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NEWTON’S INTERPOLATION FORMULAE IN MS EXCEL WORKSHEET

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ABSTRACT : It is observed that most of the numerical problems have been solved by developing algorithm using high level languages ,such as FORTRAN ,C,C++, Visual Basic ect ,which are not so easy to handle by all readers .

In this paper we have developed formulae in excel worksheet for solving numerical problems without using high level language which is very familiar and easy accessible to all .

Keywords : interpolation, difference table , excel worksheet

INTRODUCTION :

Interpolation is the process of computing intermediate values of a function from the set of given or tabulated values of the function . For example ,suppose the population of a city is given for last five years 1971,1981,1991,2001and 2011 . The process of computing the population of the city for the year 1975 is called interpolation .the process of interpolation is very much interesting and useful for all branches science , humanities ,commerce , and in technical branches . There are several methods of interpolation but the most suitable interpolation formulae are given by Newton and Lagrange . Newton introduced three interpolation formulae ,known as Newton’s forward interpolation , Newton’s backward interpolation and Newton’s general interpolation formula .

Now let us briefly discuss the Newton’s forward interpolation formula.....

Let a function $y=f(x)$ takes the values $y_0 ,y_1 , y_2 , y_3 , \dots , y_n$ when the independent variable x takes the equidistant values $x_0 ,x_1 , x_2 , x_3 , \dots , x_n$ respectively where $x_1 - x_0 =x_2 - x_1 = x_3 - x_2 = \dots = x_n - x_{n-1} =h$.The differences $y_1 - y_0 ,y_2 - y_1 , y_3 - y_2 , \dots$ are called the first differences and is denoted by $\Delta y_0 , \Delta y_1 , \Delta y_2 , \dots$ respectively .

Similarly the second and the higher order differences are as follows...

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 , \Delta^2 y_1 = \Delta y_2 - \Delta y_1 , \Delta^2 y_2 = \Delta y_3 - \Delta y_2 , \dots$$

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 , \Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1 , \Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2 , \dots$$

With these differences we construct the difference table as follows...

x	y	Δ	Δ^2	Δ^3	Δ^4
x_0	y_0				
		$\Delta y = y_1 - y_0$			
x_1	y_1		$\Delta^2 y = y_2 - y_1 - \Delta y_0$		
		$\Delta y = y_2 - y_1$		$\Delta^3 y = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y = y_3 - y_2 - \Delta y_1$		$\Delta^4 y = \Delta^3 y_1 - \Delta^3 y_0$

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		$\Delta y = y_3 - y_2$		$\Delta^2 y = \Delta y_2 - \Delta y_1$	
x_3	y_3		$\Delta y = y_2 - y_1$		
		$\Delta y = y_4 - y_3$			
x_4	y_4				

Newton's Forward Interpolation formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y_0}{2} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 y_0}{4!} + \frac{u(u-1)(u-2)(u-3)(u-4)\Delta^5 y_0}{5!} + \dots$$

Where $u = \frac{x-x_0}{h}$

For better understanding let us consider a interesting problem

The population of a city in last five decennial censuses was as follows

Year	1971	1981	1991	2001	2011
Population (in lacs)	12	19	31	47	62

The difference table is

x	y	Δ	Δ^2	Δ^3	Δ^4
1971	12				
		7			
1981	19		5		
		12		-1	
1991	31		4		-4
		16		-5	
2001	47		-1		
		15			
2011	62				

Here $x = 1975$ $x_0 = 1971$ and $h = 10$

$$S_0 \quad u = \frac{x-x_0}{h} = \frac{1975-1971}{10} = 0.4$$

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By Newton's Forward Interpolation formula.....

$$\begin{aligned}
 y &= y_0 + u\Delta y_0 + u(u-1)\Delta^2 y_0/2 + u(u-1)(u-2)\Delta^3 y_0/3! + u(u-1)(u-2)(u-3)\Delta^4 y_0/4! \\
 &\quad + u(u-1)(u-2)(u-3)(u-4)\Delta^5 y_0/5! + \dots \\
 &= 12 + 0.4 \times 7 + 0.4 \times (0.4-1) \times 5/2 + 0.4 \times (0.4-1) \times (0.4-2) \times (-1)/6 \\
 &\quad + 0.4 \times (0.4-1) \times (0.4-2) \times (0.4-3) \times (-4)/24 \\
 &= 12 + 2.8 - 0.6 - 0.064 + 0.1664 \\
 &= 14.302
 \end{aligned}$$

Hence the population of the city in the year 1975 is estimated as 14.302 lacs (approximately)

EXISTING IN-BUILT SOFTWARE FOR NUMERIC CALCULATION :

Though the interpolation method is interesting but the computational part is laborious and time consuming as it contains higher decimal places. Therefore sometimes the student feels bore.

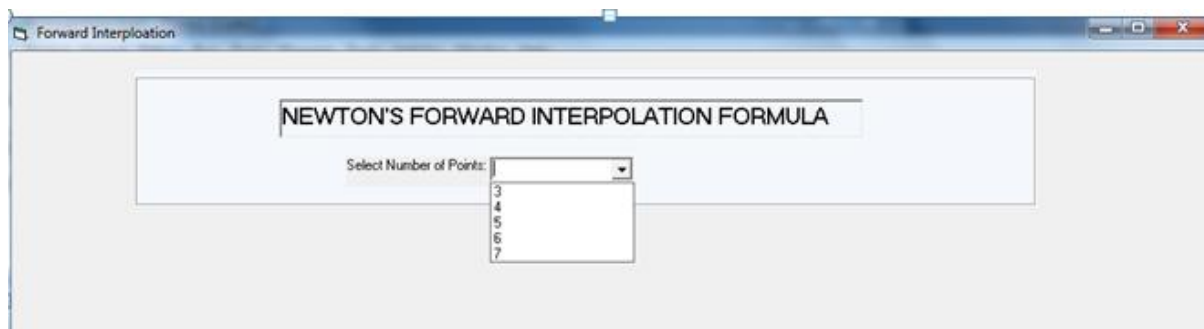
Therefore the mathematician and the computer experts introduce many suitable software for numeric computations. Such as FORTRAN, C, C++, Visual Basic, MATLAB etc. As for example let us discuss the problem in Visual Basic programme.

The same problem can be solve in high-level programming languages like Visual Basic, C, C++, java etc. We have solved this problem by using Visual Basic. Visual Basic is a third-generation event-driven programming language first released in 1991. Visual Basic was designed to be easily learned and used by beginner programmers. Visual Basic is not only a programming language, but also a complete graphical development environment. The language not only allows programmers to create simple GUI applications, but to also develop complex applications.

Visual Basic's main selling point is the ease with which it allows the user to create nice looking, graphical programs with little coding by the programmer, unlike many other languages that may take hundreds of lines of programmer keyed code. As the programmer works in the graphical environment, much of the program code is automatically generated by the Visual Basic program.

We solved the above problem in Visual Basic. We can build our own formula in Visual Basic to solve our problem. To build a formula or program in Visual Basic the reader must have the knowledge of Visual Basic. But one advantage is that it can be used as a software tool. To use the software the user does not need the knowledge of Visual Basic. It can be beneficial for organisation. Here below statements shows how to use the software:

First user needs to select number of points.



By selecting the number of points, a table will automatically appear where user can insert the value of X. After inserting the values in X, the user needs to put the value in Interpolation point field. And then user have to click the **Generate** button to see

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the values in the difference table and result in the result box.

NEWTON'S FORWARD INTERPOLATION FORMULA

Select Number of Points:

x	1971	1981	1991	2001	2011	
$f(x)$	776968	784872	792816	800800	808824	Interpolation Point: <input type="text" value="1975"/>

X	f(X)	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
1971	776968				
1981	784872	79040			
1991	792816	79440	400	0	
2001	800800	79840	400	0	0
2011	808824	80240	400		

Result:

MAIN RESULT : DEVELOPMENT OF NEW TECHNIQUES FOR NUMERIC CALCULATION BY USING MS WORKSHEET

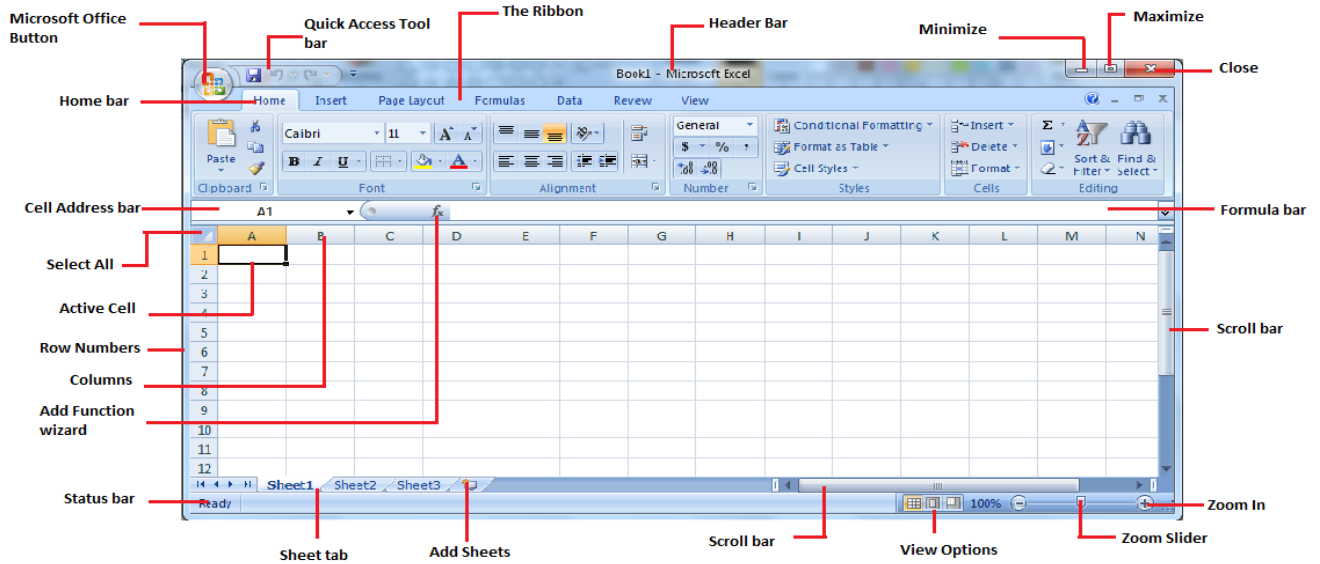
We see that to handle all these software we have to know the respective high level languages for each software. Therefore it is not easy accessible for all. But in Microsoft Office we find a very interesting and easy accessible file known as MS **excel worksheet** in which we can develop the formulae for numeric computations. And for this we need not require any high level

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language. Here following figure shows S worksheet

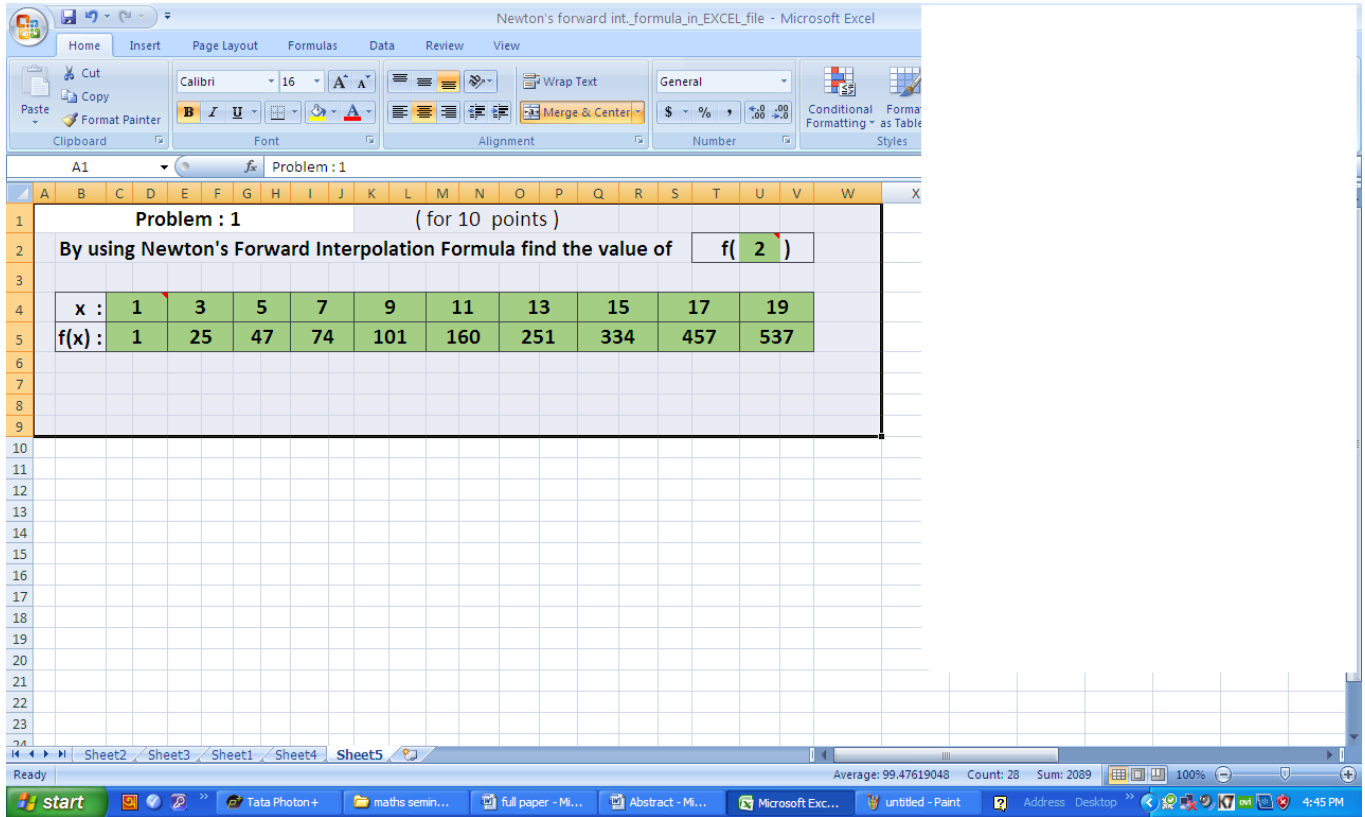


First construct the table in MS excel Worksheet by the given data as follows

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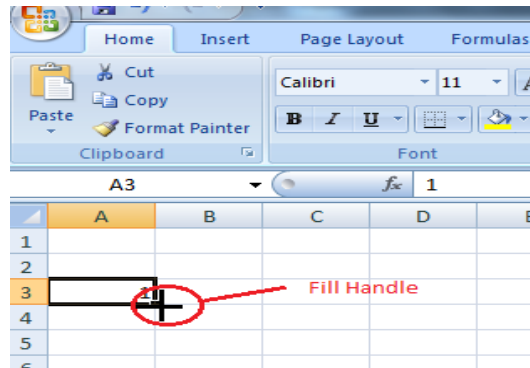
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To construct the difference table by cell linking type **=C4** in the cell A8 and type **=C5** in the cell B8. Type **=E4** and **=E5** in the cells A10 and B10 and so on . Then you will see that the values in the table carried over to the difference table .

For first difference type **=B10-B8** in the cell C9 . Bring your mouse over bottom-right corner of the cell C9 till the pointer turns to a cross shape.

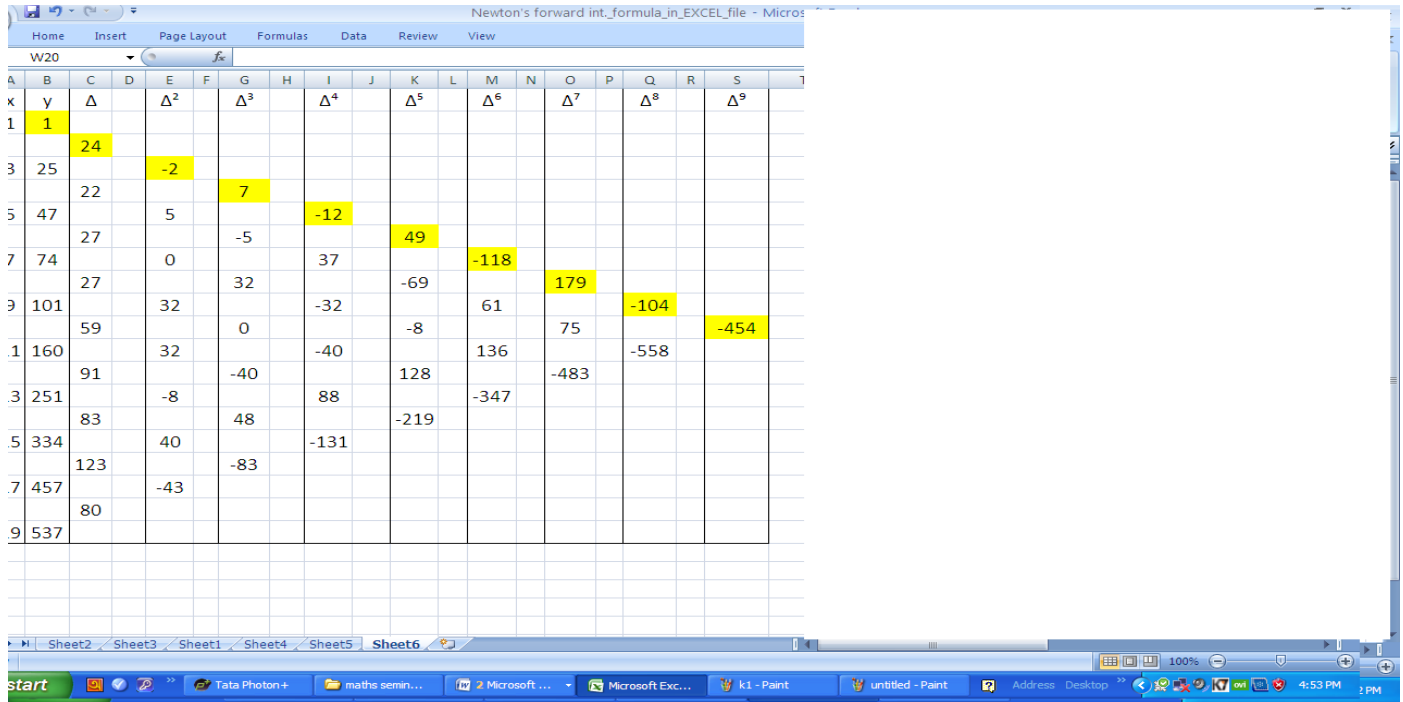


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Drag the fill handle across the cells to C 26. Delete the unnecessary cells. For the second difference type **=C11-C9** in the cell E10 and drag the fill handle .Repeat this process all differences you require . Then you will see that your difference table is constructed .If you change the values in the given table then the difference table will be automatically changed .



For the value of x type **=O2** in the cell B27 .Any value of x for which you require the interpolation put in the cell O2

For the value of u type **=(B27-A8)/(A10-A8)** in the cell C28

For step by step calculations type the formulae and sign in appropriate cells as shown below.

cell	B33	C33	D33	E33	F33	G33	H33	I33	J33	K33	L33	M33	N33	O33	P33
Type as	=B8	+	=C28	X	=C9	+	=C28	X	=C28	-	1	X	=E10	/	2!

B35	C35	D35	E35	F35	G35	H35	I35	J35	K35	L35	M35	N35	O35	P35
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=G11	/	3!

B37	C37	D37	E37	F37	G37	H37	I37	J37	K37	L37	M37	N37	O37	P37	Q37	R37	S37	T37	U37
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=I12	/	4!	

B39	C39	D39	E39	F39	G39	H39	I39	J39	K39	L39	M39	N39	O39	P39	Q39	R39
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=C28
			S39	T39	U39	V39	W39	X39	Y39	Z39	AA39	AB39				

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-	4	X	=K13	/	5!												
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B41	C41	D41	E41	F41	G41	H41	I41	J41	K41	L41	M41	N41	O41	P41	Q41	R41
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=C28
			S41	T41	U41	V41	W41	X41	Y41	Z41	AA41	AB41				
			-	4	X	=C28	-	5	X	=M14	/	6!				

B43	C43	D43	E43	F43	G43	H43	I43	J43	K43	L43	M43	N43	O43	P43	Q43	R43
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=C28
			S43	T43	U43	V43	W43	X43	Y43	Z43	AA43	AB43	AC43	AD43	AE43	AF43
			-	4	X	=C28	-	5	X	=C28	-	6	X	=O15	/	7!

B45	C45	D45	E45	F45	G45	H45	I45	J45	K45	L45	M45	N45	O45	P45	Q45	R45
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=C28
			S45	T45	U45	V45	W45	X45	Y45	Z45	AA45	AB45	AC45	AD45	AE45	AF45
			-	4	X	=C28	-	5	X	=C28	-	6	X	=C28	-	7
								AG45	AH45	AI45	AJ45	AK45	AL45	AM45	AN45	AO45
								X	=Q16	/	8!					

B47	C47	D47	E47	F47	G47	H47	I47	J47	K47	L47	M47	N47	O47	P47	Q47	R47
	+	=C28	X	=C28	-	1	X	=C28	-	2	X	=C28	-	3	X	=C28
			S47	T47	U47	V47	W47	X47	Y47	Z47	AA47	AB47	AC47	AD47	AE47	AF47
			-	4	X	=C28	-	5	X	=C28	-	6	X	=C28	-	7
								AG47	AH47	AI47	AJ47	AK47	AL47	AM47	AN47	AO47
								X	=C28	-	8	X	=S17	/	9!	

For the main calculations of Newton's forward interpolation formula type appropriate formulae in the respective cells as shown below .

A49	B49	C49	D49	E49	F49	G49	H49
=	=B33	+	=D33*F33	+	=H33*(J33-L33)*N33/2	+	=D35*(F35-H35)*(J35-L35)*N35/6
	I49	J39				K49	
	+	=D37*(F37-H37)*(J37-L37)*(N37-P37)*R37/24				+	
	L49					M49	

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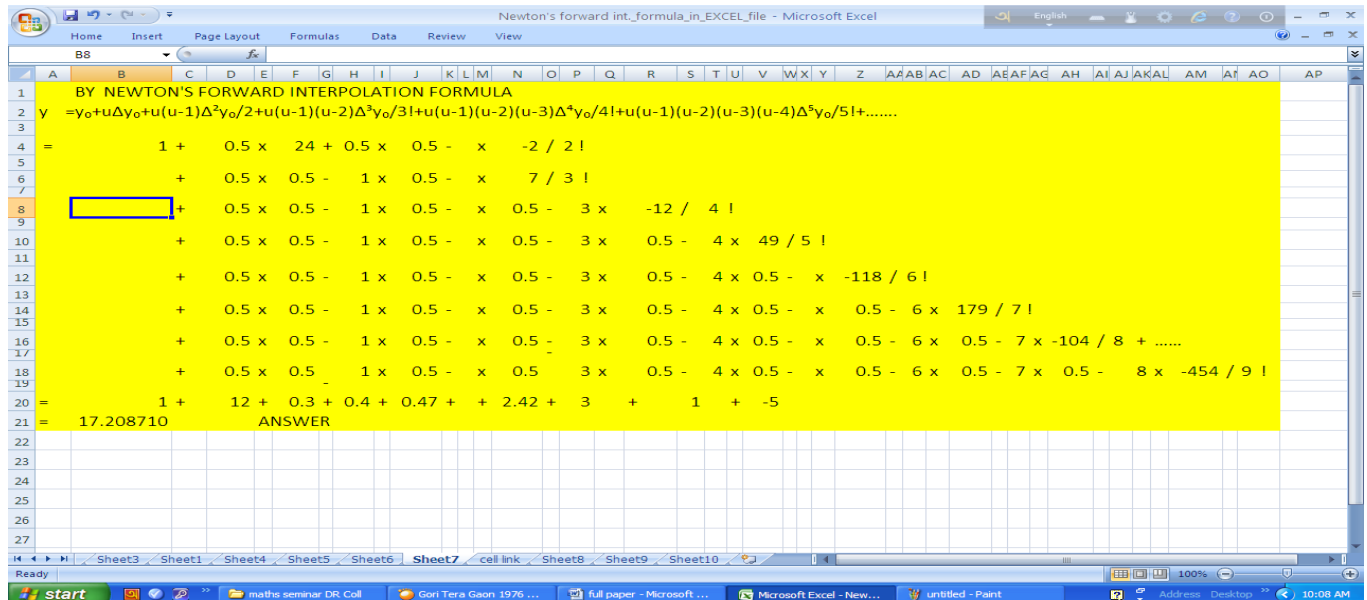
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=D39*(F39-H39)*(J39-L39)*(N39-P39)*(R39-T39)*V39/120		+	
N49			O49
=D41*(F41-H41)*(J41-L41)*(N41-P41)*(R41-T41)*(V41-X41)*Z41/720		+	
PQ49			R49
=D43*(F43-H43)*(J43-L43)*(N43-P43)*(R43-T43)*(V43-X43)*(Z43-AB43)*AD43/5040		+	
ST49			U49
=D45*(F45-H45)*(J45-L45)*(N45-P45)*(R45-T45)*(V45-X45)*(Z45-AB45)*(AD45-AF45)*AH45/40320		+	
V49			
=D47*(F47-H47)*(J47-L47)*(N47-P47)*(R47-T47)*(V47-X47)*(Z47-AB47)*(AD47-AF47)*(AH47-AK47)*AM47/362880			

For final result type as follows in the appropriate cells

A50	BCD50
=	=B49+D49+F49+H49+J49+L49+N49+P49+S49+V49

Thus you will get a self-built application for solving any problem having all total 10 points by using Newton's forward Interpolation Formula . For problems of fewer than 10 points you have only to delete the values appeared in the unnecessary cells in the difference table .



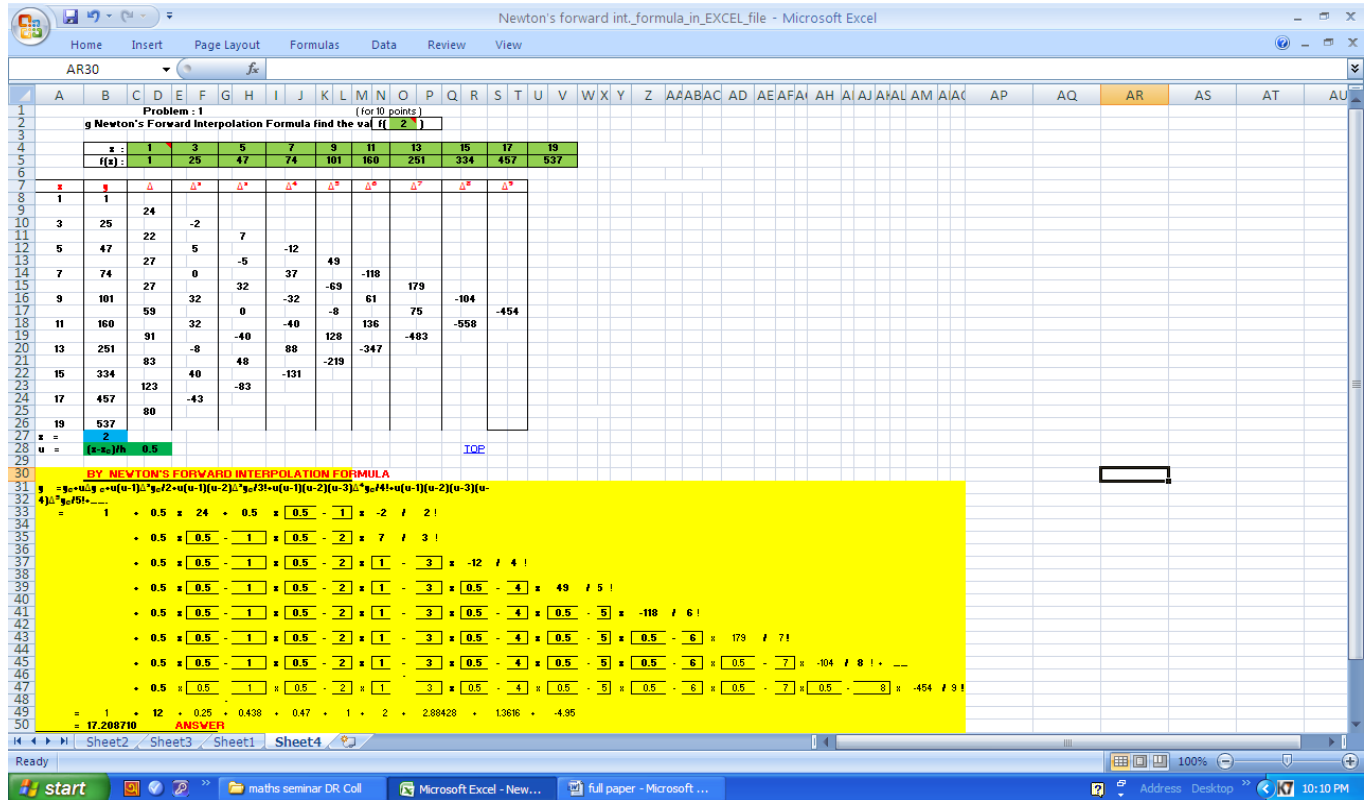
We think everyone having minimum computer knowledge can use the MS Excel for calculations and for storing data . We find that this worksheet is very much interesting and usefull for numeric calculations involving in **NUMERICAL ANALYSIS** . Specially in interpolation , solution of algebraic and transcendental equations , numerical differentiation and integrations . we can develop the process as our choice . The construction of difference table is quite easy and interesting in excel worksheets . Once we construct a difference table for higher points it can be use for every cases of fewer points also by deleting the unnecessary points . In the same way once we put a formula for higher points it can be use for all fewer points .Complete

Worksheet Page :-

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The screenshot shows an Excel spreadsheet titled "Newton's forward int_formula_in_EXCEL_file". It contains a table of data points and a detailed calculation of the Newton's Forward Interpolation formula.

x	1	3	5	7	9	11	13	15	17	19
f(x)	1	25	47	74	101	160	251	334	457	537

The calculation area shows the following steps:

$$f(x) = 1 + 0.5x + 24 \times \frac{0.5x(0.5x-1)}{2!} + 7 \times \frac{0.5x(0.5x-1)(0.5x-2)}{3!} + 49 \times \frac{0.5x(0.5x-1)(0.5x-2)(0.5x-3)}{4!} + 179 \times \frac{0.5x(0.5x-1)(0.5x-2)(0.5x-3)(0.5x-4)}{5!} + 104 \times \frac{0.5x(0.5x-1)(0.5x-2)(0.5x-3)(0.5x-4)(0.5x-5)}{6!} + 454 \times \frac{0.5x(0.5x-1)(0.5x-2)(0.5x-3)(0.5x-4)(0.5x-5)(0.5x-6)}{7!}$$

The final result is $f(2) = 17.208710$.

Similarly we can show that the solution of algebraic and transcendental equation , numerical differentiation and integration also be done in MS Excel worksheets by developing appropriate formulae.

CONCLUSION

From the above discussion it is clear that MS Excel worksheet is a quite interesting and useful for computational part of interpolation formulae, solution of algebraic and transcendental equation, numerical differentiation and integration of numerical analysis . It is much more easier than all other in-built software available in the computers . It is utmost studentfriendly and easy accessible by all users . Therefore we think a chapter “ **NUMERICAL ANALYSIS IN MS EXCEL**” shall be introduce in the degree Curriculum .

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