ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 12, December 2013

MATHEMATICAL MODEL OF ACCELERATED LIFE TESTING USING GEOMETRIC PROCESS FOR MARSHALL-OLKIN EXTENDED EXPONENTIAL DISTRIBUTION

Sadia Anwar[1], Mustafa Kamal[2], Arif-Ul-Islam[3]

Research Scholar, Dept of Statistics & Operations Research, Aligarh Muslim University, Aligarh, U.P, India [1]

Research Scholar, Dept of Statistics & Operations Research, Aligarh Muslim University, Aligarh, U.P, India [2]

Professor, Dept of Statistics & Operations Research, Aligarh Muslim University, Aligarh, U.P, India [3]

Abstract: In accelerated life testing researcher generally use a life stress relationship between life characteristic and stress to estimate the parameters of failure time distributions at use condition which is just a re-parameterization of original parameters but from statistical point of view it is easy and reasonable to deal with original parameters of the distribution directly instead of developing inference for the parameters of the life stress relationship. By assuming that the lifetimes at increasing stress levels forms a geometric process one can easily handle the original parameters of life distribution directly in accelerated life testing. In this paper a mathematical model for the analysis of constant stress accelerated life testing by using geometric process for Marshall-Olkin Extended Exponential distribution is developed. The estimates of parameters are obtained by using the maximum likelihood method for complete data. In order to get the asymptotic variance of the ML estimator, the Fisher information matrix is constructed. The asymptotic interval estimates of the parameters are then obtained by using this asymptotic variance. In the last a simulation study is performed to illustrate the statistical properties of the parameters and the confidence intervals.

Keywords: Maximum Likelihood Estimation; Reliability Function; Fisher information Matrix; Confidence Intervals; Simulation Study.

I. INTRODUCTION

Nowadays many manufactured products have a long life when using them at normal operating conditions due to their good manufacturing designs. As in life testing experiments, the time-to-failure data obtained under normal operating conditions is used to quantify the product's failure-time distribution and its associated parameters; therefore, it is very costly and time consuming or may indeed be impossible to perform a life test at normal operating conditions. A solution to such life testing problems is Accelerated Life Testing (ALT) in which the products or materials are tested at higher levels of stress to obtain information quickly on the life distribution or product performance. By this process failures which under normal conditions would occur only after a long testing can be observed quicker and the size of data can be increased without a large cost and long time.

Generally three types of stress loadings are applied in accelerated life tests: constant stress, step stress and linearly increasing stress. In constant stress loading products are operated at fixed stress levels throughout the test. In step stress, a specimen is subjected to successively higher levels of stress. At first, it is subjected to a specified constant stress for a specified length of time. If it does not fail, it is subjected to a higher stress level for a specified time. The stress on a unit is increased step by step until it fails. In this type of stress, a specimen undergoes a continuously increasing level of stress. Among all stresses, the constant stress method has been used widely and it is considered more important than other stress testing methods because most products are assumed to operate at a constant stress under normal use and hence mimics the actual use of the products. There are two types of data are obtained from ALT, first is the complete in which failure time of each sample unit is observed or known and second is the censored data in which failure time of each sample unit may not be available or observed.

Constant stress ALT has been studied by many authors, for example, Ahmad et al. [1], Islam and Ahmad [2], Ahmad and Islam [3], Ahmad et al.[4] and Ahmad [5] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring. Yang [6] proposed an optimal design of 4-level constant stress ALT plans considering different censoring times. Pan et al. [7] proposed a Bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by the logistic function. Walkins and johns [8] considered constant stress accelerated life test based on Weibull distribution with constant shape and a log linear link between scale the stress factor which is terminated by a Type-II censoring regime at one of the stress level.

The geometric process (GP) is first studied by Lam [9] in the problems of repair replacement. Lam [10] studied the geometric process model for a multistate system and concluded a replacement policy to minimize the long run average cost per unit time. Huang [11] introduced the GP model for the analysis of ALT with complete and censored exponential sample under constant stress. Kamal et al. [12] used the GP model for the analysis of ALT with complete Weibull failure data under constant stress. Kamal [13, 14] extended GP model in ALT for type-I and type-II censored Weibull failure data. Kamal et al. [15] implemented the GP model to estimate the parameters of Pareto distribution in ALT. Zarrin et al. [16] investigated the statistical properties of Inverted Weibull Distribution in ALT using GP. Saxena et al. [17] proposed ALT model for Log-Logistic distribution using GP in case of Censored Data.

In this paper a mathematical model for the analysis of constant stress ALT using GP for MOEE distribution is developed. ML method of estimation is implemented to obtain the estimates of model parameters. In addition the asymptotic confidence intervals are also generated by using fisher information matrix. The common statistical properties of parameters are inspected by a simulation study. A detailed discussion over the present model and some conclusions arise here are given in the end.

II. MODEL DESCRIPTION

A. The Geometric Process

A GP is stochastically process $\{X_n, n=1,2,...\}$ such that $\{\lambda^{n-1}X_n, n=1,2,...\}$ forms a renewal process where $\lambda>0$ is real valued and called the ratio of the GP. It is easy to show that if $\{X_n, n=1,2,...\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1}f(\lambda^{n-1}x)$ with mean μ/λ^{n-1} and variance $\sigma^2/\lambda^{2(n-1)}$.

It is clear to see that a GP is stochastically increasing if $0<\lambda<1$ and stochastically decreasing if $\lambda>1$. Therefore, GP is a natural approach to analyse the data from a series of events with trend. For more details about GP and its properties see Braun et al. [18].

B. The Marshall-Olkin Extended Exponential distribution (MOEE)

Marshall and Olkin [19] proposed a new method for adding a parameter to a family of distributions. In particular they consider a two-parameter generalization of the one-parameter exponential distribution which plays a central role in reliability and life time data analysis. The one-parameter exponential family of distributions is not broad enough to model data from various real contexts.

Suppose we have a given distribution with survival function (SF) $\overline{F}(x), -\infty<x<\infty$ then the Marshall-Olkin extended distribution is defined by the SF

$$\overline{G}(x)=\frac{\alpha\overline{F}(x)}{1-\overline{\alpha}\overline{F}(x)} \qquad -\infty<x<\infty, \alpha>0, \overline{\alpha}=1-\alpha \tag{1}$$

If the survival function of exponential distribution is $\overline{F}(x)=e^{-\theta x}$, $x,\theta>0$, and put it in equation (1), we obtain the SF

$$\overline{G}(x) = \frac{1-\overline{\alpha}}{e^{\theta x} - \overline{\alpha}}, \qquad -\infty < x < \infty, \alpha > 0, \overline{\alpha} = 1-\alpha \tag{2}$$

The distribution with the survival function (2) is called the MOEE with parameters α and θ. The probability density function (pdf) and the cumulative distribution function (cdf) and the hazard rate of the Marshall-Olkin extended exponential with the survival function (2), respectively are given by

$$g(x;\alpha,\theta) = \frac{\alpha\theta e^{\theta x}}{[e^{\theta x} - \overline{\alpha}]^2}, \qquad -\infty < x < \infty, \alpha > 0, \overline{\alpha} = 1-\alpha \tag{3}$$

$$G(x;\alpha,\theta) = \frac{e^{\theta x} - 1}{[e^{\theta x} - \overline{\alpha}]} \tag{4}$$

$$\mathrm{r}(x) = \frac{\theta e^{\theta x}}{[e^{\theta x} - \overline{\alpha}]} \tag{5}$$

For $\alpha = 1,$ the SF, cdf, pdf and hr all are reduce to those of the exponential distribution.

Marshall-Olkin extended distribution includes the original distribution as a special case and gives more flexibility to model various types of data. Marshall- Olkin extended distributions offer a wide range of the behaviour than the basic distributions from which they are derived. Alice et al. [20] discussed the Marshall-Olkin Pareto distribution and its reliability application. Ghitany et al. [21] used Marshall- Olkin Extended Weibull distribution and its application to the censored data. Gupta et al. [22] estimated the reliability from Marshall-Olkin Extended Lomax distribution. In this paper MOEE distribution is considered to model the accelerated life testing.

C. Assumptions and test procedure

i. Suppose an accelerated life test with s increasing stress levels in which a random sample of n identical items is placed under each stress level and start to operate at the same time. Let $x_{k\,i}, i = 1,2,...n, k = 0,1,2,..s$ denote the observed failure time of i^{th} test item under k^{th} stress level. Whenever an item fails, it will be removed from the test and the test is continue till all the test items failed (complete data) and the exact failure time x_{ki} of item is observed.

ii. The product life follows a MOEE distribution given by (3) at any stress

iii. Let the sequence of random variables $X_0, X_1, X_2, ..., X_n$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily. We assume $\{X_k, k = 1,2,3,.....s\}$ is a geometric process with ratio $\lambda > 0$.

iv. At any constant stress level S_k, the mean life $1/\theta_k$ is a log linear function of stress, that is, $\log(1/\theta_k) = a + bS_k$, where a and b are unknown parameters that depend on the nature of the product and the test method. When $k = 0$, the above equation depicts the relation of the mean life and the designed stress level.

D. G.P assumption in ALT.

From assumption (iv), it can easily be shown that

$$\log\left(\frac{\theta_k}{\theta_{k+1}}\right) = b\left(S_{k+1} - S_k\right) = b\Delta S \tag{6}$$

Above relation can be written as

$$\left(\frac{\theta_{k+1}}{\theta_k}\right) = e^{-b\Delta S} \qquad (7)$$

It is clear from (7) that the mean life under each increasing stress levels forms a geometric sequence with the ratio $e^{-b\Delta S}$ if the increased stress levels form an arithmetic sequence with a constant difference ΔS .

In case of MOEE distribution, the pdf of a test item at the k^{th} stress level is

$$g_{X_k}(x) = \frac{\alpha\theta_k e^{\theta_k x}}{[e^{\theta_k x} - \bar{\alpha}]^2} = \frac{\alpha\lambda\theta_{k-1} e^{\lambda\theta_{k-1} x}}{[e^{\lambda\theta_{k-1} x} - \bar{\alpha}]^2} = \frac{\alpha\lambda^k\theta e^{\theta\lambda^k x}}{[e^{\theta\lambda^k x} - \bar{\alpha}]^2}$$

This implies that

$$g_{X_k}(x) = \lambda^k g_{x_0}(\lambda^k x)$$

where $g_{x_0}(x)$ is the lifetime distribution at designed stress level with θ being the failure rate ; $\lambda = e^{-b\Delta S}$.

Therefore the pdf of a test item at the k^{th} stress level is

$$g_{X_k}(x) = \lambda^k g(\lambda^k x) = \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x}}{(e^{\theta\lambda^k x} - \bar{\alpha})^2}$$

III. MAXIMUM LIKELIHOOD ESTIMATION

The maximum likelihood estimation (MLE) is the most important and widely used method in statistics. The idea behind the maximum likelihood parameter estimation is to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples.

The likelihood function using geometric process for MOEE distribution under constant stress accelerated life testing for complete data is given by

$$L = \prod_{k=1}^{s}\prod_{i=1}^{n} \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x_{ki}}}{\left[e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right]^2} \qquad (8)$$

The log-likelihood function corresponding (8) can be rewritten as

$$l = \sum_{k=1}^{s}\sum_{i=1}^{n}\left[k\ln\lambda + \ln\alpha + \ln\theta + \theta\lambda^k x_{ki} - 2\ln\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)\right] \qquad (9)$$

MLE's of α , θ and λ are obtained by solving the equations $\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \theta} = 0$ and $\frac{\partial l}{\partial \lambda} = 0$, where

$$\frac{\partial l}{\partial \alpha} = \frac{ns}{\alpha} - 2\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{1}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)} \qquad (10)$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 12, December 2013

$$\frac{\partial l}{\partial \theta}=\frac{ns}{\theta}+\sum_{k=1}^{s}\sum_{i=1}^{n}\lambda^{k}x_{ki}-2\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{\lambda^{k}x_{ki}e^{\theta\lambda^{k}x_{ki}}}{\left(e^{\theta\lambda^{k}x_{ki}}-1+\alpha\right)} \tag{11}$$

$$\frac{\partial l}{\partial \lambda}=\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{k}{\lambda}+\theta\sum_{k=1}^{s}\sum_{i=1}^{n}k\lambda^{k-1}x_{ki}-2\theta\sum_{k=1}^{s}\sum_{i=1}^{s}\frac{k\lambda^{k-1}x_{ki}e^{\theta\lambda^{k}x_{ki}}}{\left(e^{\theta\lambda^{k}x_{ki}}-1+\alpha\right)} \tag{12}$$

It is difficult obtain a closed form solution to nonlinear equations to (10), (11) and (12). Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha},\hat{\theta}$ and $\hat{\lambda}$.

IV. FISHER INFORMATION MATRIX

The Fisher information matrix F is obtained by taking the negative second partial derivatives of the log-likelihood function and can be written as

$$F=\begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \theta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \theta \partial \alpha} & -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \theta} & -\frac{\partial l}{\partial \lambda^2} \end{bmatrix}$$

Elements of Fisher Information matrix are

$$\frac{\partial^2 l}{\partial \alpha^2}=-\frac{ns}{\alpha^2}+2\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{1}{\left(e^{\theta\lambda^{k}x_{ki}}-1+\alpha\right)^2}$$

$$\frac{\partial^2 l}{\partial \theta^2}=-\frac{ns}{\theta^2}-2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{(\alpha-1)\lambda^{2k}x_{ki}^2e^{\theta\lambda^{k}x_{ki}}}{\left(e^{\theta\lambda^{k}x_{ki}}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial \lambda^2}=-\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{k}{\lambda^2}+\theta\sum_{k=1}^{s}\sum_{i-1}^{n}k(k-1)\lambda^{k-2}x_{ki}$$
$$-2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\theta k(k-1)\lambda^{k-2}x_{ki}e^{2\theta\lambda^{k}x_{ki}}+(\alpha-1)\left\{\theta k(k-1)\lambda^{k-2}x_{ki}e^{\theta\lambda^{k}x_{ki}}+\theta^2k^2\lambda^{2k-2}x_{ki}^2e^{\theta\lambda^{k}x_{ki}}\right\}}{\left(e^{\theta\lambda^{k}x_{ki}}-1+\alpha\right)^2}\right]$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 12, December 2013

$$\frac{\partial^2 l}{\partial\alpha\partial\theta}=\frac{\partial^2 l}{\partial\theta\partial\alpha}=2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\lambda^k x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\alpha}=2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\theta k\lambda^{k-1} x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\theta\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\theta}=\sum_{k=1}^{s}\sum_{i=1}^{n}k\lambda^{k-1}x_{ki}-2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{k\lambda^{k-1}x_{ki}e^{2\theta\lambda^k x_{ki}}+(\alpha-1)\{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}+\theta k\lambda^{2k-1}x_{ki}^2e^{\theta\lambda^k x_{ki}}\}}{\left(e^{\theta\lambda^k x_{ki}}-1+\alpha\right)^2}\right]$$

V. ASYMPTOTIC CONFIDENCE INTERVAL ESTIMATES

The variance covariance and covariance matrix of the parameter can be written as

$$\sum=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda}\\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial l}{\partial\lambda^2}\end{bmatrix}^{-1}=\begin{bmatrix}AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\theta}) & ACov(\hat{\alpha}\hat{\lambda})\\ ACov(\hat{\theta}\hat{\alpha}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{\lambda})\\ ACov(\hat{\lambda}\hat{\alpha}) & ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\lambda})\end{bmatrix}$$

The $100(1-\beta)\%$ asymptotic confidence interval for α,θ and λ are then given respectively as

$$\left[\hat{\alpha}\pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\alpha})}\right],\ \left[\hat{\theta}\pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\theta})}\right]\text{ and }\left[\hat{\lambda}\pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\lambda})}\right]$$

VI. SIMULATION STUDY

In order to obtain MLEs of λ,α,θ and to study the properties of these estimates through Mean squared errors (MSEs), relative absolute biases (RABs) and confidence limits for 95% and 99% asymptotic confidence interval, a simulation study is performed.

For this purpose, first different random samples with sizes $n=100,200,\ldots,500$ are generated from MOEE distribution. The combinations (λ,α,θ) of values of the parameters are chosen to be $(1.0,1.5,2.0)$ and $(1.25,0.5,2.5)$. The number of stress levels s is assumed to be 4 and 6 throughout the study. For different sample sizes and stress level, MLEs,

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 12, December 2013

MSEs, RABs and the lower and upper CI limits of 95% and 99% confidence interval of parameters based on 500 simulations are obtained by our proposed model and summarized in Table 1, 2, 3, and 4.

Table 1: Simulation study results based on CSALT with complete data for MOEE distribution using GP with $\lambda = 1, \alpha = 1.5, \theta = 2.0$ **and** $s = 4$

n	$\hat{\lambda}$ $\hat{\alpha}$ $\hat{\theta}$	MSE	RAB	95% CL		99 % CL	
				LCL	UCL	LCL	UCL
100	1.0189 1.5182 1.9856	0.0073 0.0443 0.0048	0.0189 0.0121 0.0072	0.8549 1.1070 1.8230	1.8128 1.9293 2.0889	0.8030 0.9770 1.7810	1.2347 2.0593 2.1309
200	1.0150 1.4937 1.9162	0.0292 0.0034 0.0290	0.0150 0.0042 0.0419	0.6812 1.3794 1.6254	1.3487 1.6079 2.2069	0.5756 1.3432 1.5335	1.4543 1.6441 2.2988
300	1.0112 1.4734 1.9102	0.0028 0.0177 0.0102	0.0112 0.0177 0.0449	0.9093 1.2178 1.8182	1.1130 1.7289 2.0021	0.8771 0.1370 1.7891	1.1452 1.8097 2.0312
400	1.0098 1.4853 1.9038	0.0140 0.0342 0.0135	0.0098 0.0098 0.0481	0.7778 1.1238 1.7752	1.2417 1.8467 2.0323	0.7045 1.0095 1.7346	1.3150 1.9610 2.0729
500	1.0004 1.5157 1.9015	0.0223 0.0031 0.0257	0.0004 0.0104 0.0492	0.7077 1.4101 1.6530	1.2930 1.6212 2.1489	1.6151 1.3767 1.5746	1.3856 1.6546 2.2273

Table 2: Simulation study results based on CSALT with complete data for MOEE distribution using GP with $\lambda = 1, \alpha = 1.5, \theta = 2.0$ **and** $s = 6$

n	$\hat{\lambda}$ $\hat{\alpha}$ $\hat{\theta}$	MSE	RAB	95% CL		99 % CL	
				LCL	UCL	LCL	UCL
100	1.0532 1.4756 2.0025	0.0049 0.0137 0.0017	0.0532 0.0162 0.0012	0.9633 1.2504 1.9216	1.1430 1.7007 2.0833	0.9349 1.1791 1.8961	1.1714 1.7720 2.1088
200	1.0249 1.4832 1.9940	0.0116 0.0016 0.0303	0.0249 0.0112 0.0030	0.8193 1.4098 1.6528	1.2304 1.5565 2.3351	0.7543 1.3866 1.5449	1.2954 1.5797 2.4430
300	1.0237 1.4889 1.9993	0.0067 0.0144 0.1055	0.0237 0.0074 0.0003	0.8693 1.2545 1.3626	1.1780 1.7232 2.6359	0.8205 1.1803 1.1612	1.2268 1.7974 2.8373
400	1.0207 1.4962 1.9856	0.0127 0.0152 0.0210	0.0207 0.0025 0.0072	0.8033 1.2545 1.7029	1.2380 1.7378 2.2682	0.7345 1.1781 1.6135	1.3068 1.8142 2.3576
500	1.0134 1.4987 1.9984	0.0028 0.0148 0.0369	0.0134 0.0008 0.0008	0.9115 1.2602 1.6218	1.1152 1.7371 2.3749	0.8793 1.1848 1.5027	1.1474 1.8125 2.4940

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 12, December 2013

Table 3: Simulation study results based on CSALT with complete data for MOEE distribution using GP with $\lambda = 1.25, \alpha = 0.5, \theta = 2.5$ and $s = 4$

n	$\hat{\lambda}$ $\hat{\alpha}$ $\hat{\theta}$	MSE	RAB	95% CL		99 % CL	
				LCL	UCL	LCL	UCL
100	1.3250 0.4823 2.5986	0.0223 0.0151 0.0125	0.0600 0.0354 0.0394	1.0717 0.2438 2.4948	1.5782 0.7207 2.7023	0.9915 0.1684 2.4620	1.6584 0.7961 2.7351
200	1.3062 0.5182 2.5781	0.0190 0.0102 0.0075	0.0449 0.0364 0.0312	1.0590 0.3231 2.5021	1.5533 0.7132 2.6540	0.9808 0.2614 2.4781	1.6315 0.7749 2.6780
300	1.2967 0.4870 2.5544	0.0169 0.0038 0.0225	0.0373 0.0260 0.2176	1.0582 0.3677 2.2800	1.5351 0.6062 2.8288	0.9828 0.3300 2.1932	1.6105 0.6439 2.9156
400	1.2890 0.5088 2.5321	0.0155 0.0088 0.0038	0.0312 0.0176 0.0128	1.0570 0.3249 2.4283	1.5209 0.6926 2.6358	0.9837 0.2667 2.3955	1.5942 0.7508 2.6686
500	1.2767 0.5038 2.5108	0.0133 0.0100 0.0015	0.0213 0.0076 0.0043	1.0566 0.3078 2.4374	1.4967 0.6998 2.5841	0.9870 0.2458 2.4142	1.5663 0.7618 2.6073

Table 4: Simulation study results based on CSALT with complete data for MOEE distribution using GP with $\lambda = 1.25, \alpha = 0.5, \theta = 2.5$ and $s = 6$

n	$\hat{\lambda}$ $\hat{\alpha}$ $\hat{\theta}$	MSE	RAB	95% CL		99 % CL	
				LCL	UCL	LCL	UCL
100	1.2703 0.4928 2.5028	0.0249 0.0062 0.0303	0.0162 0.0144 0.0011	0.9635 0.3384 2.1616	1.5770 0.6471 2.8439	0.8664 0.2896 2.0537	1.6741 0.6959 2.9518
200	1.2654 0.5096 2.5432	0.0236 0.1056 0.0062	0.0123 0.0192 0.0172	0.9655 -0.1270 2.4131	1.5652 1.1462 2.6732	0.8707 -0.3284 2.3720	1.6600 1.3476 2.7143
300	1.2563 0.4915 2.5309	0.0223 0.0152 0.0031	0.0050 0.0170 0.0123	0.9636 0.2498 2.4389	1.5489 0.7331 2.6228	0.8710 0.1734 2.4098	1.6415 0.8095 2.6519
400	1.2543 0.4936 2.5221	0.0184 0.0131 0.0212	0.0034 0.0128 0.0088	0.9884 0.2692 2.2394	1.5201 0.7179 2.8047	0.9043 0.1983 2.1500	1.6042 0.7888 2.8941
500	1.2508 0.4976 1.5019	0.0231 0.0148 0.0028	0.0006 0.0048 0.0007	0.9975 0.2591 2.3981	1.5040 0.7360 2.6056	0.9173 0.1837 2.3653	1.5842 0.8114 2.6384

VII. DISCUSSION AND CONCLUSION

This paper deals with use of GP model in the analysis of CSALT plan for MOEE distribution with complete data. The MLEs, MSEs and RABs of the model parameters were obtained. Based on the asymptotic normality, the lower and upper CI limits of 95% and 99% confidence interval of the model parameters were also obtained.

From the results in Table 1, 2, 3 and 4, it is easy to find that estimates of λ, α and θ perform well. For fixed λ, α and θ we find that as sample size n increases, MSEs, RABs and the confidence intervals get narrower. This is very usual because big samples increase the efficiency of the estimators. From these results, it may be concluded that the present model work well for complete data.

.ACKNOWLEDGMENT

This work has been funded by University Grant Commission under Maulana Azad National Fellowship.

REFERENCES

[1] Ahmad, N., Islam, A., Kumar, R., and Tuteja, R.K., "Optimal design of accelerated life test plan under periodic inspection and Type I censoring: The case of Rayleigh failure law", South African Statistical Journal, Vol. 28, pp. 27-35, 1994.

[2] Islam, A., and Ahmad, N., "Optimal design of accelerated life test plan for Weibull distribution under periodic inspection and Type I censoring", Microelectronics Reliability, Vol. 34, No.9, pp. 1459-1468, 1994.

[3] Ahmad, N., and Islam, A., "Optimal design of accelerated life test plan for Burr Type XII distribution under periodic inspection and Type I censoring", Naval Research Logistics, Vol.43, pp.1049-1077, 1996.

[4] Ahmad, N., Islam, A., and Salam, A., "Analysis of optimal accelerated life test plan for periodic inspection: The case of exponential Weibull failure model", International Journal of Quality and Reliability Management, Vol.23, No.8, pp. 1019-1046, 2006.

[5] Ahmad, N., "Designing Accelerated Life Test for Generalized Exponential Distribution with Log linear Model", International Journal of Reliability and Safety, Vol.4, No. 2/3, pp. 238-264, 2010.

[6] Yang, G. B., "Optimum constant stress accelerated life-test plan, IEEE Transactions on Reliability". Vol. 43, No. 4, pp.575-581, 1994.

[7] Pan, Z., Balakrishnan, N., and Quan.Sun., "Bivariate constant-stress accelerated degradation model and inference", Communication in Statistics-Simulation and Computation, Vol.40, No. 2, 247-257, 2011.

[8] Watkins, A.J., and John, A.M., "On constant-stress accelerated life tests terminated by Type II censoring at one of the stress levels", Journal of Statistical Planning and Inference, Vol. 138, No. 3, pp.78-786, 2008.

[9] Lam, Y., "Geometric Process and Replacement Problem", Acta Mathematicae Applicatae Sinica, Vol.4, No. 4, pp.366-377, 1998.

[10] Lam, Y., "A monotone process maintenance model for a multistate system with delayed system", Journal of Applied Probability, Vol. 42, No.1, pp.1-4, 2005.

[11] Huang, S., "Statistical Inference in accelerated life testing with geometric process model", Master's thesis, San Diego State University, (2011).

[12] Kamal, M., Zarrin, S., S., Saxena, S. and Islam, A., "Weibull Geometric Process Model for the Analysis of Accelerated Life Testing with Complete Data", International Journal of Statistics and Application, Vol. 2, No.5, pp, 60-66, 2012.

[13] Kamal, M., "Application of Geometric Process in Accelerated life testing analysis with type-I Weibull censored data", Reliability: Theory & Applications, Vol.8, No.3, pp.87-96, 2013.

[14] Kamal, M., "Estimation of Weibull Parameters In Accelerated Life Testing Using Geometric Process With Type-II Censored Data", International Journal of Engineering Sciences & Research Technology, Vol. 2, No. 9, pp. 2340-2347,2013.

[15] Kamal, M., Zarrin, S. and Islam, A., "Accelerated Life Testing Design using Geometric Process for Pareto Distribution", International Journal of Advanced Statistics and Probability, Vol. 1, No. 2, pp, 25-31, 2013.

[16] Zarrin, S., Kamal, M., and Islam, A., "Constant Stress Accelerated Life Testing Analysis using Geometric Process for Inverted Weibull Distribution", Journal of Applied Statistical Research, Vol. 1, No. 2, pp. 17-28, 2013.

[17] Saxena, S., Kamal, M., Zarrin, S., and Islam, A., " Analysis of Accelerated Life Testing using Log-Logistic Geometric Process Model in case of Censored Data", International Journal of Engineering Science and Technology, Vol. 5, No. 7, pp. 1434-1442, 2013.

[18] Braun, W.J., Li, W., and Zhao, Y.Q., "Properties of the geometric and related processes", Naval Research Logistics, Vol. 52, No. 9, pp. 607–616, 2005.

[19] Marshall, A. N., Olkin, I., "A new method for adding a parameter to a family of distributions with applications to the exponential and Weibull families", Biometrica, Vol. 84, pp. 641-652, 1997.

[20] Alice, T., and Jose, K.K., "Marshall-Olkin Pareto distribution and its reliability application", IAPQR Trans., Vo. 29, No.1, pp. 1-9, 2005.

[21] Ghitany, M.E., Al-Hussaini, E.K., and Al-Jarallah, "Marshall-Olkin Extended Weibull distribution and its application to censored data", Journal of Applied Statistics, Vol. 32, No. 10, pp. 1025-1034, 2005.

[22] Gupta, R.C., Ghitany, M.E., Al-Mutairi, "Estimation of reliability from Marshall-Olkin Extended Lomax distribution", Journal of Statistical Computation and Simulation, Vol. 80, No. 8, pp. 937-947, 2010.