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Homotopy Analysis Method Applied To Determine Pressure Head In Unsaturated Soil During Infiltration Phenomenon

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Abstract: The Homotopy analysis method has been applied to Richard's equation under considering the value of diffusivity coefficient $D(\theta)$ and hydraulic conductivity $K(\theta)$ under certain assumption. The solution of this equation has been obtained by considering the guess value of pressure head in an unsaturated soil. It is concluded that the pressure head in an unsaturated soil is decreasing as depth Z is increasing for given time T > 0. The graph and a numerical value are given by using Maple coding.

Keywords: Richard's equation, Unsaturated porous media, Homotopy Analysis Method, Infiltration Phenomenon.

I. INTRODUCTION

From last few years, groundwater flow and mass transport in the unsaturated porous media has significantly increasingly and it has a great important for hydrologist, agriculturists and people related with water resources sciences. In hydrology, volumetric water content is very important phenomenon in infiltration of groundwater in an unsaturated porous media. The groundwater flow and mass transport in the unsaturated porous media is a region of underground as its top by the soils (porous media) and below by the groundwater table, where physical phenomena such as infiltration, groundwater recharge, moisture content and others are happened. Ground water recharge problem has been discussed by many researchers with different viewpoints. Swartzendruber uses Philip's [31] method to get graphical illustration of the mathematical solution for horizontal water function. Verma and Mishra [29] have obtained solution by similarity transformation of a unidimensional vertical ground water recharges through porous media. Mehta [27] has obtained an approximate solution by the method of singular perturbation technique. Hari Prasad et al. [30] had developed a numerical model for unsaturated zones for the process of gravity drainage and infiltration. Almost all unsaturated flow simulations use the Richard's equation. Richards [15] investigated the influence capillarity has on liquid infiltration in soils, and equation derived is now known as Richards' equation. He derived a governing nonlinear equation for water flow in porous media based on a continuum approach. In the derivation of Richards' equation, it is assumed the liquid infiltration is driven by capillarity and gravity, while the second fluid phase occupying the unsaturated void-spaces is largely inert and does not affect the evolution. The sophistication of the analytical and numerical methods that are available for solving the governing nonlinear equation of unsaturated flow in porous media (Richard's Equation) require specialized forms of the conductivity (hydraulic conductivity) and water diffusivity (diffusivity functions). Several investigators have described different relation between the diffusivity coefficient and hydraulic conductivity in groundwater recharge problem.

In 1958, Gradner [32] model provided a relationship between the pressure head 'h' and the volumetric water content θ as, $h(\theta)=a\cdot\theta^{-b}$ where a & b are empirical constants. The exponential hydraulic conductivity function has been widely

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used, but it is known to have a limited range of application to many real soils. Other functions developed by Brooks and Corey [33] and Van Genuchten [31] are firmly established for practical applications. Such special forms of the hydraulic functions make it possible to linearize the governing flow equations, and hence solve them analytically. Solutions to the linearized unsaturated flow equations are generally limited to the steady flow in semi-infinite, homogenous soils (Broadbridge and White, 1988; Warrick and Yeh, 1990) and to transient flow in homogeneous and layered soils (Srivastava and Yeh) [7]. In 1988,The Broadbridge and White Model [5] has adopted a function form for the diffusivity given by Philip and Knight (1974)[6] which allows for the transformation of the soil water diffusivity $D(\theta)$ as function has the form:

$$D(\theta)=\frac{a}{(b-\theta)^2} \tag{1}$$

where a and b are constant. As a second step in the solution of the nonlinear flow problem, Broadbridge and White (1988) [5] developed an expression for $K(\theta)$ which in conjunction with the assumed function for $D(\theta)$ transforms equation (8) to the weakly nonlinear Burger's equation. This expression for $K(\theta)$ is given as:

$$K(\theta)=\beta+\gamma(b-\theta)+\frac{\lambda}{2(b-\theta)} \tag{2}$$

where β, γ and λ are constants. With the suggested analytical forms for $K(\theta)$ and $D(\theta)$ and the imposed boundary conditions, the Hopf-Cole transformations are applied to reduce a nonlinear equation to a linear form that possesses an exact parametric solution [5]. In this paper, Broadbridge and White Model [5] was applied to solve infiltration phenomenon in unsaturated porous media. In 1992, Liao[12] employed the basic idea of the homotopy in topology to propose a general analytical method for nonlinear problems, namely Homotopy Analysis method. Homotopy analysis method has been applied to nonlinear fluid dynamics problems by Liao [9], [11]. The important factor of this method is the generalized Taylor expansion. Liao point out that the generalized Taylor series provided a way to control and adjust the convergence region through an auxiliary parameter $\hbar$ such that this method is particularly suitable for strongly nonlinear problems. This method does not dependence on small or large parameter and is easy to adjust the convergence region and rate of approximation of the series. In this paper, ground water recharge phenomenon is solved by Homotopy analysis method.

II. ASSUMPTION AND MATHEMATICAL STATEMENT OF THE PROBLEM

In the present model, Darcian-based unsaturated flow equation described over the large basin L (i.e. L is the length of a large basin) of homogeneous porous media, and there is no air resistance to the flow (i.e. the porous medium contains only the flowing liquid water and empty voids of air). The air in the void space is approximately at atmospheric pressure. The flowing liquid (water) is considered continuous at a microscopic level, incompressible and isothermal, where the pressure head in unsaturated soil is considered as time depended function. In this case consider the flow of infiltrated water in vertical downward direction up to length L by neglecting spreading in other directions. We obtain one dimensional nonlinear partial differential equation by combining Darcy's law [1] for unsaturated flow with the continuity equation.

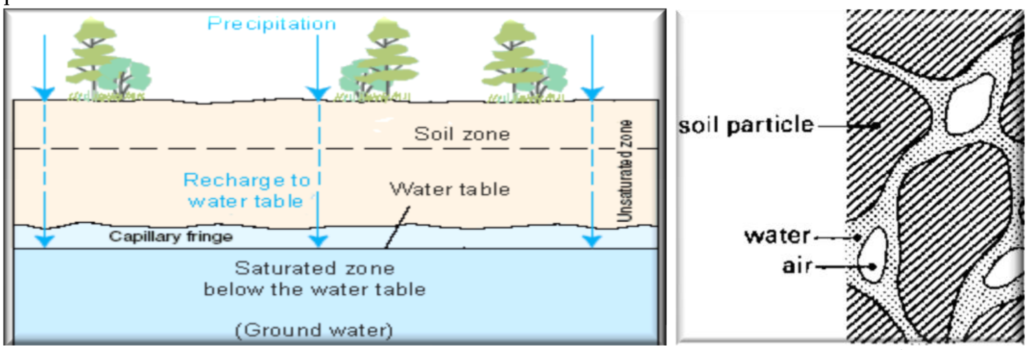

Fig 1: Representation groundwater phenomenon by spreading moisture content in the pore space of the soil.

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In this paper, the Homotopy analysis method is implemented to obtain the approximate solutions of the governing nonlinear partial second order differential equation for infiltration phenomenon, which is able to solve the one-dimensional h-based forms of Richard's equation. Its solution provides the infiltrated water into the porous media (soil) at any depth Z at time T > 0. The graph of pressure head versus time T is given at depth Z > 0, which shows at any depth Z pressure head is decreases as time T increases.

III. MATHEMATICAL STRUCTURE

When water flow through unsaturated porous media in vertically downward direction, hydraulic conductivity is varies nonlinearly with the volumetric water content: $K = K(\theta)$ The variation of the hydraulic conductivity with the volumetric water content θ in unsaturated homogeneous soil for small Reynold number the volume of flow of water described by Darcy's law as [23],

$$\vec{V} = -K(\theta)\nabla H \tag{3}$$

Where, $\vec{V}$ = The volume flux of moisture

$K(\theta)$ =The coefficient of the volumetric water content,

∇H =The gradient of the whole moisture potential

Such ground water flow satisfies the equation of continuity as follows,

$$\frac{\partial}{\partial t}(\rho_s \phi S) = -\nabla M \tag{4}$$

Where ρ_s is the bulk density of the soil on dry weight basis, M is the mass of flux of the water at any time $t \geq 0$.
Considering the fact that water is incompressible and $M = \rho\vec{V}$ and also considering the fact that the water content the of the soil is given by standard relation with saturation of soil S as $\theta = \phi S$ [9].
Where ϕ is porosity and S is a saturation of the soil.
Equation (4) reduces to,

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\nabla\left(\rho\vec{V}\right) \tag{5}$$

Where ρ is the flux density.
Using equation (3) in (5), we get

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\nabla\left(\rho\left(-K(\theta)\nabla H\right)\right) \tag{6}$$

It is also considered here as that the flow takes place only in vertical downward direction [7], equation (6) reduced to,

$$\rho_s \frac{\partial \theta}{\partial t} = \rho \frac{\partial}{\partial z}\left(K(\theta)\frac{\partial H}{\partial z}\right) \tag{7}$$

In unsaturated soil instead of pressure head h one introduces, the soil suction ψ by negative forces of capillary and pressure head is negative $\psi = |h|$. For reduced water content, only the soil suction really matter. The water thus moves inside the unsaturated soil from a point having a greater pressure head (or a lower suction value $\psi = |h|$) to another point by a smaller pressure head, until these values become the same. For unsaturated porous media, H is total soil moisture potential : $H = \psi - gz$ [9], where ψ is the pressure potential (soil matrix suction (ψ)), z is the elevation in the vertical downward direction of flow, and g is gravitational constant. Hence equation (7) will be,

$$\frac{\partial \theta}{\partial t} = \frac{\rho}{\rho_s}\frac{\partial}{\partial z}\left(K(\theta)\frac{\partial \psi}{\partial z}\right) - \frac{\rho}{\rho_s} g \frac{\partial K(\theta)}{\partial z} \tag{8}$$

The equation (8) can be written as,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial \theta}{\partial z}\right) - \frac{\rho g}{\rho_s} K'(\theta)\frac{\partial \theta}{\partial z} \tag{9}$$

Where z is depth in vertical downward direction, t is time, $\theta(z,t)$ is volumetric soil water content, $D(\theta) = \frac{\rho K}{\rho_s}\frac{\partial \psi}{\partial \theta}$ is called the diffusivity coefficient, $K(\theta)$ is the coefficient of the volumetric water content and $K'(\theta) = \frac{dK}{d\theta}$.

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The expression in equation (9) is written as θ dependent equation. This equation (9) is model based on Darcy-Buckingham approach in vertical downward direction flow of water in unsaturated soil.
We consider following new independent variables $Z=\frac{z}{L}, and\, T=\frac{\rho g}{\rho_s L}t$ has been introduced to simplify the equation (9) as,

$$\frac{\partial\theta}{\partial T}=\varepsilon\frac{\partial}{\partial Z}\left(D(\theta)\frac{\partial\theta}{\partial Z}\right)-K'(\theta)\frac{\partial\theta}{\partial Z} \tag{10}$$

where $\varepsilon=\frac{\rho_s L}{\rho g}$ is a constant parameter. Broadridge and White model (1988) [5]; the soil water diffusivity $D(\theta)$ and hydraulic conductivity $K(\theta)$ from equation (1) and (2) are written as

$$D(\theta)=\frac{a}{b^2}\left(1-\frac{\theta}{b}\right)^{-2}=\frac{a}{b^2}\left(1+\frac{2\theta}{b}\right) \tag{11}$$

since θ is very small. and

$$K(\theta)=\beta+\gamma b+\frac{\delta}{2b}+\left(\frac{\delta}{2b^2}-\gamma\right)\theta \tag{12}$$

where a, b, $\beta,\gamma\, and\, \lambda$ are constants.
Using equation (11) and (12) in (10), we get,

$$\frac{\partial\theta}{\partial T}=\varepsilon\frac{a}{b^3}\frac{\partial}{\partial Z}\left((b+2\theta)\frac{\partial\theta}{\partial Z}\right)-\left(\frac{\delta}{2b^2}-\gamma\right)\frac{\partial\theta}{\partial Z} \tag{13}$$

Let $b+2\theta=h(\theta)\Rightarrow 2\frac{\partial\theta}{\partial T}=\frac{\partial h}{\partial T}$ substituting in (13)

$$\frac{\partial h}{\partial T}=\varepsilon\frac{a}{2b^3}\frac{\partial}{\partial Z}\left(h\frac{\partial h}{\partial Z}\right)-\left(\frac{\delta}{2b^2}-\gamma\right)\frac{\partial h}{\partial Z} \tag{14}$$

$$\frac{\partial h}{\partial T}=\varepsilon A'\frac{\partial}{\partial Z}\left(h\frac{\partial h}{\partial Z}\right)-B'\frac{\partial h}{\partial Z}, \text{ where } A'=\frac{a}{2b^3}, \text{ and } B'=\left(\frac{\delta}{2b^2}-\gamma\right) \tag{15}$$

Their model describes the dependence of the pressure head 'h' in unsaturated soil. The analytical solutions describe the temporal development of the pressure head profile during rainfall. They predict the time dependence of pressure head and surface soil water potential and the shape of large time. equation (15) can be written as,

$$\frac{\partial h}{\partial T}=\varepsilon A'\left[h\frac{\partial^2 h}{\partial Z^2}+\left(\frac{\partial h}{\partial Z}\right)^2\right]-B'\frac{\partial h}{\partial Z} \tag{16}$$

The equation (16) called the equation of Richards' [11], which is widely used in the movement of groundwater in unsaturated porous media due to the pressure varies continuously in the reservoir.
Equation (16) is non-linear second order partial differential equation for one-dimensional unsteady flow in porous media, where $0\le\varepsilon\le 1$ is parameter. Mehta and Meher [8] has obtained solution of moisture content of the soil in terms of negative exponential and $h(\theta)=b+2\theta$. Hence for the sake of application of Homotopy analysis method, we choose appropriate initial guess value of solution (17) with respect to this phenomenon as,

$$\hbar(\mathrm{Z},\mathrm{T}:\varepsilon)=(1-T)e^{-Z}+\varepsilon^m \tag{17}$$

IV. THE SOLUTION WITH HOMOTOPY ANALYSIS METHOD

For one dimensional non-linear partial differential equation for ground water problem, we have assumed that the pressure head in unsaturated soil at top of large basin is expressed as,

$$\hbar(\mathrm{Z},T;\varepsilon=0)=\left(1-e^{-Z}\right)T+\varepsilon^m, \tag{18}$$

where $\varepsilon=0$ at top of the soil.
Now we apply the Homotopy analysis method to the discussed solution of equation (16) to find pressure head in unsaturated homogeneous soil . Since equation (16) is nonlinear Richard's equation. We can write it as symbolically as,

$$\square\left[\hbar(\mathrm{Z},T;\varepsilon)\right]=0 \tag{19}$$

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Where □ is a non-linear operator, $\hbar(Z,T;\varepsilon)$ is considered as unknown function which represent pressure head h in unsaturated soil of the groundwater through its pores at any depth Z for given time T > 0, for $0 \le \varepsilon \le 1$. We use auxiliary linear operator $\Im[\hbar(Z,T;\varepsilon)] = \frac{\partial \hbar(Z,T;\varepsilon)}{\partial T}$ and initial approximation of pressure head in unsaturated soil of the groundwater $h_0(Z,T) = (1-e^{-Z})T$ to construct the corresponding zeroth order deformation equation. As the auxiliary linear operator $\Im$ which satisfies $\Im[C_1] = 0$, where C_1 is arbitrary constant. This provides a fundamental rule to direct the choice of the auxiliary function $H(Z,T) \neq 0$, the initial approximation $h_0(Z,T)$, and the auxiliary linear operator $\Im$, called the rule of solution expression. Establish the zero-order deformation equation of groundwater phenomenon as [9],

$$(1-\varepsilon)\Im[\hbar(Z,T;\varepsilon) - h_0(Z,T)] = \varepsilon\hbar H(X,T)\,\square\,[\hbar(Z,T;\varepsilon)] \quad (20)$$

where $h_0(Z,T)$ denote an initial guess value of the exact solution $h(Z,T)$ which is our purpose to find it. Since $\hbar \neq 0$ is an auxiliary parameter and $H(Z,T) \neq 0$ is an auxiliary function such that $\varepsilon \in [0,1]$ is an embedding parameter. The auxiliary parameter $\hbar$ is providing a simple way to ensure the convergence of series. Thus it renamed $\hbar$ as convergence control parameter [9]. Let $\Im$ is an auxiliary linear operator with the property that,

$\Im[\hbar(Z,T;\varepsilon)] = 0$ when $\hbar(Z,T;\varepsilon) = 0$

When $\varepsilon = 0$, the zero-order deformation equation (20) becomes

$$\Im[\hbar(Z,T;\varepsilon) - h_0(Z,T)] = 0 \quad (21)$$

Which gives the first rule of solution expressed and according to the initial guess solution of h is $h_0(Z,T) = (1-e^{-Z})T$, it is straightforward to choose

$$\hbar(Z,T;0) = h_0(Z,T) \quad (22)$$

When $\varepsilon = 1$, since $\hbar \neq 0$, $H(Z,T) \neq 0$ the zero-order deformation equation (20) is equivalent to

$$\square\,[\hbar(Z,T;\varepsilon)] = 0 \quad (23)$$

which is exactly the same as the original equation (19) provided that

$$\hbar(Z,T;1) = h(Z,T) \quad (24)$$

According to (22) and (24) as the embedding parameter ε increases from 0 to 1, solution $\hbar(Z,T;\varepsilon)$ varies continuously from the initial guess value $h_0(Z,T)$ of an unsaturated soil to the solution $h(Z,T)$ and its solution is assumed by expanding $\hbar(Z,T;\varepsilon)$ in Taylor series with respect to ε as,

$$\hbar(Z,T;\varepsilon) = \hbar(Z,T;0) + \sum_{m=1}^{\infty} h_m(Z,T)\varepsilon^m \quad (26)$$

Where, $h_m(Z,T) = \frac{1}{m!}\frac{\partial^m \hbar(Z,T;\varepsilon)}{\partial \varepsilon^m}\Big|_{\varepsilon=0}$ (27)

So during infiltration water, pressure head is a function of θ but θ is function of depth Z and time T for any parametric value ε is expressed as, the pressure head in unsaturated porous media at time $T=0$, $h_0(Z,T)$ and sum of pressure head in unsaturated soil $h_1(Z,T)$, $h_2(Z,T)$,…at different time T for different value of parameter ε. Here, the series (26) is called homotopy-series; the series (26) is called homotopy series solution of $\square\,[\hbar(Z,T;\varepsilon)] = 0$ and $h_m(Z,T)$ is called the mth-order derivative of $\hbar$. Auxiliary parameter $\hbar$ in Homotopy-series (26) can be regard as iteration factor and is widely used in numerical computations. It is well known that the properly chosen iteration factor can ensure the convergence of Homotopy series (26) is depending upon the value of $\hbar$, one can ensure that convergent of Homotopy series, solution simply by means of choosing the proper value of $\hbar$ as shown by Liao [9, 10, 11, 12]. If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, the auxiliary function $H(X,T)$ are so properly chosen, the series (26) converges at $\varepsilon = 1$.

Hence the pressure head in unsaturated soil can be expressed as,

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$$h(Z,T)=h_0(Z,T)+\sum_{m=1}^{\infty}h_m(Z,T) \tag{28}$$

where $h_m(Z,T)$ can be calculated by equation (32) and (33). This must be one of the solution of original non-linear partial differential equation (16) for pressure head in unsaturated homogenous soil.

According to the definition (27), the governing equation can be deduced from the zero-order deformation equation (21), define the vector

$$\vec{h}_m=\{h_0(Z,T),_1 h_1(Z,T),...,h_n(Z,T)\}$$

Differentiating equation (21) m-times with respect to the embedding parameter ε and then setting $\varepsilon=0$ and finally dividing them by $m!$, we have the so-called m[th] order deformation equation of the pressure head in unsaturated soil $h(Z,T)$ will be as given by [9]

$$\Im\left[h_m(Z,T)-\chi_m h_{m-1}(Z,T)\right]=\varepsilon\hbar H(Z,T)R_m\left(\vec{h}_{m-1},Z,T\right) \tag{29}$$

Where $R_m\left(\vec{h}_{m-1},Z,T\right)=\frac{1}{(m-1)!}\frac{\partial^{m-1}\square\left[\hbar(Z,T;\varepsilon)\right]}{\partial\varepsilon^{m-1}}\Bigg|_{\varepsilon=0}$ (30)

And $\chi_m=\begin{cases}0, & m\le 1\\ 1, & m>1\end{cases}$ (31)

It should be emphasized that $h_m(Z,T)$ for $m\ge 1$, is governed by the linear equation (22) with the linear boundary condition that came from the original problem, which can solve by symbolic computation software Maple as bellow. The rule of solution expression as given by equation (18) and equation (21), the auxiliary function independent of ε can be chosen as $H(Z,T)=1$ as given by [9].

According to initial guess value of pressure head in unsaturated soil is given by equation (17) and taking inverse of equation (21), equation (29) becomes,

$$h_m(Z,T)=\chi_m h_{m-1}(Z,T)+\hbar\Im^{-1}\left[R_m\left(\vec{h}_{m-1},Z,T\right)\right] \tag{32}$$

$$R_m\left(\vec{h}_{m-1},Z,T\right)=\frac{1}{(m-1)!}\frac{\partial^{m-1}\square\left[\hbar(Z,T;\varepsilon)\right]}{\partial\varepsilon^{m}}\Bigg|_{\varepsilon=0} \tag{33}$$

In this way, we get $h_m(Z,T)$ for m=1, 2, 3, … successively by using Maple software as,

$$h_1(Z,T)=\frac{1}{6}hT\left(-4T^2+12T+3Te^Z-12e^Z-12\right)e^{-2Z} \tag{34}$$

$$h_2(Z,T)=\frac{1}{60}hT\begin{pmatrix}-36hT^4+180T^3h+25hT^3e^Z-360hT^2-160hT^2e^Z-5hT^2e^{2Z}+270hT+30Te^{2Z}\\+45hTe^{2Z}+240hTe^Z-90e^{2Z}-60he^Z-60he^{2Z}-20T^2e^Z+60Te^Z-60e^Z\end{pmatrix}e^{-3Z} \tag{35}$$

......

Using initial guess value of the pressure head in unsaturated soil from equation (18) and successive value of pressure head in unsaturated soil at different deformation of pressure head is given by (34), (35)..etc. We get pressure head of infiltration soil is,

$$h(Z,T)=\left\{\begin{matrix}(1-T)e^{-Z}+\frac{1}{6}hT\left(-4T^2+12T+3Te^Z-12e^Z-12\right)e^{-2Z}\\+\frac{1}{60}hT\begin{pmatrix}-36hT^4+180T^3h+25hT^3e^Z-360hT^2-160hT^2e^Z-5hT^2e^{2Z}+270hT+30Te^{2Z}\\+45hTe^{2Z}+240hTe^Z-90e^{2Z}-60he^Z-60he^{2Z}-20T^2e^Z+60Te^Z-60e^Z\end{pmatrix}e^{-3Z}+....\end{matrix}\right\} \tag{36}$$

V. NUMERICAL AND GRAPHICAL SOLUTION

The numerical and graphical presentations of equation (36) in the present work has been carried out using Maple coding. Fig 2 (a) represents the graphs of pressure head $h(Z,T)$ vs. depth Z, and time T = 0.1, 0.2, 0.3, 0.4, 0.5 is fixed, fig 2(b) represents the graphs of pressure head $h(Z,T)$ vs. depth Z, and time T = 0.6, 0.7, 0.8, 0.9, 1.0 is fixed and Table I indicate the numerical values of pressure head for different time T and depth Z. The figure 2 & 3 and the table 1 indicates the graphical representations of the infiltration phenomenon in homogeneous soil to obtain pressure head in unsaturated soil. The convergence of the Homotopy series (24) is dependent upon the value of convergence-parameter

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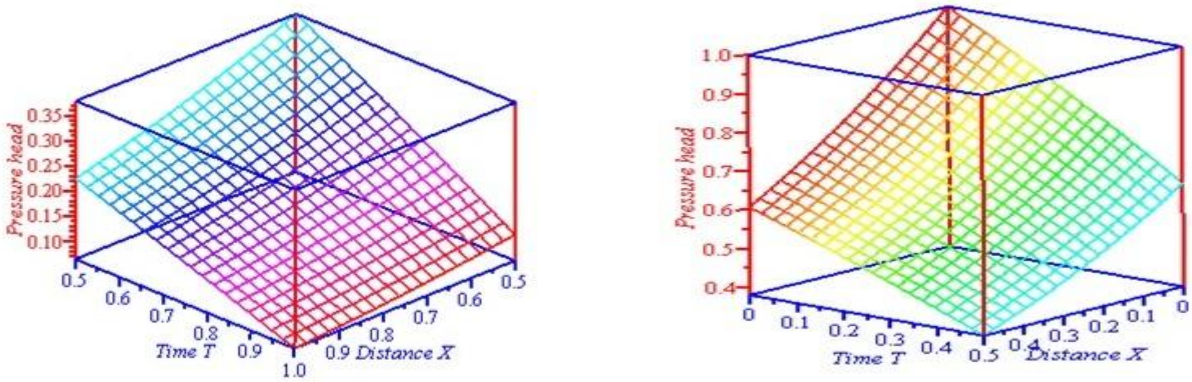
$\hbar$ [6, 9, 10, 17]. Therefore, we choose the proper value of the convergence-parameter $\hbar = -0.05$ to obtain convergent Homotopy-series solution [9].

Table I : Pressure head $h(Z,T)$ for different time T for fixed depth Z = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

Time T	Pressure Head									
	Z=0.1	Z=0.2	Z=0.3	Z=0.4	Z=0.5	Z=0.6	Z=0.7	Z=0.8	Z=0.9	Z=1.0
0.1	0.8459	0.7644	0.6908	0.6243	0.5643	0.5101	0.4612	0.4169	0.3769	0.3409
0.2	0.7839	0.7073	0.6384	0.5763	0.5203	0.4699	0.4245	0.3835	0.3465	0.3131
0.3	0.7189	0.6477	0.5838	0.5263	0.4747	0.4283	0.3865	0.3489	0.3149	0.2845
0.4	0.6509	0.5856	0.5269	0.4746	0.4276	0.3854	0.3474	0.3134	0.2827	0.2552
0.5	0.5804	0.5212	0.4683	0.4211	0.3789	0.3411	0.3073	0.2769	0.2496	0.2251
0.6	0.5072	0.4546	0.4078	0.3661	0.3289	0.2957	0.2659	0.2395	0.2157	0.1943
0.7	0.4317	0.3859	0.3455	0.3095	0.2776	0.2492	0.2238	0.2012	0.1809	0.1629
0.8	0.3541	0.3155	0.2816	0.2516	0.2251	0.2016	0.1807	0.1621	0.1456	0.1308
0.9	0.2745	0.2434	0.2162	0.1924	0.1715	0.1529	0.1367	0.1223	0.1095	0.0982
1.0	0.1933	0.1699	0.1497	0.1321	0.1169	0.1036	0.0919	0.0818	0.0729	0.0649



(a) (b)

Fig 2: Represents pressure head in unsaturated soil $h(Z,T)$ vs. depth Z and time T for auxiliary parameter $\hbar = -0.05$ and auxiliary function $H(Z,T) = 1$ [9] (a) for $0 < Z < 0.5,\ and\ 0 < T < 0.5$ (b) for $0.5 \le Z \le 1\ and\ 0.5 \le T \le 1$.

VI. CONCLUSION AND DISCUSSION

The equation (36) represents solution of nonlinear Richard's equation (16). Which gives pressure head of infiltrated water in unsaturated porous media. the solution is obtained by guessing its value $h(Z,T:\varepsilon) = (1-e^{-Z})T + \varepsilon^m$ and its successive pressure head of infiltrate water in an unsaturated deformed porous media is obtain by equation (34) and (35). The solution (36) of Richard's equation (16) is expressed in term of infinite series. The solution is expressed as exponential function of Z and time T with given guess vale $h(Z,T:\varepsilon) = (1-e^{-Z})T + \varepsilon^m$

It is concluded that the pressure head from top to bottom is decreasing as depth of unsaturated soil Z is increasing for given time T > 0. The figure 2 number (a) and (b) also shows pressure head verses depth Z for given T > 0 and pressure head verses time T for given Z > 0. From figure, it is concluded that the pressure head h in unsaturated soil is decreasing as depth Z as well as time T is increasing, which is practically and experimentally fact.

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