\omega_o}$$

Final form of the equation is

$$\frac{dQ}{dT}+Q^2=h \qquad \text{where, } h=h(Q,\Omega) \qquad (6)$$

b. Equations of Motion for Pump and Prime Mover for Flow Coastdown

On applying the conservation of angular momentum principle to the rotating elements of the pump and prime mover, we get:

$$\frac{d\omega}{dt}=-\frac{M}{J} \qquad (7)$$

The minus sign shows that the torque exerted by fluid on the impeller is always opposite to that of the prime mover. The polar moment of inertia in Eqn. (7) is that of all the rotating parts of the prime mover and pump, including the fluid that rotates with the impeller.

Introducing the following dimensionless quantities in Eqn. (7),

$\left(\frac{\omega}{\omega_o}\right)=\left(\frac{N}{N_o}\right)=\Omega$ and $\left(\frac{M}{M_o}\right)=m$ we get

$$\frac{d\Omega}{dt}=-\left(\frac{60}{2\pi N_o}\right)\left(\frac{M_o}{J}\right)m \qquad (8)$$

Suppose the pump is operating at normal speed and suddenly power failure occurs, then the time in which the pump speed is reduced to half of its initial speed (pump half time), can be obtained as follows:

$$\tau_{1/2}=\frac{\omega_o J}{M_o} \qquad (9)$$

Now, we introduce a new variable called system half time. The significance of system half time is that it is the ratio of loop half time to pump half time. It is given by:

$$\alpha=\frac{t_{1/2}}{\tau_{1/2}} \qquad (10)$$

Equation (8) can be normalized with respect to normalized time 'T' as:

$$\frac{d\Omega}{dT} = -\left(\frac{60}{2\pi N_o}\right)\left(\frac{M_o}{J}\right) \times m \times t_{1/2} \quad (11)$$

Simplifying (11) using (9) and (10), the resulting equation is:

$$\frac{d\Omega}{dT} = -\left(\frac{t_{1/2}}{\tau_{1/2}}\right) m \quad (12)$$

Final form of the equation is

$$\frac{d\Omega}{dT} + \alpha m = 0 \qquad \text{where, } m = m(Q, \Omega) \quad (13)$$

c. Equations of Motion for Startup Transient

Application of the conservation of angular momentum principle to the rotating elements yields,

$$J\left(\frac{d\omega}{dt}\right) + c\omega^2 = c\omega_o^2 \quad (14)$$

Assuming no retarding impeller torque, the pump equation becomes:

$$J\left(\frac{d\omega}{dt}\right) = c\omega_o^2 \quad (15)$$

The time required for the pump to start-up is obtained by putting $\omega = \omega_0$ is given by:

$$\tau_{1/2} = \frac{J}{c\omega_o} \quad (16)$$

Normalizing Eqn. (15) using the dimensionless quantities we get:

$$\frac{d\Omega}{dT} + \alpha m = \alpha \qquad \text{where, } m = m(Q, \Omega) \quad (17)$$

3. NUMERICAL METHOD

The coast down transients can be obtained by using the pump characteristic curves. In this case, the simultaneous integration of Equations (6) and (13) are carried out numerically with initial conditions as Q (0) = 1.0 and Ω (0) = 1.0. The value of h = h(Q, Ω) and m = m(Q, Ω) required for the integration can be obtained from the complete pump characteristic curves.

Similarly, the startup transients can also be obtained by using the pump characteristic curves. In this case, the simultaneous integration of Equations (6) and (17) are carried out numerically with initial conditions as Q (0) = 0.0 and Ω (0) = 0.0. A detailed solution procedure is described here.

It is convenient to write equations (6), (13) and (17) in the following form:

$$\frac{dQ}{dT} = h - Q^2 \qquad \text{where, } h = h(Q, \Omega) \quad (18)$$

$$\frac{d\Omega}{dT} = -\alpha m \qquad \text{where, } m = m(Q, \Omega) \quad (19)$$

$$\frac{d\Omega}{dT} = -\alpha m + \alpha \qquad \text{where, } m = m(Q, \Omega) \quad (20)$$

The quantities Q and Ω in the above equations are the instantaneous values at any particular time. The quantities m and h are available as functions of Q and Ω from the pump characteristic curves. As mentioned earlier for coastdown transients, Equations (6) and (13) need to be solved simultaneously. Similarly, for start-up, Equations (6) and (17) need to be solved simultaneously. The procedure described below is applicable to both these transients. Step wise integration of relevant equations is as follows:

(1) At the instant of power failure/pump startup (t=0), set the initial conditions as

$h_O = \Omega_O = m_O = Q_O = 1.0$ during coast-down

$h_O = \Omega_O = m_O = Q_O = 0.0$ during start-up

(2) Assume that during a small interval of time ΔT the discharge remains constant, at Q_O Therefore, $\Delta Q_1 = 0$

(3) Calculate $\Delta\Omega_1 = -(\alpha)m_O\Delta T + \alpha'$, where $\alpha' = 0$ for coast down and $\alpha' = \alpha$ for startup.

(4) At the end of first time step, $Q_1 = Q_O$ and $\Omega_1 = \Omega_O + \Delta\Omega_1$

(5) Calculate $\left(Q/\Omega\right)_1$ and $\left(\Omega/Q\right)_1$

(6) If the initial conditions are found to be in section 1, then the values of $\left(h/\Omega^2\right)_1$ and $\left(m/\Omega^2\right)_1$ can be read from the curves corresponding to $\left(Q/\Omega\right)_1$, so that applying the affinity laws we get,

$$h_1 = \left(h/\Omega^2\right)_1\Omega^2 \quad ; \quad m_1 = \left(m/\Omega^2\right)_1\Omega^2$$

(7) This concludes the first time step iteration.

(8) The procedure for the subsequent time steps are different because of the expected change in the value of Q

(9) Now the 4th order Runge-Kutta method is used to compute the value of Ω_2 and Q_2

(10) At each nth time step calculate values of $\left(Q/\Omega\right)_n$ and $\left(\Omega/Q\right)_n$. Depending on this value, chose appropriate section of the graph to extract the data.

(11) Then, applying the affinity laws, 'h' and 'm' can be calculated as follows:

$$h_n = \left(h/\Omega^2\right)_n\Omega^2; m_n = \left(m/\Omega^2\right)_n\Omega^2 \text{ for } 0 < \left(\frac{Q}{\Omega}\right) < 1.0$$

$$h_n = \left(h/Q^2\right)_n Q^2; m_n = \left(m/Q^2\right)_n Q^2 \text{ for } 0 < \left(\frac{\Omega}{Q}\right) < 1.0$$

(12) Continue steps (10) and (11) till the entire transient is obtained.

4. RESULTS AND DISCUSSION

For the numerical analysis, geometrical details and pump data was taken from Yokomura [2] and the details are given in Table 1.

TABLE 1. SYSTEM DATA FOR VARIOUS FLOW LOOP CONFIGURATIONS

Sl. No	Parameters	System Details
1	Type of reactor/ system	Scaled down model of water-cooled reactor (Yokomura)
2	Number of loops	Two parallel loop
3	NR (rpm)	1470
4	H_0 (m)	41.9
5	Q_0 (m^3/s)	0.125
6	η_o (%)	75.6
7	I_P ($kg\text{-}m^2$)	3.7
8	α	0.262
9	$\sum(L/A)$ (m^{-1})	959

a. Flow Coastdown Transient

Equations (6) and (13) were solved analytically without using pump characteristics to validate experimental data of Yokomura [2]. The comparison of flow transient is shown in Figure 2. Very good agreement is obtained except at the final stages of coastdown where the deviation is about 7%. This deviation is expected at the terminal stages, as the present model assumes that the pump head is always positive (proportional to square of pump speed) and therefore excludes the possibility of turbining.

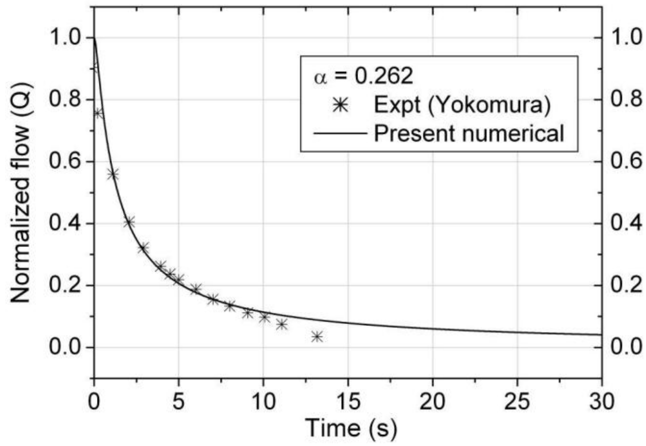

FIGURE 2. COMPARISON OF COAST-DOWN PREDICTION WITH EXPERIMENTAL DATA OF YOKOMURA

a. Pump Start-up Transient

Following the coastdown studies, the pump start-up transient was taken up. For the startup transients, no specific experimental investigation was available. Therefore, the present predictions are benchmarked against the analytical/numerical predictions of Yokomura. For this, equations (6) and (17) were solved analytically without using pump characteristic curves. Flow transient obtained by the present model is found to overlap with the analytical model of Yokomura as shown in Figure 3.

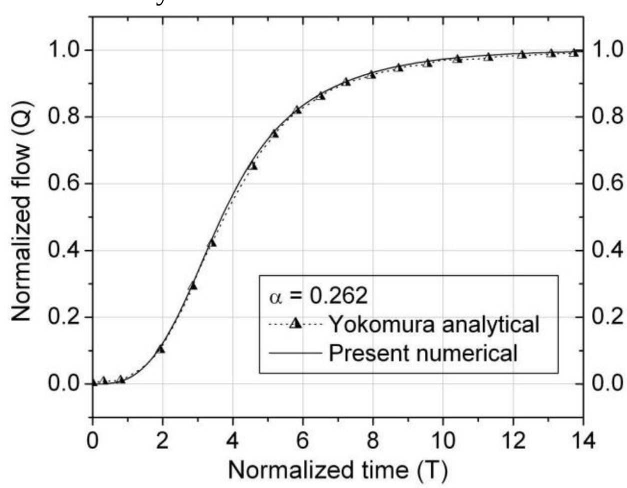

FIGURE 3. COMPARISON OF START-UP PREDICTION WITH ANALYTICAL MODEL OF YOKOMURA

Based on the case studies presented above for coastdown and start-up transients, it is clear that the model is capable of predicting these transients well and can be employed for further studies. Before the parametric studies are presented, it is essential to study the time step sensitivity of the numerical solution. Choice of an appropriate time step value is critical to obtain a fairly good prediction. This is discussed in the next section.

b. Time Step Sensitivity Study

Numerical simulations for pump coastdown and startup were conducted using different time step (Δt) values in the range of 0.5s to 0.01s. Figures.4a and 4b show the transients for startup and coastdown respectively. It is seen that as the time step is reduced from 0.5s to 0.01s, the transients tend to approach a similar trend. The deviation in normalized flow values for Δt of 0.01s and 0.1s is found to be less than 2% in case of startup (see Figure 4a). For the coastdown case, the transients with Δt of 0.01s, 0.1s and 0.2s overlap (see Figure 4b). Based on this sensitivity analysis, for computational economy, time step of 0.1s was chosen for all the subsequent studies.

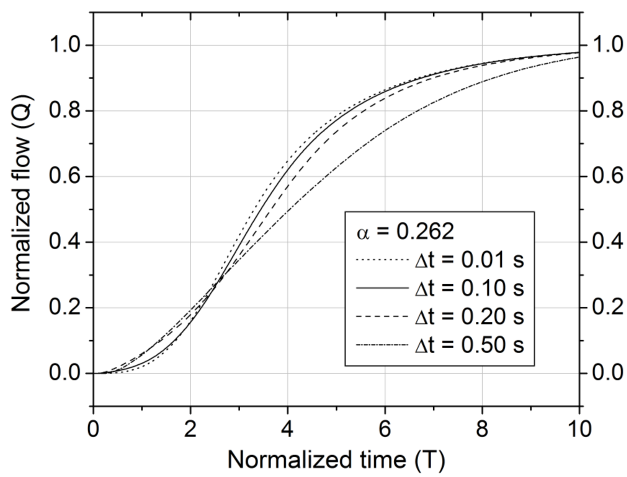

FIGURE 4a. EFFECT OF TIME STEP ON STARTUP TRANSIENTS

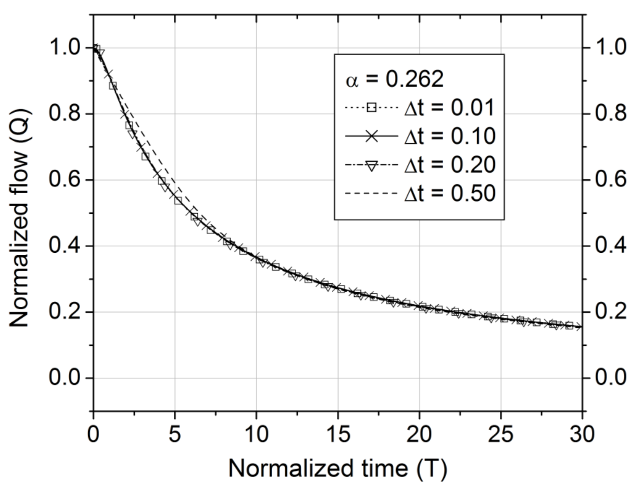

FIGURE 4b. EFFECT OF TIME STEP ON COASTDOWN TRANSIENTS

c. Solution using Complete Pump Characteristic Curves

As mentioned earlier, integration of equations (6) and (13) are done numerically using the pump characteristic curves. The value of head developed (h) and torque (m) required for the integration can be obtained from these curves. For the purpose of computation, polynomials were used to approximate the characteristic curves in the range where it operates in the normal mode. The polynomials used to represent the characteristic diagrams for a typical single suction pump is shown in Table 2. Similar curves were used for other pumps as well. Due to space constraints these are not shown here. Table 3 gives the pump specific speed and maximum efficiency of various pumps.

TABLE 2. POLYNOMIAL FIT FOR SINGLE SUCTION CENTRIFUGAL PUMP PERFORMANCE CURVE

	$Y = \sum_{i=0}^{4} A_i X^i$ Voith pump (N_S =1935)			
	$X = \left(\frac{\Omega}{Q}\right)$ for $0 < \left(\frac{\Omega}{Q}\right) < 1.0$		$X = \left(\frac{Q}{\Omega}\right)$ for $0 < \left(\frac{Q}{\Omega}\right) < 1.0$	
Y	$\left(\frac{h}{Q^2}\right)$	$\left(\frac{m}{Q^2}\right)$	$\left(\frac{h}{\Omega^2}\right)$	$\left(\frac{m}{\Omega^2}\right)$
A_0	-0.925	-0.600	1.150	0.400
A_1	1.355	2.360	0.483	0.747
A_2	2.090	-2.520	-3.033	-1.107
A_3	-3.280	3.040	5.067	2.453
A_4	1.760	-1.280	-2.667	0.160

TABLE 3. PUMP SPECIFIC SPEED AND OVERALL EFFICIENCY

Parameters	Single suction	Double suction	Radial	Axial
Specific speed	1935	1500	1800	13500
Maximum efficiency (%)	84	84	83	79

The solution algorithm described in section 3 was employed along with the curve fit data for each pump to obtain the coastdown and startup transients shown in Figures 5a and 5b. It is essential to mention that the curve fit data corresponds to normal operating conditions of the pumps i.e., the head, torque, flow and speed are all positive. However, the solution algorithm is valid for abnormal operating conditions also.

Apart from demonstrating the use of characteristic curves to obtain the solution, there is another aspect which is highlighted by Figures 5a and 5b. The use of pumps with different specific speed basically alters the effective energy ratio (α) of the system. The loop with an axial flow pump (Ns=13500) has very low α and therefore the coastdown is slow. Similarly, it takes a longer time to attain steady state conditions. The other pumps considered namely radial, single suction and double suction centrifugal pumps have much lower but comparable specific speeds and therefore a high α. In the loop with these pumps, the decay is much faster and startup is also quick. The trends shown in these figures are as expected i.e., as N_s decreases, the decay and startup are faster and vice versa.

Figure 5c shows the variation of normalized flow rate with the normalized pump speed for coastdown transients. It is evident that for each of the chosen pumps, a linear relationship exists between these two variables.

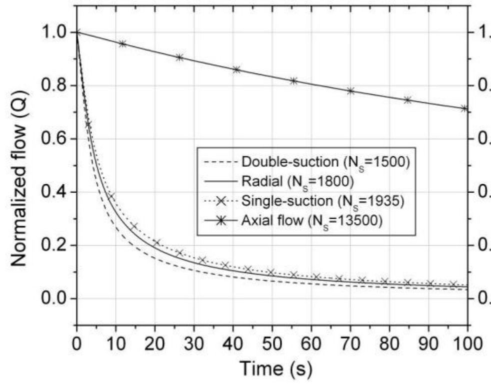

FIGURE 5a. EFFECT OF SPECIFIC SPEED ON NORMALIZED FLOW DURING COAST-DOWN

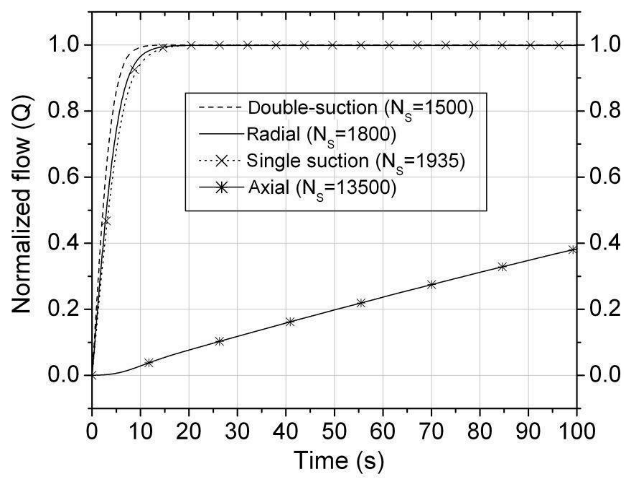

FIGURE 5b. EFFECT OF SPECIFIC SPEED ON NORMALIZED FLOW DURING START-UP

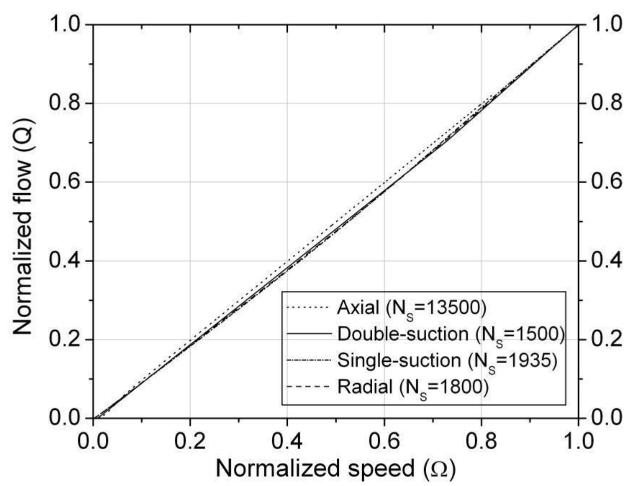

FIGURE 5c. NORMALIZED FLOW RATE VS. NORMALIZED SPEED

5. CONCLUSION

A detailed mathematical model for flow transients in a single loop is presented. The model can be applied to investigate pump coastdown as well as start-up transients, with and without the use of pump characteristic curves. A solution algorithm is developed and described in detail for solving the governing equations using pump characteristic curves. Validation and benchmarking exercises have been conducted using available experimental and analytical data for various operating conditions. Fairly good agreement is found. The influence of pump parameters like specific speed and type of pump on the flow transients are clearly brought out by these studies.

REFERENCES

[1] Burgreen, D., 1959. "Flow coast-down in a loop after pumping power cut-off". Nucl. Sci. Eng,. 6, pp. 306-312.

[2] Yokomura, T., 1969. "Flow coast down in centrifugal pump systems". Nucl. Eng. Des,. 10, pp. 250-258.

[3] Kazem Farhadi, Anis Bousbia-salah, and Franscesco D'Auria, 2007. "A model for the analysis of pump start-up transients in Tehran Research Reactor". Progress in Nuclear Energy, 49, pp. 499-510.

[4] Kazem Farhadi, 2010. "Analysis of flow coastdown for an MTR-pool type research reactor". Progress in Nuclear Energy, 52, pp. 573–579.