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Fixed point Theorem in Intuitionistic Fuzzy metric space

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ABSTRACT: In this paper ,we generalize fuzzy metric space in term of fixed point theorem in modified Intuitionistic fuzzy metric space using weakly compatible mapping along with property (CLR_S) and (CLR_T).

Keywords*:* fixed point, intuitionistic fuzzy metric, implicit relation, compatibility.

I. INTRODUCTION

Aanassov[2] Introduction in studied the concept of Intuitionistic fuzzy set as a noted generalization of fuzzy set which has inspired intense research activity around this newly introduced notion. Recently Park[7] using the idea of Intuitionistic fuzzy sets. Defined Intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces due to George and Veeramani[4] and also proved some basic results which include Baire's theorem and uniform limit theorem besides some other core results. Afterthat ,Saadati and Park[8] defined precompact sets in Intuitionistic fuzzy metric spaces and proved that any subset of an Intuitionistic fuzzy metric space is compact if and only if it is precompact and complete.They also defined topologically complete Intuitionistic fuzzy metrizable space and proved that any G_δ set in a complete Intuitionistic fuzzy metric space is a topologically complete Intuitionistic fuzzy metrizable space and vice versa.George and veeramani [11] modified the concept of fuzzy metric space due to Kramosi and Michalek [6] and defined a Hansdorff topology on modified fuzzy metric space which often used
In current researches. Grabiee [5] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively.
The theory of fuzzy sets was initiated by Zadeh [13]. In the last four decades, like all other aspects of Mathematics various authors have introduced the concept of fuzzy metric in several ways.

II. PREMILIRIES

Definition 2.1.[10] A binary operation $\ast : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t –norm if ([0, 1], $\ast$) is an abelian topological monoid with unit 1 such that $a \ast b \leq c \ast d$ whenever $a \leq c$ and $b \leq d$ for a, b, c, d $\in$ [0, 1].

Examples of t-norms are $a \ast b = ab$ and $a \ast b = \min\{a, b\}$

Lemma 2.2. [3] Consider the set L^* and operation $\leq_L^*$ defined by

$L^* = \{(x_1, x_2): (x_1, x_2) \in [0,1]^2 \text{ and } x_1+x_2 \leq 1\}$,

$(x_1,x_2) \leq L^* (y_1,y_2) \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$ –v $(x_1,x_2),(y_1,y_2) \in L^*$.Then($L^* \leq_{L^*}$) is a complete lattice.

Example 2.3. [3] A triangular norm(t-norm) on $L^*$is a mapping $T:L^*\times L^* \rightarrow L^*$satisfying the following conditions:

(i) $T(x,1_L^*)=x$,
(ii) $T(x,y)=T(y,x)$,
(iii) $T(x,T(y,z))=T(T(x,y),z)$,
$x \leq_L^* x'$ and $y \leq_L^* y' \rightarrow T(x,y) \leq_L^* T(x',y')$,

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for all x,y ,z,x' and y'$\in L^*$.

Definition2.4. [3] A continuous t-norm T on L^* is called continuous t-representable if and only if there exist a continuous t-norm * and a continuous t-conorm ◊ on [0,1] such that, for all $x=(x_1,x_2), y=(y_1,y_2)$◊ L^*,
$T(x, y)=(x_1*y_1, x_2 \lozenge y_2)$.
Now we define a sequence $\{T^n\}$ recursively by $\{T^1=T\}$ and $T(x^{(1)},\ldots,x^{(n+1)})=T(T^{n-1}(x^{(1)},\ldots,x^{(n)},x^{(n+1)})$ for n≥2and $x^{(i)}\in L^*$.

Definition2.5 [9] Let M,N be fuzzy sets from $X^2\times(0,\infty)$to [o,1] such that $M(x,y,t)+N(x,y,t)\leq 1$ for all $x,y \in X$ and t>0. The 3-tuple $(X,M_{M,N},T)$ is said to be a modified intuitionistic fuzzy metric space(i.e modified IFMS)if X is arbitrary non empty set,T is a continuous t-representable and $M_{M,N}$ is an intuitionistic fuzzy set from $X\times(0,\infty)\rightarrow L^*$satisfying the following conditions:

(i) $M_{M,N}(x,y,t)>_{L^*} 0_{L^*}$,
(ii) $M_{M,N}(x,y,t)=1_{L^*}$ if and only if x=y,
(iii) $M_{M,N}(x,y,t)= M_{M,N}(y,x,t)$
(iv) $M_{M,N}(x,y,t+s)\geq_{L^*} T(M_{M,N}(x,y,t), M_{M,N}(x,y,s))$,
(v) $M_{M,N}(x,y,.):(0,\infty)\rightarrow L^*$is continuous. for every $x,y \in X$ and t,s>0.
In this case $M_{M,N}$ is called a modified intuitionistic fuzzy metric.Here
$M_{M,N}(x,y,t)=(M (x,y,t), N(x,y,t))$

Example 2.6 Let (X,d) be a mertic space.Denote $T(a,b)=(a_1b_1,\min\{a_2+b_2,1\})$ for all $a=(a_1,a_2)$ and $b=(b_1,b_2)\in L^*$and let M and N be fuzzy sets on $X^2\times(0,\infty)$. Then an intuitionistic fuzzy metric can be defined as:

$M_{M,N}(x,y,t)=(M (x,y,t), N(x,y,t))= \left(\frac{ht^n}{ht^n+md\,(x,y)},\frac{md\,(x,y)}{ht^n+md\,(x,y)}\right)$, for all $h,m,n,t \in R^+$ so that $X,M_{M,N},T)$ is a modified IFMS.

Definition 2.7 [3] A negator on L^* is a decreasing mapping $N:L^*\rightarrow L^*$ satisfying $N(0_{L^*})=1_{L^*}$and $N(1_{L^*})=0_{L^*}$. A negator on [0,1] is a decreasing mapping N:[0,1]→[o,1] satisfying N(o)=1and N(1)=0.In what follows,N_s denotes the standard negator on [0,1] defined as $N_s(x)=1-x$ for all $x \in [0,1]$.

Definition 2.8 Let $(X,M_{M,N},T)$ be a modified IFMS. For t>0, define the open ball B(x,r,t) with centre $x \in X$ and radius 0<r<1, as
$B(x,r,t)=\{y \in X: M_{M,N}(x,y,t)>_{L^*}(N_s(r),r)\}$.
A subset $A\subseteq X$ is called open if for each $x \in A$,there exist t>0 and 0<r<1 such that B(x, r ,t)⊂ A. Let $TM_{M,N}$ denote family of all open subset of X. Then $TM_{M,N}$ is called the topology induced by modified intuitionistic fuzzy metric $M_{M,N}$.

Definition 2.9 A sequence $\{x_n\}$ in a modified IFMS $(X,M_{M,N},T)$ is called a Cauchy sequence if for each $0<\in<1$ and t>0,there exist $n_0 \in N'$ such that
$M_{M,N}(x_n y_m,t)>L^*(N_s(\in),\in)$ and for each $n,m\geq n_0$ where N_s is the standard negator .
The sequence $\{x_n\}$ is said to be convergent to x∈X in the intuitionistic fuzzy metric space $(X,M_{M,N},T)$ and is generally denoted by $x_n\rightarrow M_{M,N}$ x if $M_{M,N}(x_n,x,t)\rightarrow 1_{L^*}$whenever $n\rightarrow\infty$ for every t>0 .An IFMS is said to be complete if and only if every Cauchy sequence convergent.

Lemma2.10 [9] Let $M_{M,N}$ be an intuitionistic fuzzy metric. Then for any t>0. $M_{M,N}(x,y,t)$ is non-decreasing with respect to t, in $(L^*\leq_{L^*})$, for all $x,y \in X$.

Lemma2.11 [9] Let (X, $M_{M,N}$,T) be a modified IFMS. Then $M_{M,N}$ is a continuous function on X×X×(0,∞).

Definition2.12 Let f and g be mapping from a modified IFMS (X,$M_{M,N}$,T) into itself. Then the mapping are said to be compatible if $\lim_{n\to\infty}M_{M,N}(fgx_n, gfx_n,t)=1_L^*$, for all t>0 Whenever $\{x_n\}$ is a sequence in X such that , $\lim_{n\to\infty}fx_n=\lim_{n\to\infty} gx_n=x \in X$

Definition2.13 Let f and g be mapping from a modified IFMS (X,$M_{M,N}$,T) into itself.Then the mapping are said to noncomptible there exist at least one sequence $\{x_n\}$ in X such that $\lim_n\to\infty fx_n=\lim_n\to\infty gx_n=x\in X$ but $\lim_{n\to\infty}M_{M,N}(fgx_n,gfx_n,t) \neq 1_L^*$ or nonexistent for at least one t>0 .

Definition2.14 Let f and g be self mappings of a nonempty set X. Then the mappings are said to be weak compatible if they commute at their coincidence point. i.e. fx=gx implies fgx=gfx.

Definition2.15 Let f and g be two self mappings of a modified IFMS (X,$M_{M,N}$,T).We say that f and g have the property (E.A.) if there exists a sequence $\{x_n\}$ in X such that for all t>0 $\lim_n\to\infty M_{M,N}(fx_n,u,t)=\lim_{n\to\infty}M_{M,N}(gx_n,u,t)=1_L^*$,for some u $\in X$ and t>0.

Definition2.16[14] Two mapping $f: X \to X$ *and* $g: X \to X$ are said to satisfy property (CLRg) if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}f(x_n)=\lim_{n\to\infty}g(x_n) = g(p)$, for some p in X.
Similarly , we can have the property CLR_T and the property CLR_S if in the definition (2.16) the mapping $g: X \to X$ has been replaced by the mapping T: $X \to X$ and S: $X \to X$ respectively.

Example2.17 Let (X,$M_{M,N}$,T) be a modified IFMS. where x=R and $M_{M,N}(x,y,t)=(\frac{t}{t+|x-y|},\frac{|x-y|}{t+|x-y|})$ for all t>o and x,y $\in X$.Define self-mappings f and g on X as follows:
fx=2x+1 and gx=x+2.
Consider the sequence $\{x_n=1+1/n, n=1,2,\ldots\}$. Thus we have
$\lim_{n\to\infty} M_{M,N}(fx_n,3, t) = \lim_{n\to\infty} M_{M,N}(gx_n,3, t) =1_L^*$, for every t>0
Then f and g share the property (E.A.).

Definition2.18 Two pairs (f,S) and (g,T) of self mappings of a modified IFMS (X ,$M_{M,N}$,T) are said to satisfy the common property (E.A.) if there exist two sequence $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty}M_{M,N}(fx_n,u,t)= \lim_{n\to\infty}M_{M,N}(Sx_n,u,t) = \lim_{n\to\infty}M_{M,N}(gy_n,u,t)= \lim_{n\to\infty}M_{M,N}(Ty_n,u,t)=1_L^*$,for some u $\in X$ and t>0.

Implicit Relatoins 2.19 Let $\phi:[0,1]^4\to[0,1]^4$ be continuous and non increasing in second ,third and fourth argument respectively and $\Phi(u,v,v,v)\geq_L{}^*0$, implies that u≥v.

Example2.20 Define $\Phi(t_1,t_2,t_3,t_4)$: $(L^*)^4\to L^*$ as $\Phi(t_1,t_2,t_3,t_4)$= t- a $\min\{t_2,t_3,t_4\}$.
Where a>1.

III. MAIN RESULTS

Theorem 3.1 Let F,G,S and T be four self-mapping of a fuzzy metric space(X,$M_{M,N}$,T) satisfying the following conditions:
(i) Let F,G,S and T,be self- mapping of a modified IFMS $(X, M_{M,N}, t)$ for all natural number 'r' the condition:

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$$\emptyset\begin{pmatrix} M_{M,N}(Fx,Gy,kt), min\left(r+M_{M,N}(Gy,Ty,t), M_{M,N}(Sx,Gy,t), M_{M,N}(Sx,Ty,t)\right), \\ M_{M,N}(Sx,Ty,t), M_{M,N}(Fx,Ty,t) \end{pmatrix} \geq_{L^*} 0$$

(II) $F(X) \subseteq T(X)$ and the pair (F,S) satisfies CLR_S property.
(iii) $G(X) \subseteq S(X)$ and the pair (G,T) satisfies CLR_Tproperty.
(iv) S(x) and T(x) are closed subsets of X.
The pairs $(F,S)$and (G,T)have a coincidence point , more over F,G,S and T have a unique common fixed point in X. Provided the pair (F,S) and (G,T) are weakly compatible.

Proof: The pair (F,S) satisfy CLR_S property therefore

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Sx_n = Sx$$

$F(X) \subseteq T(X)$ *therefore* a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Ty_n$ =Sx

Now we show that $\lim_{n\to\infty} Gy_n$ =Sx.Now let $\lim_{n\to\infty} Gy_n$ = z .We have

$$\emptyset\begin{pmatrix} M_{M,N}(Fx,Gy,kt), min\left(r+M_{M,N}(Gy,Ty,t), M_{M,N}(Sx,Gy,t), M_{M,N}(Sx,Ty,t)\right), \\ M_{M,N}(Sx,Ty,t), M_{M,N}(Fx,Ty,t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Fx_{n,}Gy_{n,}kt), min\left(r+M_{M,N}(Gyn,Tyn_{,}t), M_{M,N}(Sx_{n,}Gy_{n,}t), M_{M,N}(Sx_{n,}Ty_{n,}t)\right) \\ , M_{M,N}(Sx_{n,}Ty_{n,}t), M_{M,N}(Fx_{n,}Ty_{n,}t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Sx,z,kt), min\left(r+M_{M,N}(z,z,t), M_{M,N}(Sx,z,t), M_{M,N}(Sx,Sx,t)\right) \\ , M_{M,N}(Sx,Sx,t), M_{M,N}(Sx,Sx,t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Sx,z,kt), min(r+1, M_{M,N}(Sx,z,t),1)1,1\right) \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Sx,z,kt), M_{M,N}(Sx,z,t)1,1\right) \geq L*0$$

Since ϕ is non increasing in third and fourth argument therefore

$$\emptyset\left(M_{M,N}(Sx,z,kt), M_{M,N}(Sx,z,t)M_{M,N}(Sx,z,t), M_{M,N}(Sx,z,t)\right) \geq_{L^*} 0$$

Therefore by implicit relation

$$M_{M,N}(Sx,z,kt) \geq M_{M,N}(Sx,z,t)$$

Hence

$$Sx = z$$

As S(x) is a closed subset of X. $\lim_{n\to\infty} Sx_n$ = z ∈ $S(X)$ there exist a, $u \in X$ such that $Su = z$ put x= u and y=y_n in (i)

$$\emptyset\begin{pmatrix} M_{M,N}(Fu,Gy_n,kt), min\left(r+M_{M,N}(Gy_n,Ty_n,t), M_{M,N}(Su,Gy,t), M_{M,N}(Su,Ty_n,t)\right), \\ M_{M,N}(Su,Ty_n,t), M_{M,N}(Fu,Ty_n,t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Fu,z,kt), min\left(r+M_{M,N}(z,z,t), M_{M,N}(z,z,t), M_{M,N}(z,z,t)\right), \\ M_{M,N}(z,z,t), M_{M,N}(Fu,z,t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Fu,z,kt), min(r+1,1,1),1, M_{M,N}(Fu,z,t)\right) \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Fu,z,kt),1,1, M_{M,N}(Fu,z,t)\right) \geq_{L^*} 0$$

Since ϕ is non increasing in third and fourth argument therefore

$$\emptyset\left(M_{M,N}(Fu,z,kt), M_{M,N}(Fu,z,t), M_{M,N}(Fu,z,t), M_{M,N}(Fu,z,t)\right) \geq_{L^*} 0$$

Therefore by implicit relation

$$M_{M,N}(Fu,z,kt) \geq M_{M,N}(Fu,z,t)$$

hence

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Fu=z=su

Since T(X) is a closed subset of X. $\lim_{n\to\infty} Ty_n$ = z there exist a, $w \in X$ such that Tw=z. Now we claim that $M_{M,N}(Gw, z, t) = 1$.Put $x = x_n$ and y =w in (i)

$$\emptyset\begin{pmatrix} M_{M,N}(Fx_n, Gw, kt), min\left(r + M_{M,N}(Gw, Tw, t), M_{M,N}(Sx_n, Gw, t), M_{M,N}(Sx_n, Tw, t)\right), \\ M_{M,N}(Sx_n, Tw, t), M_{M,N}(Fx_n, Tw, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(z, Gw, kt), min\left(r + M_{M,N}(Gw, z, t), M_{M,N}(z, Gw, t), M_{M,N}(z, z, t)\right), \\ M_{M,N}(z, z, t), M_{M,N}(z, z, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\big(M_{M,N}(z, Gw, kt), min\big(r + M_{M,N}(Gw, z, t), M_{M,N}(z, Gw, t)1\big), 1,1\big) \geq_{L^*} 0$$

$$\emptyset\big(M_{M,N}(z, Gw, kt), M_{M,N}(Gw, z, t), 1,1\big) \geq L * 0$$

Since ϕ is non increasing in third and fourth argument therefore

$\emptyset\left(M_{M,N}(z, Gw, kt), M_{M,N}(Gw, z, t), M_{M,N}(Gw, z, t), M_{M,N}(Gw, z, t)\right) \geq_{L^*} 0$

Therefore by implicit relation we have

$$M_{M,N}(z, Gw, kt) \geq M_{M,N}(Gw, z, t)$$

hence

$$z = Gw.$$

Since Fu=Su and (F, S) is weakly compatible $Fz = FSu = SFu = Sz$

We assert that $M_{M,N}(Fz, z, t) = 1$.Put x = z and y =w in (i)

$$\emptyset\begin{pmatrix} M_{M,N}(Fz, Gw, kt), min\left(r + M_{M,N}(Gw, Tw, t), M_{M,N}(Sz, Gw, t), M_{M,N}(Sz, Tw, t)\right), \\ M_{M,N}(Sz, Tw, t), M_{M,N}(Fz, Tw, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Fz, z, kt), min\left(r + M_{M,N}(z, z, t), M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t)\right), \\ M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Fz, z, kt), min\left(r + 1, M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t)\right), M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t)\right) \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Fz, z, kt), M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t), M_{M,N}(Fz, z, t)\right) \geq_{L^*} 0$$

Therefore $M_{M,N}(Fz, z, kt) \geq M_{M,N}(Fz, z, t)$

Fz=z

Also Gw=Tw and the pair (G, T) is weakly compatible therefore Gz = GTw = TGw =Tz

. We show that z is the common fixed point of the pair pair (G, T).We assert that $M_{M,N}(Gz, z, t) = 1$.

We have put x = u and y = z in (i)

$$\emptyset\begin{pmatrix} M_{M,N}(Fu, Gz, kt), min\left(r + M_{M,N}(Gz, Tz, t), M_{M,N}(Su, Gz, t), M_{M,N}(Su, Tz, t)\right), \\ M_{M,N}(Su, Tz, t), M_{M,N}(Fu, Tz, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(z, Gz, kt), min\big(r + M_{M,N}(Gz, z, t), M_{M,N}(z, Gz, t), 1\big) \\ , M_{M,N}(z, Gz, t), M_{M,N}(z, Gz, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(z, Gz, kt), M_{M,N}(z, Gz, t), M_{M,N}(z, Gz, t), M_{M,N}(z, Gz, t) \\ , \end{pmatrix} \geq_{L^*} 0$$

Therefore $M_{M,N}(z, Gz, kt) \geq M_{M,N}(Gz, z, t)$

Hence z is the common fixed point.

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Theorem 3.2 Let f,g,S and T be self mappings of a modified IFMS (X,M,T) satisfying the conditions(i-iv) of theorem 3.1.suppose that

(v) S(X) (orT(X)) is a closed subset of X.

Then the pairs (f,S) and (g,T) have a coincidence point. Moreover, f,g,S and T have a unique common fixed point in X provided that the pairs (f,S) and (g,T) are weakly compatible.

Proof:

The pair (F, S) satisfy CLR_S property therefore

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Sx_n = Sx$$

$F(X) \subseteq T(X)$ *therefore* a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Ty_n$ =Sx

Now we show that $\lim_{n\to\infty} Gy_n$ =Sx.Now let $\lim_{n\to\infty} Gy_n$ = z .We have

$$\emptyset\begin{pmatrix} M_{M,N}(Fx, Gy, kt), min\left(r + M_{M,N}(Gy, Ty, t), M_{M,N}(Sx, Gy, t), M_{M,N}(Sx, Ty, t)\right), \\ M_{M,N}(Sx, Ty, t), M_{M,N}(Fx, Ty, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Fx_{n,}Gy_{n,}kt), min\left(r + M_{M,N}(Gyn, Tyn_{,}t), M_{M,N}(Sx_{n,}Gy_{n,}t), M_{M,N}(Sx_{n,}Ty_{n,}t)\right) \\ , M_{M,N}(Sx_{n,}Ty_{n,}t), M_{M,N}(Fx_{n,}Ty_{n,}t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\begin{pmatrix} M_{M,N}(Sx, z, kt), min\left(r + M_{M,N}(z, z, t), M_{M,N}(Sx, z, t), M_{M,N}(Sx, Sx, t)\right) \\ , M_{M,N}(Sx, Sx, t), M_{M,N}(Sx, Sx, t) \end{pmatrix} \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Sx, z, kt), min\left(r + 1, M_{M,N}(Sx, z, t), 1\right)1,1\right) \geq_{L^*} 0$$

$$\emptyset\left(M_{M,N}(Sx, z, kt), M_{M,N}(Sx, z, t)1,1\right) \geq L * 0$$

Since ϕ is non increasing in third and fourth argument therefore

$$\emptyset\left(M_{M,N}(Sx, z, kt), M_{M,N}(Sx, z, t)M_{M,N}(Sx, z, t), M_{M,N}(Sx, z, t)\right) \geq_{L^*} 0$$

Therefore by implicit relation

$$M_{M,N}(Sx, z, kt) \geq M_{M,N}(Sx, z, t)$$

Hence

$$Sx = z$$

As S(x) is a closed subset of X.Therefore

$$\lim_{n\to\infty} Sx_n = z$$

As S(X) is a closed subset of X.Then show that the pairs(f,S) has a coincidence point say u,i.e. fu=Su. Since fu $\in f(X)$ and f(X)⊂ T(X),there exists w $\in X$ such that fu=Tw. Now we assert that $M_{M,N}$(gw,z,t)=1.If not then using inequality (i) ,we have

$$\emptyset\begin{pmatrix} M_{M,N}(Fx_n, Gw, kt), min\left(r + M_{M,N}(Gw, Tw, t), M_{M,N}(Sx_n, Gw, t), M_{M,N}(Sx_n, Tw, t)\right), \\ M_{M,N}(Sx_n, Tw, t), M_{M,N}(Fx_n, Tw, t) \end{pmatrix} \geq_{L^*} 0$$

making n→∞,reduces to

$$\emptyset\ (M_{M,N}(z,Gw,t),1,1,M_{M,N}(Gw,z,t),)\geq_L{}^* 0$$

This implies

$$M_{M,N}(z, Gw, kt) \geq M_{M,N}(z, Gw, t)$$

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Therefore w is a coincidence point of the pair(G,T) .

Example 3.3 Let $(X,M_{M,N},T)$ be an intuitionistic fuzzy metric space, where in X-(0,128), $T(a,b)=\{(a_1b_1,\min\{a_1+b_2,1\})$ for all $a=(a_1,a_2)$ and $b=(b_1,b_2) \in L^*$ with

$M_{M,N}(x,y,t)=\left(\frac{t}{t+|x-y|},\frac{|x-y|}{t+|x-y|}\right)$, define self mappings f, g ,S and T on X by

$$f(x)=\begin{cases}2x4-1 \text{ if } 1\le x\le 2\\ 7, \text{ if otherwise}\end{cases} \quad \text{and } g(x)=\begin{cases}2x6-1 \text{ if } 1\le x\le 2\\ 3, \text{ if otherwise}\end{cases}$$

$$S(x)=\begin{cases}x2 \text{ if } 1\le x\le 2\\ 2 \text{ if otherwise}\end{cases} \quad \text{and } T(x)=\begin{cases}x3 \text{ if } 1\le x\le 2\\ 4 \text{ if otherwise}\end{cases}$$

Define $F(t_1,t_2,t_3,t_4)=t_{1-}\ \Psi[\min\{t_2,t_3,t_4\}]$,where $\Psi(s)>_L{}^* s$ for all $s \in L^*\setminus\{0,1\}$ and $F \in\Psi$.

Now, for all x, y ∈ X and t>0, we have

$$\Psi[\{M_{M,N}(Sx,Ty,t),M_{M,N}(fx,Sx,t),M_{M,N}(gy,Ty,t),$$
$$M_{M,N}(Sx ,gy, t),M_{M,N}(fx ,Ty,t)\}]\le_L{}^* \ M_{M,N}(fx , gy,t).$$

Which demonstrates the verification of the esteemed implicit function.

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