ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

Fuzzy Optimization Technique in EOQ Model Using Nearest Interval Approximation Method

A.Faritha Asma [1], E.C.Henry Amitharaj [2]

Assistant Professor, Department of Mathematics, Government Arts College, Trichy, Tamil Nadu, India [1]

Associate Professor, Department of Mathematics, Bishop Heber College, Trichy, Tamil Nadu, India[2]

ABSTRACT: This paper discusses an Economic Order Quantity (EOQ) model with shortage, where the setup cost, the holding cost, the shortage cost are considered as fuzzy numbers. The fuzzy parameters are then transformed into corresponding interval numbers. Minimization of the interval objective function (obtained by using interval parameters) has been transformed into a classical multi-objective EOQ problem. The order relation that represents the decision maker's preference among the interval objective function has been defined by the right limit, left limit ,center and half –width of an interval. This concept is used to minimize the interval objective function. The problem has been solved by fuzzy programming technique. Finally, the proposed method is illustrated with a numerical example

KEYWORDS: Inventory, Interval number, EOQ, Fuzzy sets, Fuzzy optimization technique, Multi-objective Programming.

I. INTRODUCTION

In traditional mathematical problems, the parameters are always treated as deterministic in nature. However, in practical problem, uncertainty always exists. In order to deal with such uncertain situations fuzzy model is used [1],[15].in such cases, fuzzy set theory, introduced by Zadeh [14] is acceptable. There are several studies on fuzzy EOQ model. Lin et al. [7] have developed a fuzzy model for production inventory problem. Katagiri and Ishii[5] have proposed an inventory problem with shortage cost as fuzzy quantity

This paper discusses a fuzzy EOQ model with shortage. Demand, Holding cost, ordering cost, shortage cost are taken as triangular fuzzy numbers, and expression for fuzzy cost is established. For minimizing the cost function we transformed the fuzzy objective function into interval objective function. Now, this single objective function is then converted to multi-objective problem by defining left limit, right limit and center of the objective function. This multi-objective is then solved by fuzzy optimization technique. Linear membership function is considered here. This model is illustrated by a numerical example

The article is organized as follows: In section 1 preliminary definitions of fuzzy set, interval number, basic interval arithmetic optimization in interval situation and nearest interval approximation is briefly described. Section 2 contains model formulation. The fuzzy optimization technique is section 3. In section 4 the process is illustrated by a numerical example and in the last section the entire work is concluded.

II. RELATED WORK

For several years, classical economic order quantity (EOQ) problems with different variations were solved by many researches and had been published since 1915**.** The main area in which research articles have been published , may be classified as crisp and fuzzy inventory models, relating to Economic order quantity, economic production quantity, optimization, defuzzyfication. The research papers relating to the optimization problems and defuzzyfications are of special interest. The major assumption in the classical EOQ model is that the demand rate is constant and deterministic.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

The main objective of this paper is to construct fuzzy EOQ model and to determine the optimum order quantity so as to minimize the average total cost.

Preliminaries

Definition 1:

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) A={(x, $\mu_A(x)$) ; xϵ X.}, here μ_A:X→ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of xϵ X in the fuzzy set A.

Definition 2:

Let $\Re$ be the set of all real numbers. An interval, may be expressed as

$$\bar{a} = [a_L, a_R] = \{x : a_L \le x \le a_R, a_L \in \Re, a_R \in \Re\} \quad (1)$$

where a_L and a_R are called the lower and upper limits of the interval $\bar{a}$, respectively.

If $a_L = a_R$ then $\bar{a}$ = [a_L , a_R] is reduced to a real number a, where a= a_L= a_R. alternatively an interval $\bar{a}$ can be expressed in mean-width or center-radius form as $\bar{a}$ = $\langle m(\bar{a}), w(\bar{a})\rangle$,where $m(\bar{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\bar{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\bar{a}$ .the set of all interval numbers in $\Re$ is denoted by I($\Re$).

Optimization in interval environment

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as Minimize $\bar{Z}(x) = \dfrac{\sum_{i=1}^{n}[a_{L_i}, a_{R_i}]\prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l}[b_{L_i}, b_{R_i}]\prod_{j=1}^{n} x_j^{q_j}}$ (2) subject to x_j>0, j=1,2,…,n and x ϵ S⊂$\Re$ where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}$, $0 < b_{L_i} < b_{R_i}$ and r_i ,q_j are positive numbers. Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\bar{Z}(x)$ can be written in the form $\bar{Z}(x) = [Z_L(x), Z_R(x)]$

where

$$Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i}\prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i}\prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(3)}$$

$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i}\prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i}\prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(4)}$$

The center of the objective function

$$Z_C(x) = \frac{1}{2}[Z_L(x) + Z_R(x)] \quad \textbf{(5)}$$

Thus the problem (2) is transformed in to

minimize $\{Z_C(x), Z_R(x); x \in S\}$ **(6)**

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

subject to the non-negativity constraints of the problem, where Z_C,Z_R are defined by (4) and (5).

III. NEAREST INTERVAL APPROXIMATION METHOD

According to Gregorzewski [3] we determine the interval approximation of a fuzzy number as: Let $\tilde{A}$ =(a_1,a_2,a_3) be an arbitrary triangular fuzzy number with a α-cuts [$A_L(\alpha)$,$A_R(\alpha)$] and with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}; a_1 \le x < a_2 \\ \frac{a_3-x}{a_3-a_2}; a_2 < x < a_3 \\ 0; otherwise \end{cases} \quad (7)$$

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 \left[a_1 + (a_2 - a_1)\alpha\right]d\alpha = \frac{a_1+a_2}{2} \quad (8)$$

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 \left[a_3 - (a_3 - a_2)\alpha\right]d\alpha = \frac{a_2+a_3}{2}$$

Therefore, the interval number considering $\tilde{A}$ as triangular fuzzy number is $\left[\frac{a_1+a_2}{2}, \frac{a_2+a_3}{2}\right]$.

Model formulation:

In this model, an inventory with shortage is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations:

C_1: Holding cost per unit time per unit quantity.
C_2: Shortage cost per unit time per unit quantity.
C_3: Setup cost per period
D: The total number of units produced per time period.
Q_1: The amount which goes into inventory
Q_2: The unfilled demand
Q: The lot size in each production run.

Assumptions:

(i) Demand is known and uniform.
(ii) Production or supply of commodity is instantaneous.
(iii) Shortages are allowed.
(iv) Lead time is zero.

Let the amount of stock for the item be Q_1 at time t=0 in the interval (0,t(=t_1+t_2)), the inventory level gradually decrease to meet the demands. By this process the inventory level reaches zero level at time t_1 and then shortages are allowed to occur in the interval (t_1,t). The cycle repeats itself.(Fig. 1)

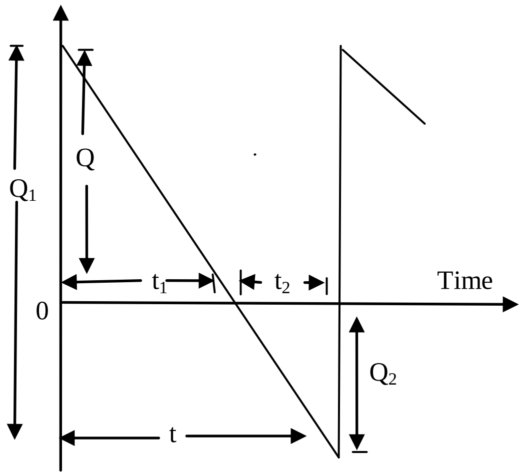
ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

Fig .1 Inventory level



The order level Q>0 which minimizes the average total cost (Q) per unit time is given by

$$\min C(Q) = \frac{1}{2}C_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_2\left(\frac{Q_2^2}{Q}\right) + C_3\left(\frac{D}{Q}\right) \quad \textbf{(9)}$$

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e .of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demand and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demand $\tilde{D} = (D-\alpha, D, D+\beta)$ fuzzy holding cost $\tilde{C}_{1`} = (C_1-\alpha, C_1, C_1+\beta)$,fuzzy shortage cost, fuzzy ordering cost $\tilde{C}_3 = (C_3-\alpha, C_3, C_3+\beta)$ Replacing the real valued variables D,C_1,C_2 & C_3 by the triangular fuzzy variables $\tilde{D}, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$ we get,

$$\tilde{C}(Q) = \frac{1}{2}\tilde{C}_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}\tilde{C}_2\left(\frac{Q_2^2}{Q}\right) + \tilde{C}_3\left(\frac{\tilde{D}}{Q}\right) \quad \textbf{(10)}$$

Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [3] , the fuzzy numbers are transformed into interval numbers as

$\tilde{D} = (D-\alpha, D, D+\beta)$ =[D_L,D_R]

$\tilde{C}_{1`} = (C_1-\alpha, C_1, C_1+\beta)$ =[C_{1L},C_{1R}]

$\tilde{C}_{2`} = (C_2-\alpha, C_2, C_2+\beta)$ = [C_{2L},C_{2R}]

$\tilde{C}_3 = (C_3-\alpha, C_3, C_3+\beta)$ =[C_{3L},C_{3R}].

Using the above expression (10) becomes

$$\tilde{C}(Q) = \left[\, f_{L,}\, f_R \,\right] \quad \textbf{(11)}$$

Where,

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

$$f_L = \frac{1}{2}C_{1L}\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_{2L}\left(\frac{Q_2^2}{Q}\right) + C_{3L}\left(\frac{D_L}{Q}\right) \quad \textbf{(12)}$$

$$f_R = \frac{1}{2}C_{1R}\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_{2R}\left(\frac{Q_2^2}{Q}\right) + C_{3R}\left(\frac{D_R}{Q}\right) \quad \textbf{(13)}$$

The composition rules of intervals are used in these equations.
Hence the proposed model can be stated as
Minimize $\{f_L(Q), f_R(Q)\}$, **(14)**
Generally, the multi-objective optimization problem(14), in case of minimization problem, can be formulated in a conservative sense from (3) as
Minimize $\{f_C(Q), f_R(Q)\}$, **(15)**
Subject to Q≥0.Where $f_C = \dfrac{f_L + f_R}{2}$.
Here the interval valued problem (14) is represented as
Minimize $\{f_L(Q), f_C(Q), f_R(Q)\}$,
Subject to Q≥0 **(16)**
The expression (16) gives a better approximation than those obtained from (14).
Fuzzy programming technique for solution:
To solve multi-objective minimization problem given by (16), we have used the following fuzzy programming technique.

For each of the objective functions $f_L(Q), f_C(Q), f_R(Q)$,we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(Q), f_C(Q), f_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $f_k(Q)$, k=L,C,R. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique is given below.

Step 1: Solve the multi-objective cost function as a single objective cost function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The initial fuzzy model of (10) can then be stated as, in terms of the aspiration levels for each objective, as follows: find Q satisfying $f_k \tilde{<} L_k, k = L, C, R$ subject to the non negativity conditions

Step 4: Define fuzzy linear membership function $\left(\mu_{f_k}; k = L, C, R\right)$ for each objective function is defined by

$$\mu_{f_k} = \begin{cases} 1; \; f_k \le L_k \\ 1 - \dfrac{f_k - L_k}{d_k}; L_k \le f_k \le U_k \\ 0; \; f_k \ge U_k \end{cases} \quad \textbf{(17)}$$

Step 5: After determining the linear membership function defined in(17) for each objective functions following the problem (16) can be formulated an equivalent crisp model

Max α,

$$\alpha \le \mu_{f_k}(x); k = L, C, R.$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

α≥0,Q≥0.

IV. NUMERICAL EXAMPLE

In this section, the above mentioned algorithm is illustrated by a numerical example.

Here the parameters demand, ordering cost, holding cost and shortage cost are considered as triangular fuzzy numbers (TFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [3].

Let C_1=5, C_2=25, C_3=100, D=5000 units.

Taking these as triangular fuzzy numbers we have,

$\tilde{C}_1$=(3,5,7), $\tilde{C}_2$=(21,25,31), $\tilde{C}_3$=(85,103,109), $\tilde{D}$=(4000,5000,6000).

The fuzzy numbers $\tilde{D},\tilde{C}_1,\tilde{C}_2,\tilde{C}_3$ are transformed into interval numbers as,

fuzzy numbers $\tilde{D},\tilde{C}_1,\tilde{C}_2,\tilde{C}_3$ are transformed into interval numbers as,

$\tilde{D}$=[D_L,D_R]=[4500,5500]

$\tilde{C}_1$=[C_{1L},C_{1R}]=[4,6]

$\tilde{C}_2$= [C_{2L},C_{2R}]=[23,28]

$\tilde{C}_3$=[C_{3L},C_{3R}]=[94,106]

Individual minimum and maximum of objective functions f_L, f_C, f_R are given in Table 1.

Objective functions	Optimize f_L	Optimize f_C	Optimize f_R
f_L	$f_L{}'$ =1697.84	$f_L{}''$ =1699.78	$f_L{}'''$ =1691.69
f_C	$f_C{}'$ =2046.83	$f_C{}''$ =2044.59	$f_C{}'''$ =2032.53
f_R	$f_R{}'$ =2407.86	$f_R{}''$ =2401.66	$f_R{}'''$ =2400.29

Now we calculate

$L_L=\min\left(f_L{}', f_L{}'', f_L{}'''\right)$=1691.69; $U_L=\max\left(f_L{}', f_L{}'', f_L{}'''\right)$=1699.78

$L_C=\min\left(f_C{}', f_C{}'', f_C{}'''\right)$=2032.53 $U_C=\max\left(f_C{}', f_C{}'', f_C{}'''\right)$=2046.83

$L_R=\min\left(f_R{}', f_R{}'', f_R{}'''\right)$=2400.29 $U_R=\max\left(f_R{}', f_R{}'', f_R{}'''\right)$=2407.86

Using the equation (18), we formulate the following problem as:

Max α

$$\frac{1}{2}(4)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(23)\left(\frac{Q_2^2}{Q}\right)+94\left(\frac{4500}{Q}\right)+(8.09)\alpha\le 1699.78$$

$$\frac{1}{2}(5)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(25.5)\left(\frac{Q_2^2}{Q}\right)+100\left(\frac{5000}{Q}\right)+(14.3)\alpha\le 2046.83$$

IJIRSET

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

$$\frac{1}{2}(6)\left(\frac{Q_1^2}{Q}\right)+\frac{1}{2}(28)\left(\frac{Q_2^2}{Q}\right)+106\left(\frac{5500}{Q}\right)+(7.57)\alpha \le 2407.86 \quad (18)$$

Results and Discussions:

The solutions obtained from (18) are given in table 2 and 3.

Using non- linear programming technique eq.(18) solved and the optimum value of α is found. It is given in table 2.

Table 2: optimum value of α

Maximum α
0.1379

The optimum results for the total average cost, the economic order quantity and the level of inventory are found and given in table 3.

Table 3: The optimum results

f_L^*	f_C^*	f_R^*	Q^*	Q_1^*
1698.66	2044.85	2403.24	492.09	414.17

The comparison is done between crisp and fuzzy model and it is represented in table 4.

Table 4:comparison table

model	C_1	C_2	C_3	Q*	Q_1*	C*(Q)	A
Crisp	5	25	100	489.9	408.2	2041.2	--
Crisp	5	20	50	353.6	282.8	1414.2	--
Crisp	3	20	50	437.8	380.7	1142.1	--
Fuzzy	[4,6]	[23,28]	[94,106]	492.09	412.17	[1698.7,2403.2]	0.1379

V. CONCLUSION

In this paper, we have presented an inventory model with shortage, where carrying cost, shortage cost, ordering or setup cost and demand are assumed as fuzzy numbers instead of crisp or probabilistic in nature to make the inventory model more realistic. At first, expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is approximated by interval number. After that the problem of minimizing the cost function is transformed into a multi-objective inventory problem, where the objective functions are left limit, right limit and the center of the interval function. Fuzzy optimization technique is then used to found out the optimal results. A numerical example illustrates the proposed method.

REFERENCES

[1] Bellman, R.E., Zadeh , L.A.; Decision-making in a fuzzy environment. Management science;(1970);17(4),141164.

[2] Chen, S.H., Wang, C.C., Chang, S.M.; Fuzzy economic production quantity model for items with imperfect quality. International Journal of Innovative Computing , Information and Control ;(2007);3(1),85-95.

[3] Grzegorzewski,P.; Nearest interval approximation of a fuzzy number Fuzzy Sets and Systems; (2002); 130,321-330.

[4] Ishibuchi, H.,Tanaka, H., Multi objective programming in optimization of the interval objective function. European Journal of Operational Research; (1990);48,219-225

[5] Katagiri,H.,Ishii,H.; Some inventory problems with fuzzy shortage cost. Fuzzy Sets and Systems; 2000;111(1),87-97.

[6] Li, L., Kabadi, S.N.,Nair, P.K.; Fuzzy models for single-period inventory problem. Fuzzy Sets and Systems;2002; 132(3), 273-289.

[7] Lin, D.C., Yao, J.S.; Fuzzy economic production for production inventory. Fuzzy Sets and Systems; 2000;111, 465-495.

[8] Mahapatra, G. S., Roy, T.K.; Fuzzy multi-objective mathematical programming on reliability optimization model. Applied Mathematics and Computation; 2006;174(1), 643-659.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

[9] Park, K.S.;Fuzzy set theoretic interpretation of economic order quantity. IEEE Transactions on systems, Man and Cybernetics, SMC;(1987);17,1082-1084.
[10] Susovan Chakraborty, Madhumagal Pal, Prasun Kumar Nayak, Solution of interval-valued manufacturing inventory models with shortages, International J. of Engineering and Physical Sciences, 4(2) (2010) 96-101.
[11] Susovan Chakraborty, Madhumagal Pal, Prasun Kumar Nayak, Multi section technique to solve interval-valued purchasing inventory models without shortages, Journal of Information and Computing Sciences, 5(3), (2010) 173-182.
[12] Tang, S.; Interval number and fuzzy number linear programming. Fuzzy Sets and Systems; (1994); 66, 301-306.
[13] Wolfe, M.,A.; Interval mathematics, algebraic equations and optimization; Journal of Computational and Applied Mathematics; (2000); 124, 263-280.
[14] Zadeh, L.A.; Fuzzy sets. Information and control;(1965); 8,338-352.
[15] Zimmermann, H.J.; Description and optimization of fuzzy system. International Journal of general system; (1976) ; 2(4),209-215.
[16] Zimmermann, H.J.; Fuzzy linear programming with several objective functions. . Fuzzy Sets and Systems; (1978); 1, 46-55.