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LRS Bianchi Type –I Universe with Magnetized Wet Dark Fluid

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Abstract: Some locally rotationally symmetric (LRS) Bianchi type I cosmological model of universe filled with dark energy from a wet dark fluid in the presence and absence of magnetic field is in investigated in general theory of relativity . We assume F_{23} is non vanishing component of F_{ij}. We obtain exact solutions to the field equation, where a relation between metric potential $b = a^n$ is considered. The geometrical and kinematical properties of the models and the behaviours of the anisotropy of the dark energy have been carried out.

Keywords: Bianchi type- I space- time, Magnetic field Wet dark fluid, Dark energy.

I.INTRODUCTION

The nature of the dark energy component of the universe [1-3] remains one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence [4-6], k-essence [7-9], phantom energy [10-12]. Modifications of the Friedmann equation such as Cardassian expansion [13,14] as well as what might be derived from brane cosmology [15-17] have also been used to explain the acceleration of the universe. A particular case of the linear Equation of state has used in the cosmological context by Xanthopuolos [18], he considered space-times with two hypersurface orthogonal, space-like, commuting killing fields.

The current standard model of cosmology implies the existence of dark energy which accounts for about 70% of the total energetic content of the universe, which ac-cording to the observations is spatially flat [19]. Several models have been proposed to explain dark energy [20-28]. An alternative consists of to consider a phenomenological decaying dark energy density with continuous creation of matter [28] or photons [29,30].The dark energy might decay slowly in the course of the cosmic evolution and thus provide the source term for matter and radiation. Different such models have been discussed and strong constraints come from accurate measurements of the CMB. Although some authors [31] have suggested cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles.

Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structures we see in the universe today. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields and therefore, their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FRW universe as a special case are the type-I space-times.

In this work, we use Wet Dark Fluid (WDF) as a model for dark energy. The solution of the field equations for (LRS) Bianchi type I space- time are found. Some physical and kinematical parameter are also evaluated for the solution. A brief summary is given in the last section .

We consider string cosmology for Bianchi Type-I metric

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 (dy^2 + dz^2) \tag{1}$$

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where a and b are functions of t only.
The energy momentum tensor of the source is given by

$$T_{ij} = {}^{WDF}T_{ij} + {}^{EM}T_{ij} \tag{2}$$

where

$${}^{WDF}T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j + p_{WDF\,ij} \tag{3}$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = -1 \tag{4}$$

In commoving system of coordinates, from (above two) , we obtain

$${}^{WDF}T_i^1 = {}^{WDF}T_2^2 = {}^{WDF}T_3^3 = p_{WDF}, \qquad {}^{WDF}T_4^4 = -\rho_{WDF} \tag{5}$$

Here $E_i^{\ j}$ is the electromagnetic field given by

$$E_i^{\ j} = \frac{1}{4}\left(F_{j\alpha}F^{i\alpha}g^{\alpha\beta} - \frac{1}{4}g_i^{\ j}F^{\alpha\beta}F_{\alpha\beta}\right)$$

where F_{ij} is the electromagnetic field tensor which satisfies the Maxwell equation

$$F_{[ij;\alpha]} = 0 \qquad \left(F^{ij}\sqrt{-g}\right)_{;j} = 0 \tag{6}$$

In commoving co-ordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equation (6), the only non-vanishing component of F_{ij} is the equation

$$F_{23} = constant = H \tag{7}$$

The non-vanishing components of ${}^{EM}T_{ij}$ corresponding to the line element (1) are as follows:

$${}^{EM}T_i^1 = \frac{H^2}{8\pi b^4} = -{}^{EM}T_2^2 = {}^{EM}T_3^3 = {}^{EM}T_4^4 \tag{8}$$

The Einstein field equations are

$$R_i^{\ j} - \frac{1}{2}Rg_i^{\ j} = 8\pi T_i^{\ j} \tag{9}$$

The non- vanishing components of the Einstein field equations are

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} = 8\pi\rho_{WDF} + \frac{H^2}{\mu^2 b^4} \tag{10}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} = -8\pi p_{WDF} + \frac{H^2}{\mu^2 b^4} \tag{11}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -8\pi p_{WDF} - \frac{H^2}{\mu^2 b^4} \tag{12}$$

The physical quantities that are of the importance in cosmology are proper volume R^3, expansion scalar θ and shear σ^2 as follows

$$R^3 = ab^2 \tag{13}$$

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \tag{14}$$

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$$\sigma^2 = \frac{2}{3}\left(\frac{\dot b}{b} - \frac{\dot a}{a}\right)^2 \tag{15}$$

Here we have three field equations connecting four unknown quantities $a, b, \rho_{WDF}, p_{WDF}$. Therefore, in order to obtain exact solutions we must need one more relation connecting the unknown quantities, we assume the relation

$$b = a^n \tag{16}$$

From (11) and (12) , we have

$$\frac{\ddot b}{b} - \frac{\ddot a}{a} + \left(\frac{\dot b}{b}\right)^2 - \frac{\dot a \dot b}{ab} = 2\frac{H^2}{\mu^2 b^4} \tag{17}$$

Using equation (16) in (17), we obtain

$$(n-1)\frac{\ddot a}{a} + 2n(n-1)\left(\frac{\dot a}{a}\right)^2 = 2\frac{H^2}{\mu^2 a^{4n}}$$

Which again leads to

$$\ddot a + 2n\frac{\dot a^2}{a} = 2\frac{H^2}{\mu^2 (n-1)} a^{-4n+1} \tag{18}$$

Let us consider

$$\dot a = f(a)$$

$$\ddot a = ff' \qquad where\ f' = \frac{df}{da} \tag{19}$$

Using (19), (18) reduces to

$$ff' + 2n\frac{f^2}{a} = \frac{2H^2}{\mu^2 (n-1)} a^{-4n+1} \tag{20}$$

After simplifying (20) , we obtain

$$f^2 = \frac{2H^2}{\mu^2 (n-1)} a^{2-4n} + k_1 a^{-4n} \tag{21}$$

Where k_1 is integrating constant

Using (19), (21) becomes

$$\frac{da}{dt} = \left[\frac{2H^2}{(n-1)\mu^2} a^{2-4n} + k_1 a^{-4n}\right]^{\frac{1}{2}} \tag{22}$$

Then the metric(1) becomes as

$$ds^2 = -\left[\frac{2H^2}{(n-1)\mu^2} a^{2-4n} + k_1 a^{-4n}\right]^{-1} da^2 + a^2 dx^2 + b^2 (dy^2 + dz^2) \tag{23}$$

Using suitable transformation $a = T$

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$$ds^2 = -\left[\frac{2H^2}{(n-1)\mu^2}T^{2-4n} + k_1 T^{-4n}\right]^{-1} dT^2 + T^2 dx^2 + T^{2n}(dy^2 + dz^2) \tag{24}$$

For the model of equation (24), the other physical and geometrical parameters can be easily obtained.
The scalar of expansion , the shear scalar , spatial volume , Pressure and density of WDF are given respectively by

$$\theta = (1+2n)\left[\frac{2H^2}{(n-1)\mu^2}T^{-4n} + k_1 T^{-(4n+2)}\right]^{\frac{1}{2}} \tag{25}$$

$$\sigma^2 = \frac{2}{3}(n-1)^2\left[\frac{2H^2}{(n-1)\mu^2}T^{-4n} + k_1 T^{-(4n+2)}\right] \tag{26}$$

$$V = T^{2n+1} \tag{27}$$

$$p_{WDF} = \frac{1}{8\pi}\left\{\frac{(2n^2+n-1)}{(n-1)}\frac{H^2}{\mu^2}T^{-4n} + n(n+2)k_1 T^{-(4n+2)}\right\} \tag{28}$$

$$\rho_{WDF} = \frac{1}{8\pi}\left\{\frac{(n^2+3n+1)}{(n-1)}\frac{H^2}{\mu^2}T^{-4n} + n(n+2)k_1 T^{-(4n+2)}\right\} \tag{29}$$

The spatial volume V tend to zero when $T \to 0$ and increases as $T \to \infty$.

The expansion and shear scalar are infinite at $T = 0$ and decreases with the increases in cosmic time. Thus, the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe.

The anisotropy parameter of the expansion $\lim_{T\to\infty}(\frac{\sigma}{\theta}) \neq 0$ is found to be constant. Thus, the model does not approach to isotropy for the future evolution of the universe.

In the absence of magnetic field $H \to 0$ then the metric (24) reduces to the form

$$ds^2 = -\left[k_1^{-1}T^{4n}\right]dT^2 + T^2 dx^2 + T^{2n}(dy^2 + dz^2) \tag{30}$$

For the model (30) the other physical and geometrical parameters can be easily obtained. The expression for scalar of expansion , the shear scalar , spatial volume and the expression for the pressure and density of WDF are respectively given by

$$\theta = (1+2n)\left[k_1 T^{-(4n+2)}\right]^{\frac{1}{2}} \tag{31}$$

$$\sigma^2 = \frac{2}{3}(n-1)^2\left[k_1 T^{-(4n+2)}\right] \tag{32}$$

$$V = T^{2n+1} \tag{33}$$

$$p_{WDF} = \frac{1}{8\pi}\left\{n(n+2)k_1 T^{-(4n+2)}\right\} = \rho_{WDF} \tag{34}$$

The scalar of expansion and shear are infinite at $T = 0$ and deceases with the increases in cosmic time. Thus the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe.

The spatial volume V tends to zero when $T \to 0$ *and* $V \to \infty$ *when* $T \to \infty$.

We observe that pressure and density of WDF is same .

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The anisotropy parameter of the expansion $\lim_{T\to\infty}(\frac{\sigma}{\theta})\neq 0$ is found to be constant. Thus, the model does not approach to isotropy for the future evolution of the universe.

II.CONCLUSION

In summary, we presented LRS Bianchi type-I string cosmological models in the form of Wet Dark Fluid in the presence and absence of magnetic field. We adopt a relation between metric potentials $b = a^n$. The solution describes a shearing non-rotating model with a big bang start. In the absence of magnetic field, pressure and density of WDF is same.

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