

Five Dimensional Exact Solutions of Bianchi Type-I Space-Time in $f(R,T)$ Theory of Gravity

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ABSTRACT: In this paper, we have obtained two five dimensional exact solutions of Bianchi type-I space-time in $f(R,T)$ theory of gravity using assumption of constant deceleration parameter and variation law of Hubble parameter which correspond to two different cosmological models. The first solution yields a singular model for $n \neq 0$ while the second gives a non-singular model for $n = 0$. The physical behavior for both the models have been discussed using some physical quantities..

KEYWORDS: $f(R,T)$ theory of gravity, five dimensional Bianchi type-I space-time, Vacuum field equations.

I. INTRODUCTION

Einstein general relativity is the basis to explain most of the gravitational phenomena but it does not seem to resolve some of the important problems in cosmology such as the accelerating expansion of the universe. It is now proved from observational and theoretical fact that our universe is in the phase of accelerated expansion. In order to explain the accelerated expansion, number of cosmological models have been proposed by different authors. $f(T)$ theory of gravity where T is the scalar torsion has been proposed to explain current accelerated expansion without involving dark energy. Ratbay M. [1] has shown that the acceleration of the universe can be understood by $f(T)$ gravity models. M. Sharif et al [2] considered spatially homogeneous and anisotropic Bianchi type-I universe in $f(T)$ gravity theory. Wei H. et al [3] tried to constrain $f(T)$ theories with the fine structure constant. Bamba K. et al [4] studied the cosmological evolution of the equation of state for dark energy with the combination of exponential, logarithmic and $f(T)$ theories. $f(R)$ theory of gravity is the another example of gravity. Many authors have investigated $f(R)$ gravity in different context. Carrol et al. [5] explained the presence of late time cosmic acceleration of the universe in $f(R)$ gravity. Nojiri and Odintsov [6,7] proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflation and late time acceleration. Bertolami et al [8] have proposed a generalization of $f(R)$ modified theory of gravity, by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m . As a result of coupling the motion of the massive particles is non-geodesic and extra force orthogonal to the four velocity arises. Multamaki and Vilja [9,10] investigated static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions by taking fluid respectively. Capozziello et al [11] used Noether symmetries to study spherically symmetric solutions in $f(R)$ theory of

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gravity. Sharif and Shamir [12] studied exact vacuum solutions in Bianchi type-I and V space-times in $f(R)$ theory of gravity. Shamir [13] discussed the plane symmetric vacuum Bianchi type-III cosmology in $f(R)$ gravity. Aktas et al [14] have studied anisotropic models in $f(R)$ gravity. Adhav [15] discussed the Kantowski-sachs string cosmological model in $f(R)$ theory. Reddy et al [16] studied vacuum solutions of Bianchi type- I & V models in $f(R)$ gravity with a special form of deceleration parameter.

II. RELATED WORK

Harko et. al. [17] proposed a new generalized theory known as $f(R, T)$ theory of gravity. Gravitational Lagrangian involves the arbitrary function of the scalar Curvature R and the trace of the energy momentum tensor T . Adhav K.S. [18] studied the exact solution of $f(R, T)$ field equations for locally rotationally symmetric Bianchi type-I space time. Houndjo [19] reconstructed $f(R, T)$ gravity by taking $f(R, T) = f_1(R, T) + f_2(T, R)$. M. Farasat Shamir et al [20] obtained the exact solutions of Bianchi types-I & V models in $f(R, T)$ by using the assumption of constant deceleration parameter and variation of law of Hubble parameter. Reddy D. R. K. [21] discussed the LRS Bianchi type-II universe in $f(R, T)$ theory.

Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. There is nothing in the equation of relativity which restricts them to four dimensions. Kaluza and Klein [22, 23] have done remarkable work by introducing an idea of higher dimension space-time. Many researchers inspired to enter into the field of higher dimension theory to explore knowledge of universe. Wesson [24, 25] and D.R. K. Reddy [26] have studied several aspects of five dimensional space-time in variable mass theory and bi-metric theory of relativity respectively. Lorentz and Petzold [27], Ibanez and Verdager [28], Khadekar and Gaikwad [29], Adhav et al [30] have studied the multidimensional cosmological models in general relativity and in other alternative theories of gravitation. Ladke L. S. [31] studied the Bianchi type-I (Kasner form) cosmological model in $f(R)$ theory of gravity in five dimensions.

Motivated with the above research work we have obtained five dimensional exact solutions of Bianchi type-I space-time assuming constant deceleration parameter and variation law of Hubble's parameter proposed by Berman M. S. [32] which corresponds to two different cosmological models. Physical aspects of both the models are also discussed.

III. FIVE DIMENSIONAL FIELD EQUATIONS IN $f(R, T)$ THEORY OF GRAVITY

The action for $f(R, T)$ in five dimensions is given by

$$S = \int \left(\frac{1}{16\pi G} f(R, T) + L_m \right) \sqrt{-g} d^5x, \quad (1)$$

Where $f(R, T)$ is an arbitrary function of Ricci scalar R and T is trace energy momentum tensor of matter T_{ij} , L_m is matter lagrangian density.

The five dimensional field equations in $f(R, T)$ theory of gravity are given by

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$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij} \square) f(R,T) = kT_{ij} - f_T(R,T)(T_{ij} + \theta_{ij}) \quad (i, j = 1, 2, \dots, 5), \quad (2)$$

where $f_R(R,T) \equiv \frac{\partial f_R(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f_T(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} L_m)}{\partial g^{ij}}$, $\theta_{ij} = -pg_{ij} - 2T_{ij}$

$\square \equiv \nabla^i \nabla_i$, ∇_i is the covariant derivative.

The energy momentum tensor for perfect fluid yields

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (3)$$

where ρ and p are energy density and pressure of the fluid respectively.

Contracting the above field equations (2), we have

$$f_R(R,T)R + 4\square f_R(R,T) - \frac{5}{2}f(R,T) = kT - f_T(R,T)(T + \theta), \quad (4)$$

Also above field equations (2), take the form

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij} \square) f_R(R,T) = kT_{ij} + f_T(R,T)(T_{ij} + pg_{ij}), \quad (5)$$

Harko set. al. [17] gives three class of models out of which we used $f(R,T) = R + 2f(T)$ for this models equation (5) can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = kT_{ij} + 2f'(T)T_{ij} + [f(T) + 2pf'(T)]g_{ij}, \quad (6)$$

where overhead prime denotes derivative w.r.to. T .

We also choose

$$f(T) = \lambda T, \text{ where } \lambda \text{ is constant.} \quad (7)$$

IV. EXACT SOLUTIONS OF BIANCHI TYPE-I SPACE TIME IN V_5

In this section we find exact solutions of five dimensional Bianchi type-I space time in $f(R)$ theory of gravity. The line element of Bianchi type-I space-time in V_5 is given by

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$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2) - C^2 (dz^2 + du^2), \tag{8}$$

where A, B and C are functions of t only.

The corresponding Ricci scalar is

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2 \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2 \frac{\dot{B}\dot{C}}{BC} + 2 \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C^2} \right], \tag{9}$$

where overhead dot means derivative with respect to t .

From equation (6), we have obtain field equations as

$$\frac{\dot{A}\dot{B}}{AB} + 2 \frac{\dot{B}\dot{C}}{BC} + 2 \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C^2} = (10\pi + 3\lambda)\rho - 2\lambda p, \tag{10}$$

$$\frac{\ddot{B}}{B} + 2 \frac{\ddot{C}}{C} + 2 \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}^2}{C^2} = \lambda\rho - (10\pi + 4\lambda)p, \tag{11}$$

$$\frac{\ddot{A}}{A} + 2 \frac{\ddot{C}}{C} + 2 \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C^2} = \lambda\rho - (10\pi + 4\lambda)p, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \lambda\rho - (10\pi + 4\lambda)p, \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \lambda\rho - (10\pi + 4\lambda)p. \tag{14}$$

The left hand side of equation (13) and (14) are identical because of the metric function C is common along z and u directions. The system of these four non-linear differential equations consist of five undefined functions i.e. A, B, C, p and ρ . Hence to find deterministic solution one more condition is necessary, so we consider well known relation between Hubble parameter H and average scale factor a given as

$$H = la^{-n}, \text{ where } l > 0 \text{ and } n \geq 0 \tag{15}$$

Subtracting equation (11) from equation (12), equation (12) from equation (13), equation (11) from equation (13), we have

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$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{2\ddot{C}}{C} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0, \quad (16)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0, \quad (17)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) = 0. \quad (18)$$

On solving above equations, we get

$$\frac{B}{A} = d_1 \exp \left[c_1 \int \frac{dt}{a^4} \right], \quad (19)$$

$$\frac{C}{B} = d_2 \exp \left[c_2 \int \frac{dt}{a^4} \right], \quad (20)$$

$$\frac{A}{C} = d_3 \exp \left[c_3 \int \frac{dt}{a^4} \right], \quad (21)$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy the relation

$$c_1 + c_2 + c_3 = 0, \quad d_1 d_2 d_3 = 1. \quad (22)$$

Using equation (19), (20) and (21), the metric functions are

$$A = ap_1 \exp \left[q_1 \int \frac{dt}{a^4} \right], \quad (23)$$

$$B = ap_2 \exp \left[q_2 \int \frac{dt}{a^4} \right], \quad (24)$$

$$C = ap_3 \exp \left[q_3 \int \frac{dt}{a^4} \right], \quad (25)$$

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Where $p_1 = (d_1^{-3} d_2^{-2})^{1/4}$, $p_2 = (d_1 d_2^{-2})^{1/4}$, $p_3 = (d_1 d_2^2)^{1/4}$, (26)

and $q_1 = -\frac{3c_1 + 2c_2}{4}$, $q_2 = \frac{c_1 - 2c_2}{4}$, $q_3 = \frac{c_1 + 2c_2}{4}$, (27)

satisfying the relations

$$p_1 p_2 p_3^2 = 1, \quad q_1 + q_2 + 2q_3 = 0. \tag{28}$$

V. SOME IMPORTANT PHYSICAL QUANTITIES

In this section we define some important physical quantities.

The average scale factor and the volume scale factor are defined respectively as under

$$a = (ABC^2)^{\frac{1}{4}}, \quad V = a^4 = ABC^2. \tag{29}$$

The generalized mean Hubble parameter H is defined by

$$H = (\ln a)_t = \frac{\dot{a}}{a} = \frac{1}{4} [H_1 + H_2 + H_3 + H_4], \tag{30}$$

Where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = H_4 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z and u axes respectively.

The mean anisotropy parameter \bar{A} is given by

$$\bar{A} = \frac{1}{4} \sum_{i=1}^4 \left(\frac{\Delta H_i}{H} \right)^2, \tag{31}$$

where $\Delta H_i = H_i - H$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u_{;i}^i = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{2\dot{C}}{C}, \tag{32}$$

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$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \tag{33}$$

Where $\sigma_{ij} = \frac{1}{2} [\nabla_j u_i + \nabla_i u_j] - \frac{1}{4} \theta g_{ij}$, (34)

From equation (15) and (30), we have

$$\dot{a} = la^{1-n}, \tag{35}$$

After integrating equation (35), we have

$$a = (nlt + k_1)^{1/n}, \quad n \neq 0 \tag{36}$$

and $a = k_2 \exp(lt), \quad n = 0,$ (37)

where k_1 and k_2 are constants of integration.

Thus we have two values of the average scale factors which correspond to two different models of the universe.

VI. FIVE DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n \neq 0$

In this section we study the five dimensional model of the universe for $n \neq 0$. For this singular model average scale factor a given as $a = (nlt + k_1)^{1/n}$

The metric coefficients A, B and C turn out to be

$$A = p_1 (nlt + k_1)^{1/n} \exp \left[\frac{q_1 (nlt + k_1)^{\frac{n-4}{n}}}{l(n-4)} \right], \quad n \neq 4 \tag{38}$$

$$B = p_2 (nlt + k_1)^{1/n} \exp \left[\frac{q_2 (nlt + k_1)^{\frac{n-4}{n}}}{l(n-4)} \right], \quad n \neq 4 \tag{39}$$

$$C = p_3 (nlt + k_1)^{1/n} \exp \left[\frac{q_3 (nlt + k_1)^{\frac{n-4}{n}}}{l(n-4)} \right], \quad n \neq 4 \tag{40}$$

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The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt + k_1}, V = (nlt + k_1)^{4/n}. \tag{41}$$

The mean anisotropy parameter \bar{A} turns out to be

$$\bar{A} = \frac{q_1^2 + q_2^2 + 2q_3^2}{4l^2(nlt + k_1)^{(8-2n)/n}}. \tag{42}$$

The deceleration parameter q in cosmology is the measure of the cosmic accelerated expansion of the universe and is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = n - 1, \tag{43}$$

which is a constant.

A positive sign of q , i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q , i.e., $0 < n < 1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q = 0$, i.e., $n = 1$. Also, recent observations of SN Ia, reveal that the present Universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$.

The expansion θ and shear scalar σ^2 are given by

$$\theta = \frac{4l}{nlt + k_1} \text{ and } \sigma^2 = \frac{q_1^2 + q_2^2 + 2q_3^2}{2(nlt + k_1)^{8/n}}, \tag{44}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[\begin{array}{l} 2(3\lambda + 10\pi) \left\{ \frac{6l^2}{(nlt + k_1)^2} + \frac{q_1q_2 + 2q_2q_3 + 2q_3q_1 + q_3^2}{(nlt + k_1)^{8/n}} \right\} \\ -3\lambda \left\{ \frac{4l^2(1-n)}{(nlt + k_1)^2} + \frac{q_1^2 + q_2^2 + 2q_3^2}{(nlt + k_1)^{8/n}} \right\} \end{array} \right], \tag{45}$$

The pressure of the universe becomes

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$$p = \frac{-1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[(\lambda + 10\pi) \left\{ \frac{6l^2}{(nlt + k_1)^2} + \frac{q_1q_2 + 2q_2q_3 + 2q_3q_1 + q_3^2}{(nlt + k_1)^{8/n}} \right\} + \frac{3}{2}(3\lambda + 10\pi) \left\{ \frac{4l^2(1-n)}{(nlt + k_1)^2} + \frac{q_1^2 + q_2^2 + 2q_3^2}{(nlt + k_1)^{8/n}} \right\} \right], \tag{46}$$

VII. FIVE DIMENSIONAL MODEL OF THE UNIVERSE WHEN $n = 0$.

In this section we study the five dimensional model of the universe for $n = 0$.

For this non-singular model average scale factor a given as $a = k_2 \exp(lt)$

Here the metric coefficients take the form

$$A = p_1 k_2 \exp(lt) \exp \left[-\frac{q_1 \exp(-4lt)}{4lk_2^4} \right], \tag{47}$$

$$B = p_2 k_2 \exp(lt) \exp \left[-\frac{q_2 \exp(-4lt)}{4lk_2^4} \right], \tag{48}$$

$$C = p_3 k_2 \exp(lt) \exp \left[-\frac{q_3 \exp(-4lt)}{4lk_2^4} \right]. \tag{49}$$

The mean generalized Hubble parameter becomes

$$H = l, \tag{50}$$

while the volume scale factor turns out to be

$$V = k_2^4 \exp(4lt), \tag{51}$$

The mean anisotropy parameter \bar{A} becomes

$$\bar{A} = \left[\frac{q_1^2 + q_2^2 + 2q_3^2}{4l^2 k_2^8} \right] \exp(-8lt), \tag{52}$$

while the quantizes θ and σ^2 are given by

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$$\theta = 4l \text{ and } \sigma^2 = \left[\frac{q_1^2 + q_2^2 + 2q_3^2}{2k_2^8} \right] \exp(-8lt) , \tag{53}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[\begin{array}{l} 2(3\lambda + 10\pi) \left\{ 6l^2 + \frac{q_1q_2 + 2q_2q_3 + 2q_3q_1 + q_3^2}{k_2^8 \exp(8lt)} \right\} \\ - 3\lambda \left\{ 4l^2 + \frac{q_1^2 + q_2^2 + 2q_3^2}{k_2^8 \exp(8lt)} \right\} \end{array} \right] , \tag{54}$$

The pressure of the universe becomes

$$p = \frac{-1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[\begin{array}{l} (\lambda + 10\pi) \left\{ 6l^2 + \frac{q_1q_2 + 2q_2q_3 + 2q_3q_1 + q_3^2}{k_2^8 \exp(8lt)} \right\} \\ + \frac{3}{2} (3\lambda + 10\pi) \left\{ 4l^2 + \frac{q_1^2 + q_2^2 + 2q_3^2}{k_2^8 \exp(8lt)} \right\} \end{array} \right] . \tag{55}$$

VIII. CONCLUSION

In this paper, we have studied the expansion of universe and obtained two five dimensional exact solutions of Bianchi type-I space time in $f(R,T)$ theory of gravity using assumption of constant deceleration parameter and variation law of Hubble parameter. These solutions lead to two different models of the universe. The first solution represents to a singular model for $n \neq 0$ with power law expansion and second solution gives a non-singular model for $n = 0$ with exponential expansion of the universe. The terms of cosmological importance for both the models are discussed below.

i) five dimensional Singular model of the universe for $n \neq 0$

The model has a point singularity at $t = -\frac{k_1}{nl}$. The metric coefficients A, B, C and volume scale factor V vanish at this point of singularity. The mean generalized Hubble parameter H , expansion scalar θ , shear scalar σ^2 and mean anisotropy parameter \bar{A} are all infinite at this point of singularity. Above observational data suggest that universe starts its expansion with zero volume and it will continue to expand.

ii) five dimensional Non-singular model of the universe for $n = 0$

For this model average scale factor $a = k_2 \exp(lt)$. This five dimensional model of the universe is non-singular because of exponential behavior of the model and there is no singularity. Mean generalized Hubble parameter H and expansion scalar θ are constant. Shear scalar σ^2 and anisotropy parameter \bar{A} are finite for finite values of t . The metric

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coefficients A, B, C and volume of the universe increase exponentially with the cosmic time. This shows that the universe expansion take place with zero volume from infinite past. It is seen that, the result obtained here are similar to the results obtained earlier by M. Farasat Shamir et. al (20).

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