

ELASTICITY OF INTERNET TRAFFIC DISTRIBUTION IN COMPUTER NETWORK IN TWO MARKET ENVIRONMENT

Diwakar Shukla*¹, Sharad Gangele², Kapil Verma³ and Pankaja Singh⁴

*1 Deptt. of Mathematics and Statistics, Sagar University, Sagar M.P. 470003, India.
diwakarshukla@rediffmail.com¹

²Deptt. of Computer Science, M. P. Bhoj (Open) university, Kolar Road, Bhopal, M.P, India.
sharadgangele@gmail.com²

³ Deptt. of Computer Science, M. P. Bhoj (Open) university, Kolar Road, Bhopal, M.P, India.
kapil_mca100@rediffmail.com³

⁴ Deptt. of Physics & Computer Science, Govt. Motilal Vigyan Mahavidhyalaya Bhopal, M.P.
pankajasingh@gmail.com⁴

Abstract: The Internet service is managed by operators and each one tries to capture larger proportion of Internet traffic. This tendency causes inherent competition in the market. The location of the market is also an important factor. This paper assumes two different markets and two operators are in competition. It is found that elasticities value depend on market position. The priority position market has higher level. This paper present Elasticities analysis of traffic sharing pattern among operators. Simulation study is performing to analyze the Elasticities impact on traffic sharing.

Keywords: Markov chain model, Transition probability, Initial preference, Blocking probability, Call-by-call basis, Internet service providers [operators or ISP], Quality of service (QOS), Transition probability matrix.

INTRODUCTION

We assume a situation that there are two markets situated at distant apart in a city. Both the markets have Internet café with connection of two operators, O_u ($u=1,3$) and O_v ($v=2,4$). A user has a choice to pickup one market based on his liking and then selected the favourite operators in the Internet café. Both operators are in competitions to occupy more and more proportion of internet users. The network of both operators is suffering from blocking. The matter of interest is to know how blocking probability affects the customer proportion in the setup of two markets. Elasticities means rate of change of one variable with respect to other when many other parameters are kept constant. The traffic sharing by two operators is a variable and needs to examine in the light of Elasticities. This paper presents Elasticities based analysis of internet traffic sharing in multi operator and multi markets environment.

A REVIEW

Shukla *et al.* (2007) discussed analysis of internet traffic distribution between two markets using Markov chain model in computer networks. This contribution has initiated the problem of traffic sharing in two-market environment. Shukla *et al.* (2009) has extended the above approach by incorporating the share loss analysis of internet traffic distribution. Medhi (1991) discussed the basic fundamentals of Markov chain model. Shukla *et al.* (2009 b) presented all comparison analysis of internet traffic sharing using Markov chain model which is an extension of Naldi (2002). Catledge and Pitkow (1995) discussed a contribution on characterization of browsing strategies. Pirolli and pitkow

(1996) suggested usable structure for web in light of many users. A similar study performed by pitkow (1997) regarding search of reliable usage data on www. Naldi (2001) presented Markov chain model based study in a multioperator environment. The detail distribution of Markov chain model is in Medhi (1991) and web browsing details are in Han and Kamber (2001). Shukla *et al.* (2007) discussed stochastic model for space decision switches for computer network. Shukla *et al.* (2007 a, b, c) suggested the use of Markov chain model in networking and operating system analysis. Shukla and Jain (2007) used Markov chain model for the analysis of multilevel Queue Scheduler in the operating system. Shukla and Singhai (2010 a) discussed traffic share analysis of message flow in three crossbar architecture space division switches. Deshpande & Karypis (2004) discussed selective Markov chain model for predicting webpage access. Shukla *et al.* (2010 a, b, c, d, e, f, g, h) discussed different aspects on Markov chain model in determining the system behavior. Shrivastava *et al.* (2000) presented a thought oriented contribution on web page mining discovery and application of usage patterns from web data.

MARKOV CHAIN MODEL

Let $\{X_n, n \geq 0\}$ be a Markov chain model. As per Fig 3.1, let O_1, O_2, O_3 and O_4 be operators (ISP) in the two competitive Market-I (M_1) and Market-II (M_2). User chooses a market first, and then enters into a cyber-café situated inside. Where computer terminals of different operators are available to access the Internet. Operators are grouped as O_u ($u=1,3$) and O_v ($v=2,4$) for market-I and

market-II Let $\{X^{(n)}, n \geq 0\}$ be a Markov chain having transitions over the state space M_1, M_2 and $\{O_1, O_2, O_3, O_4, Z_1, Z_2, A\}$

State O₁: First operator in market-I,

State O₂: Second operator in market-I,

State O₃: Third operator in market-II,

State O₄: Fourth operator in market-II,

State Z₁: Success (link) in market-I (M_1)

State Z₂: Success (link) in market- II (M_2)

State A: Abandon the attempt process.

The $X^{(n)}$ stands for the state of random variable X at n^{th} attempt of connectivity ($n \geq 0$) made by the user. Some underlying assumptions of the Markov chain model are:

(a) A User (or Customer or CU) first select the Market-I with probability q and Market-II with probability $(1-q)$, (see Fig 3.1)

(b) After choosing a market, User enters in the cyber-café (shop), chooses the first operator O_u with probability p or to O_v with $(1-p)$.

(c) Blocking probability experienced by the operator O_u are L_1 & L_3 and by O_v are L_2 & L_4

(d) The connectivity attempts by user between operators are on call-by-call basis, if the call for O_u is blocked in k^{th} attempt ($k > 0$) then in $(k + 1)^{\text{th}}$ attempt user shifts to O_v . If this also fails, user switches to O_u in $(k+2)^{\text{th}}$.

(e) Whenever call connects through either of operators O_u or O_v , we say system reaches to the state of success in n attempts.

(f) User can terminate the attempt process which is marked as system to the abandon state Z at n^{th} attempts with probability p_A (either O_u or from O_v).

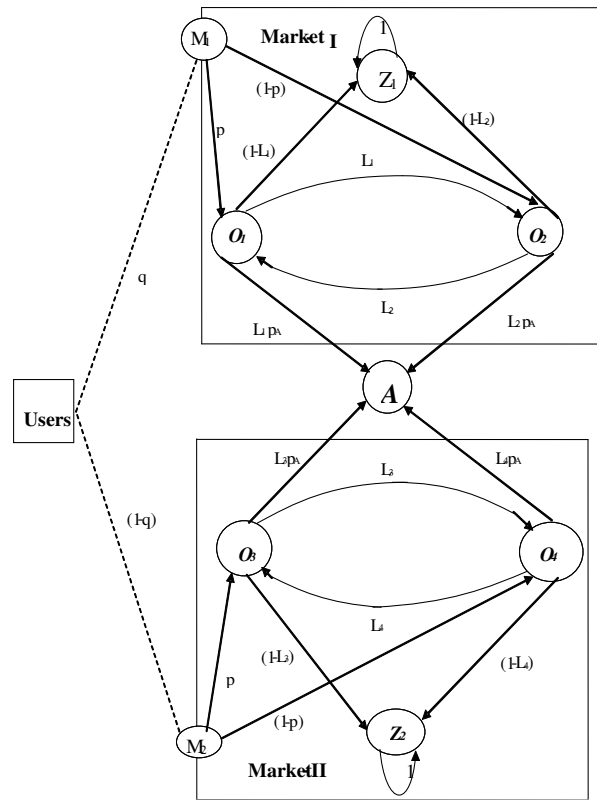


Figure.1 Transition Diagram of model.

Fig.3.1 Explains the transition mechanism with transition probability matrix in (3.1)

	States									
	$X^{(n)}$									
	O_1	O_2	O_3	O_4	Z_1	Z_2	A	M_1	M_2	
$X^{(n-1)}$	O_1	0	$L_1(1-p_A)$	0	0	$1-L_1$	0	$L_1 p_A$	0	0
O_2	$L_2(1-p_A)$	0	0	0	$1-L_2$	0	$L_2 p_A$	0	0	
O_3	0	0	0	$L_3(1-p_A)$	0	$1-L_3$	$L_3 p_A$	0	0	
O_4	0	0	0	$L_4(1-p_A)$	0	$1-L_4$	$L_4 p_A$	0	0	
Z_1	0	0	0	0	1	0	0	0	0	
Z_2	0	0	0	0	0	1	0	0	0	
A	0	0	0	0	0	0	1	0	0	
M_1	p	$1-p$	0	0	0	0	0	0	0	
M_2	0	0	p	$1-p$	0	0	0	0	0	

SOME USEFUL RESULTS FOR n^{th} CONNECTIVITY ATTEMPTS

Theorem 1.0 : The odd and even n^{th} step probability for O_1 in Market –I is:

$$P[X^{(n)} = O_1]_{M_1} = q (1 - p) (1 - p_A) [(1 - p_A)^{n-2}]$$

(Even)

$$P[X^{(n)} = O_1]_{M_1} = q p [(1 - p_A)^{n-1}]$$

(Odd)

Proof: For $n=0$ at M_1 , we have $P[X^{(0)}=M_1]=q$

The start is either from O_1 or from O_2 after choosing M_1 , therefore,

$$P[X^{(1)} = O_1] = p[X^{(0)} = M_1]P[X^{(1)} = O_1 / X^{(0)} = M_1] = qp$$

$$P[X^{(1)} = O_2] = P[X^{(0)} = M_1]P[X^{(1)} = O_2 / X^{(0)} = M_1] = q(1-p)$$

$$P[X^{(2)} = O_1] = P[X^{(0)} = O_2]P[X^{(2)} = O_1 / X^{(1)} = O_2] = q(1-p)(L_2)(1-p_A)$$

$$P[X^{(2)} = O_2] = P[X^{(1)} = O_1]P[X^{(2)} = O_2 / X^{(1)} = O_1] = qpL_1(1-p_A)$$

$$P[X^{(3)} = O_1] = P[X^{(2)} = O_2]P[X^{(3)} = O_1 / X^{(2)} = O_2] = qpL_2(1-p_A)$$

$$P[X^{(3)} = O_2] = P[X^{(2)} = O_1]P[X^{(3)} = O_2 / X^{(2)} = O_1] = q(1-p)L_1L_2(1-p_A)$$

$$P[X^{(4)} = O_1] = P[X^{(3)} = O_2]P[X^{(4)} = O_1 / X^{(3)} = O_2] = q(1-p)L_1L_2^2(1-p_A)^3$$

$$P[X^{(4)} = O_1] = P[X^{(3)} = O_2]P[X^{(4)} = O_1 / X^{(3)} = O_2] = q(1-p)L_1L_2^2(1-p_A)^3$$

$$P[X^{(4)} = O_2] = P[X^{(3)} = O_1]P[X^{(4)} = O_2 / X^{(3)} = O_1] = qpL_1^2L_2(1-p_A)$$

The continuation provides proof of theorem for n^{th} odd and even attempts.

Theorem 1.1: The n^{th} step transitions probability for O_2 in Market -I is:

$$P[X^{(n)} = O_2]_{M_1} = qp(1-p_A)[(1-p_A)^{n-2}] \text{ (Even)}$$

$$p[x^{(n)} = O_2]_{M_1} = q(1-p)[(1-p_A)^{n-1}] \text{ (Odd)}$$

Theorem 1.2: The n^{th} step transitions probability for O_3 in Market-II is:

$$P[X^n = O_3]_{M_2} = (1-q)(1-p)L_4(1-p_A)^{n-2}$$

$$p[X^n = O_3]_{M_2} = (1-q)p[(1-p_A)^{n-1}] \text{ (Odd)}$$

Theorem 1.3: The n^{th} step transitions probability for O_4 in Market-II is:

$$P[X^n = O_4]_{M_2} = (1-q)pL_3(1-p_A)^{n-2}$$

$$P[X^{(n)} = O_4]_{M_2} = (1-q)(1-p)[(1-p_A)^{n-1}] \text{ (Odd)}$$

QUALITY OF SERVICE [QOS]

There are two types of users as:
Faithful User [FU]

A user who is faithful to an operator O_u only otherwise he goes to abandon state but does not attempt for O_v . The converse of it may as he attempts for O_v only and goes to state A otherwise.

Impatient User [IU]

A user who attempts between the two Operators O_u and O_v only all the time until call complete or otherwise abandons the process.

$$B_f = q[pL_1 + (1-p)L_2] + (1-q)[pL_3 + (1-p)L_4]$$

ELASTICITY STUDIES OVER LARGE ATTEMPT

Let p_1 be the traffic sharing by the first operator, p_2 be the traffic sharing by the second operator using Markov chain model using Naldi (2002), Shukla et al. (2007) we can obtain the expressions of traffic sharing as:

$$\bar{p}_{1M_1} = \frac{(1-L_1)q}{1-L_1L_2(1-p_A)^2} [p + (1-p)L_2(1-p_A)]$$

$$\bar{p}_{2M_1} = \frac{(1-L_2)q}{1-L_1L_2(1-p_A)^2} [(1-p) + pL_1(1-p_A)]$$

$$\bar{p}_{3M_2} = \frac{(1-L_1)(1-q)}{1-L_1L_2(1-p_A)^2} [p + (1-p)L_2(1-p_A)]$$

$$\bar{p}_{4M_2} = \frac{(1-L_2)(1-q)}{1-L_1L_2(1-p_A)^2} [(1-p) + pL_1(1-p_A)]$$

If $y=f(x, z)$ is function then elasticity of y with respect to z is

$$\left(\frac{\partial y}{\partial z}\right) \text{ Where } x \text{ is a constant.}$$

Similarly with respect to x is

$$\left(\frac{\partial y}{\partial x}\right) \text{ Where } z \text{ is a constant.}$$

Differentiate with respect to L_1 we get

$$e_{1(.)} = \left(\frac{\partial \bar{p}_{1M_1}}{\partial L_1}\right)_{L_2, p, q, p_A} = \frac{[q\{p + (1-p)L_2(1-p_A)\}]}{\{L_1L_2(1-p_A)^2 - 1\} + \{(1-L_1)L_2(1-p_A)^2\}} \frac{1}{\{1-L_1L_2(1-p_A)^2\}^2}$$

Differentiate with respect to L_2 we get

$$e_2(.) = \left(\frac{\partial \bar{p}_{1.M_1}}{\partial L_2} \right)_{L_1, p, q, p_A} = \frac{\{(-L_1 L_2(1-p_A)^2) \cdot \{(1-p)(1-p_A)\} + \{p+(1-p)L_2(1-p_A)\}\}}{[1-L_1 L_2(1-p_A)^2]^2} \cdot \{L_1(1-p_A)^2\}$$

Differentiate with respect to L_1 we get

$$f_1(.) = \left(\frac{\partial \bar{p}_{2.M_1}}{\partial L_1} \right)_{L_2, p, q, p_A} = \frac{(1-L_2)q\{1-L_1 L_2(1-p_A)^2\}\{(p(1-p_A)\} + \{(1-p)+pL_1(1-p_A)\}\{L_2(1-p_A)^2\}}{[1-L_1 L_2(1-p_A)^2]^2}$$

Differentiate with respect to L_2 we get

$$f_2(.) = \left(\frac{\partial \bar{p}_{2.M_1}}{\partial L_2} \right)_{L_1, p, q, p_A} = \frac{q(1-p)+pL_1(1-p_A)\{[L_1 L_2(1-p_A)^2-1] + (1-L_2)L_1(1-p_A)^2\}}{[1-L_1 L_2(1-p_A)^2]^2}$$

Differentiate with respect to L_1 we get

$$g_1(.) = \left(\frac{\partial \bar{p}_{3.M_2}}{\partial L_1} \right)_{L_2, p, q, p_A} = \frac{(1-q)\{p+(1-p)L_2(1-p_A)\} + \{[L_1 L_2(1-p_A)^2-1] + (1-L_1)L_2(1-p_A)^2\}}{\{1-L_1 L_2(1-p_A)^2\}^2}$$

Differentiate with respect to L_2 we get

$$g_2(.) = \left(\frac{\partial \bar{p}_{3.M_2}}{\partial L_2} \right)_{L_1, p, q, p_A} = \frac{\{(1-L_1)(1-q)\{1-L_1 L_2(1-p_A)^2\} + \{p+(1-p)L_2(1-p_A)\}L_1(1-p_A)^2\}}{[1-L_1 L_2(1-p_A)^2]^2}$$

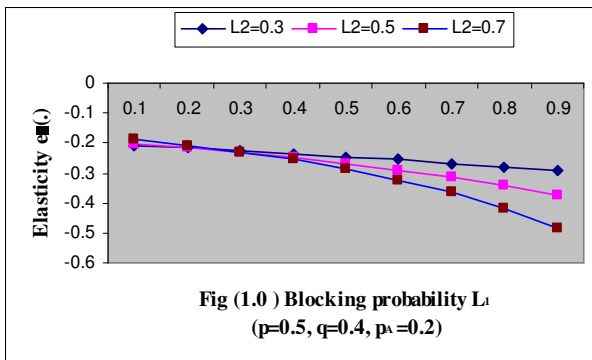
Differentiate with respect to L_1 we get

$$h_1(.) = \left(\frac{\partial \bar{p}_{4.M_2}}{\partial L_1} \right)_{L_2, p, q, p_A} = \frac{\{(1-q)(1-L_2)\{1-L_1 L_2(1-p_A)^2\} + \{p(1-p_A) + \{(1-p)+pL_1(1-p_A)\}L_2(1-p_A)^2\}}{[1-L_1 L_2(1-p_A)^2]^2}$$

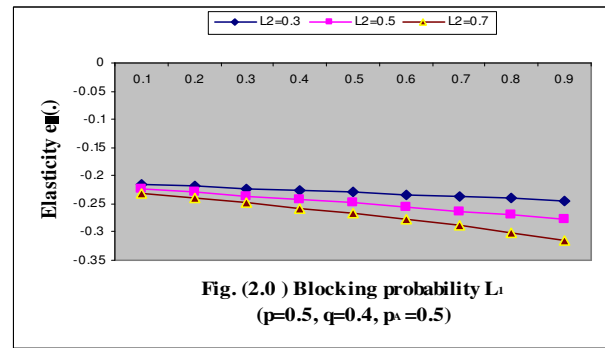
Differentiate with respect to L_2 we get

$$h_2(.) = \left(\frac{\partial \bar{p}_{4.M_2}}{\partial L_2} \right)_{L_1, p, q, p_A} = \frac{(1-q)\{(1-p)+pL_1(1-p_A)\} + \{[L_1 L_2(1-p_A)^2-1] + (1-L_2)L_1(1-p_A)^2\}}{[1-L_1 L_2(1-p_A)^2]^2}$$

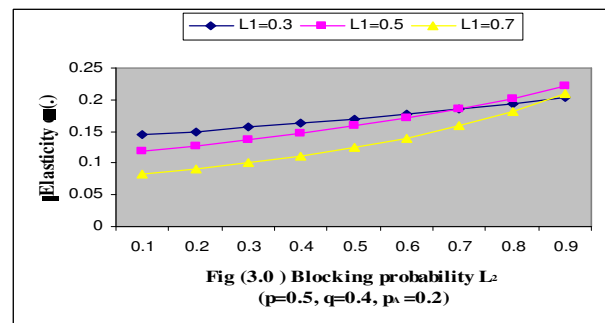
SIMULATION STUDY



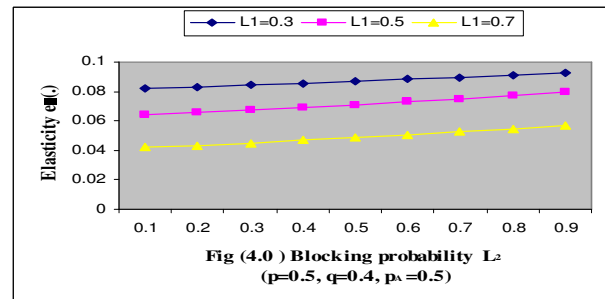
In view to fig. 1.0 the elasticities of the traffic share of the first operator in the first market is going down with the increasing level blocking probability. However if opponent operator also bears the same in increasing patterns, then the elasticity curve is further lower down.



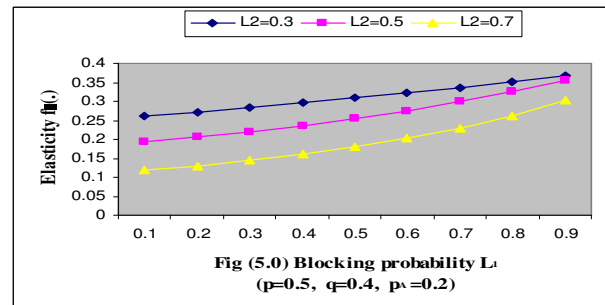
According to fig. 2.0, when p_A probability is little high the similar pattern exists at the higher down fall level. If opponent bears higher level of blocking then the elasticity has negative trend.



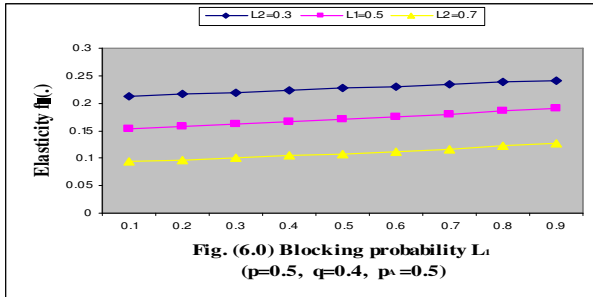
When to consider the opponent blocking probability as a base (fig 3.0) the trend is linear and by increasing L_2 the elasticity bears positive sign. If L_1 is also high with L_2 , the rate of linear increment becomes small.



But when p_A is little high (fig. 4.0) the curves are sifted toward higher values with similar trend. This is observed in the first market.



While to consider traffic sharing of second operator, the elasticities pattern is in upward trend as in case of first market. With the increase of opponent L_2 this pattern remains the same but elasticity value is lower.



When p_A probability is high (fig. 6.0) the more stable pattern is found. In this case elasticities are looking like independent to L_1 variation.

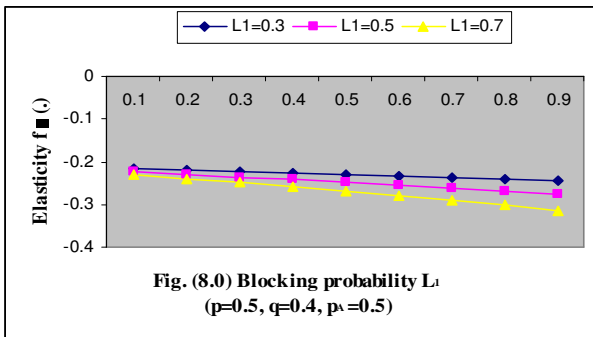
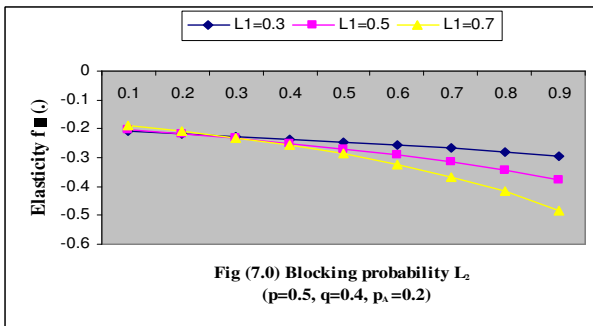


Fig. (7.0, 8.0) are similar type but having L_2 probability as variant. These graphs are similar to figure (5.0, 6.0) but differ for traffic share p_2 of O_2 .

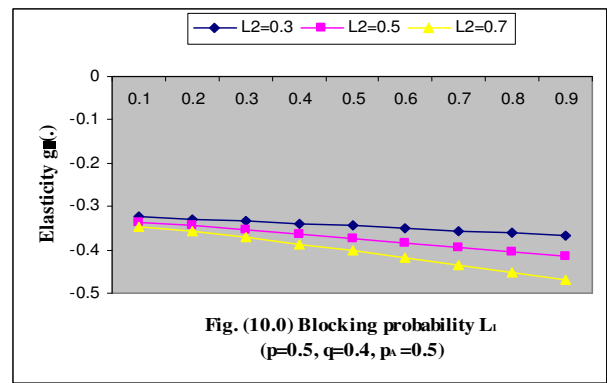
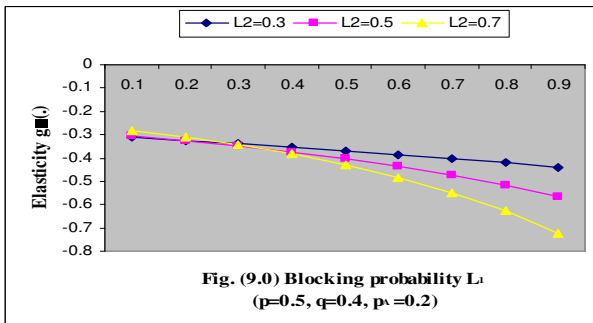


Fig. (9.0, 10.0) are showing elasticities pattern for third operator bearing the blocking probability L_1 . The trend is downward and sharper than earlier cases. For little high L_2 level this goes further down.

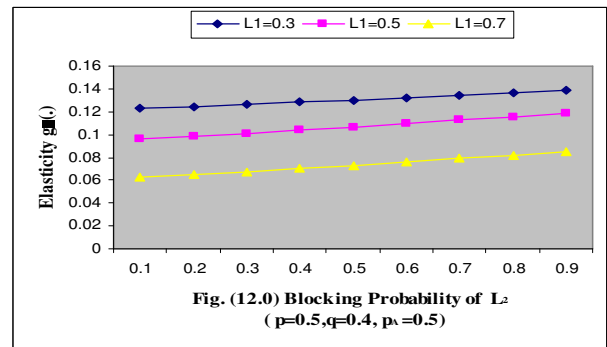
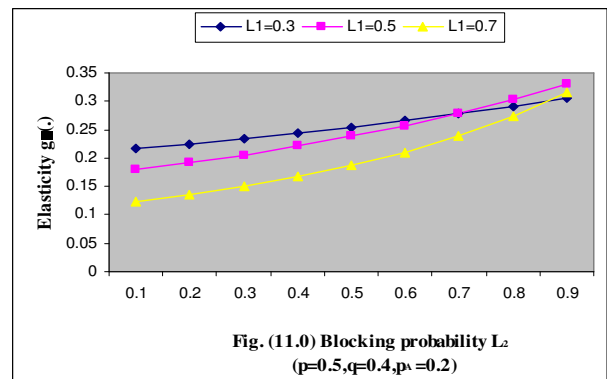
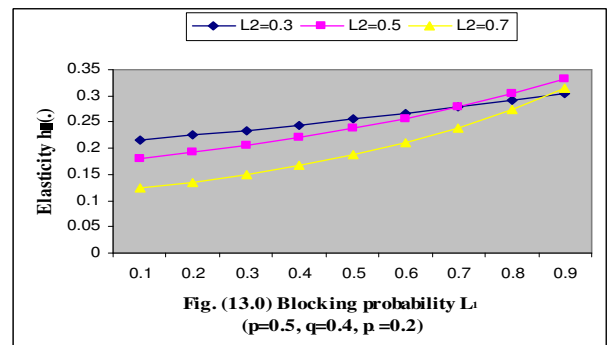


Fig. (11.0-12.0) are similar but with respect to L_2 probability for P_3 of operator O_3 is high in the second market. Both the curves are having increasing Elasticities over L_2 and decreasing probability level over L_1 .



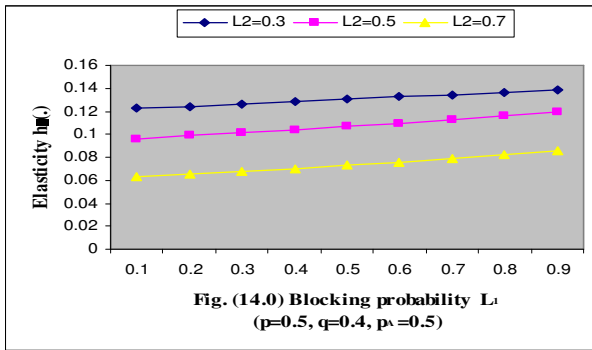


Fig. (13.0-14.0) are similar to fig. (11.0, 12.0) and showing the positive value of elasticity. When p_A is high then value of positive elasticity is also high.

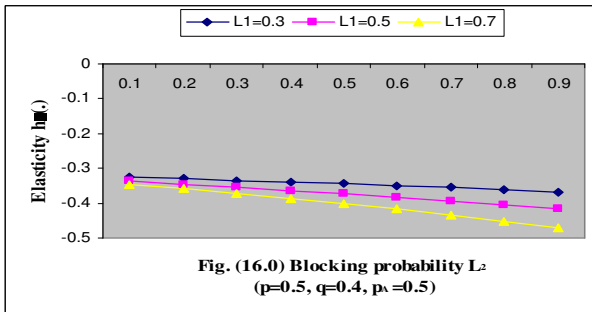
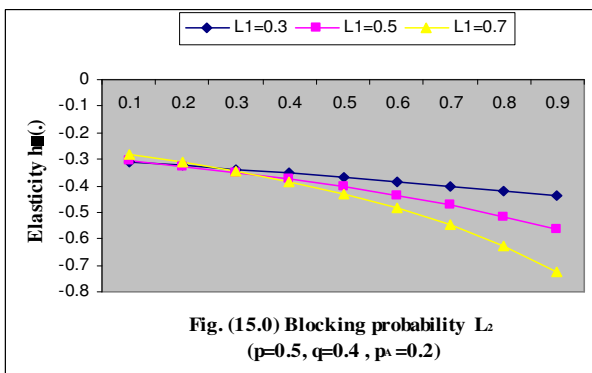


Fig. (15.0, 16.0) are matching to fig. (9.0, 10.0) and the pattern of elasticity is downward showing a sharp decrement in the tendency.

CONCLUDING REMARKS

The Elasticities of traffic share of operator depends on blocking probability. These are negative in trend but when opponent blocking is high the negativity becomes high. Elasticities value depends on market position. If a market is of high priority, it has higher elasticities level. The abandon probability affects the elasticity level. High abandon chance produces stable pattern of traffic share independent to the blocking probability.

REFERENCES

[1]. Abuagla Babiker Mohd and Dr. Sulaiman bin Mohd Nor “Towards a Flow-based Internet Traffic Classification for

Bandwidth Optimization”, International Journal of Computer Science and Security (IJCSS), 3(2):146-153, 2009

[2]. Ankur Agarwal “System-Level Modeling of a Network-on-Chip”, International Journal of Computer Science and Security (IJCSS), 3(3):154-174, 2009.

[3]. D. Shukla, Virendra Tiwari, Sanjay Thakur, Arvind Kumar Deshmukh “Share Loss Analysis of Internet Traffic Distribution in Computer Networks” International Journal of Computer Science and Security (IJCSS) Volume 3, Issue 5:414-426, 2009

[4]. C. Yeian, and J. Lygeres., “Stabilization of a class of stochastic differential equations with markovian switching, System and Control Letters”, issue 9:819-833, 2005.

[5]. D. Shukla, S. Gadewar and R.K. Pathak “A Stochastic model for Space division switches in Computer Networks”, International Journal of Applied Mathematics and Computation, Elsevier Journals, 184(2): 235-269, 2007.

[6]. D. Shukla, Saurabh Jain, Rahul Singhai and R.K. Agarwal “A Markov Chain model for the analysis of Round Robin Scheduling Scheme”, International Journal of Advanced Networking and Applications (IJANA), 01(01):01-07, 2009.

[7]. D. Shukla, R.K. Pathak and Sanjay Thakur “Analysis of Internet traffic distribution between two markets using a Markov chain model in computer networks”, Proceedings of National Conference on Network Security and Management (NCSM-07), pp. 142-153, 2007.

[8]. D. Shukla and Sanjay Thakur, “Crime based user analysis in Internet traffic sharing under cyber crime”, Proceedings of National Conference on Network Security and Management (NCSM-07), pp. 155-165, 2007.

[9]. D. Shukla and Sanjay Thakur, “Rest state analysis in Internet traffic sharing under a Markov chain model”, Proceedings of 2nd National Conference on Computer Science & Information Technology, pp. 46-52, 2008.

[10]. Emanuel Perzen “Stochastic Processes”, Holden-Day, Inc., San Francisco, and California, 1992.

[11]. J. Medhi, “Stochastic Models in queuing theory”, Academic Press Professional, Inc., San Diego, CA, 1991.

[12]. J. Medhi, “Stochastic Processes”, Ed. 4, Wiley Eastern Limited (Fourth reprint), New Delhi ,1992.

[13]. M. Naldi, “Internet Access Traffic Sharing in A Multi-user Environment”, Computer Networks, 38:809-824, 2002.

[14]. M. Newby and R. Dagg, “ Optical inspection and maintenance for stochastically deteriorating systems: average cost criteria”, Jour. Ind. Stat. Asso., 40(2):169-198, 2002

[15]. Rinkle Agarwal and Lakhwinder Kaur “On Reliability Analysis of Fault-tolerant Multistage Interconnection

Networks“, International Journal of Computer Science and Security (IJCSS), 2(4):01-08, 2008.

[16]. D. Shukla, Virendra kumar Tiwari and Sanjay Thakur “Cyber Crime Analysis for Multi-Dimensional Effects in Computer Networks”, Journal of Global Research in Computer Science (IGRCS), Volume 1, No. 4:31-37, 2010.

AUTHOR'S BIOGRAPHY



Dr. Diwakar Shukla is working as an Associate Professor in the Department of Mathematics and Statistics, Sagar University, Sagar, M.P. and having over 21 years experience of teaching to U.G. and P.G. classes. He obtained M.Sc.(Stat.) Ph.D.(Stat.), degrees from Banaras Hindu University, Varanasi and served the Devi Ahilya University, Indore, M.P. as a Lecturer over nine years and obtained the degree of M. Tech. (Computer Science) from there. During Ph.D., he was junior and senior research fellow of CSIR, New Delhi qualifying through Fellowship Examination (NET) of 1983. Till now, he has published more than 80 research papers in national and international journals and participated in more than 35 seminars / conferences at national level. He also worked as a selected Professor to the Lucknow University, Lucknow, U.P., for one year and visited abroad to Sydney (Australia) and Shanghai (China) for conference participation. He has supervised twelve Ph.D. theses in Statistics and Computer Science both; and five students are presently enrolled for their doctoral degree under his supervision. He is member of 10 learned bodies of Statistics and Computer Science both at national level. The area of research he works for are Sampling Theory, Graph Theory, Stochastic Modeling, Computer Network and Operating Systems.



Mr. Sharad Gangele has completed M.Sc.(Statistics) in 2005 from Devi Ahilya university, Indore,(M.P.), P. G. D.C. A. in 2006 from M. C. R. P. V, Bhopal,(M.P.) . M. Sc. (Computer Science) in 2008 from M.C.R.P.V., Bhopal,(M.P.). He is pursuing research in the field of stochastic modeling of computer network system. He has authored and co-authored some research papers. He has presented/ participate three international / National Conferences. His current research interest is stochastic modeling of computer network system and Internet traffic sharing analysis.

Mr. Kapil Verma has completed M.C.A degree from Dr. H. S. Gour Central University, Sagar [M.P.]. He is presently working as a guest Lecturer in the Department of Computer Science & Applications in Govt. Autonomous Girls P. G. Excellence College Sagar, M.P. He is pursuing research in the field of Probability Based Mathematical Modeling of Internet Traffic in

Computer Network. He has authored and co-authored some research papers. He presented two research papers in National/International conferences. His current research interest is to study the internet traffic sharing under various traffic variants.

Dr. Pankaja Singh is working as a professor in the Department of Physics, Govt. Motilal Vigyan Mahavidhyalaya, Bhopal, M.P, and having over 28 years experience of teaching to U.G. and P.G. classes. He has published more than 13 research papers in national and international journals and participated in more than 30 seminars / conferences at national and international level. He also published two books in field of computer science.

