ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 8, August 2014

Effect of Rotation on Marangoni Convection in a Relatively Hotter or Cooler Liquid Layer

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ABSTRACT: The effect of Coriolis force on the onset of surface tension driven convection in a relatively hotter or cooler liquid layer confined between two horizontal boundaries in the presence of uniform rotation is considered theoretically. The modified linear stability analysis is performed to obtain the eigenvalue system of eighth order. A Fourier series method is used to obtain the characteristic value equation analytically for the Marangoni number M which is then computed numerically. It is established that the rotation has stabilizing character irrespective of whether the liquid layer is relatively hotter or cooler.

KEYWORDS: Conducting, Convection, Linear stability, Stationary, Surface tension.

I. INTRODUCTION

The phenomenon of the onset of surface tension induced convection in a thin horizontal liquid layer heated from below with free upper surface was first established experimentally by Block [1] and theoretically by Pearson [2]. They established that the patterned hexagonal cells observed by Bénard [3, 4], and explained by Rayleigh [5] in terms of buoyancy (discussed in detail by Chandrasekhar [6]), were in fact due to temperature dependent surface tension. Convection driven by surface tension gradients is now commonly known as Bénard-Marangoni convection, in contrast to buoyancy driven Rayleigh-Bénard convection. Bénard-Marangoni convection has received a great deal of research activities because it has many applications in geophysics, oceanography, atmospheric sciences, chemical engineering of paints and detergents. Quantitative disagreement between experiment and theory has indicated that gravity was present in Bénard's experiments as well as in other experiments involving convection in a fluid layer with free surface in a laboratory on the earth, therefore, Nield [7] considered the combined effects of surface tension and buoyancy on the onset of convection in a fluid layer heated from below with free upper surface and found that the two effects causing instability reinforce one another and that as the thickness of the fluid layer decreases, the surface tension effects become more dominant. Further contributions made by many researchers namely, Scriven and Sternling [8], Smith [9], Davis [10] and Takashima [11, 12] have refined Pearson's model by incorporating more realistic conditions. For a detailed study of Bénard-Marangoni convection one may be referred to the work of Normand et al. [13], Koschmieder [14] and Schatz et al. [15].

II. RELATED WORK

The onset of Bénard-Marangoni convection in a relatively hotter or cooler liquid layer has been analyzed by Gupta and Shandil [16] and established that irrespective of the thermal nature (conducting or insulating) of the lower boundary, the critical Marangoni number significantly depends on whether liquid layer is relatively hotter or cooler, and hotter the liquid layer more the postponement of the onset of convection. In view of the stabilizing nature of rotating fluids, a fact that has already been established by Chandrasekhar [6] for the buoyancy driven convection, by Vidal and Acrivos [17] for the surface tension driven convection and by Namikawa et al [18] for the convective instability induced by both surface tension and buoyancy. In the present study, therefore, we consider theoretically the possible influence of such a Coriolis force on the onset of convection driven by surface tension in a relatively hotter or cooler liquid layer. A Fourier series method is used to obtain the characteristic value equation analytically. The numerical results are obtained for a wide range of values of the Taylor number Ta. It is shown numerically that both the critical Marangoni number M_c and the critical wave number a_c increase monotonically with Ta for a fixed value of the parameter $\alpha_2 T_0$ where T_0 and

ISSN: 2319-8753
International Journal of Innovative Research in Science, Engineering and Technology
(An ISO 3297: 2007 Certified Organization)
Vol. 3, Issue 8, August 2014

α_2 being the appropriately chosen mean temperature and coefficient of specific heat (at constant volume) variation due to temperature variation respectively of the fluid layer.

III. FORMULATION OF THE PROBLEM

We consider an infinite horizontal liquid layer of uniform thickness *d*, heated from below and kept rotating about a vertical axis with a constant angular velocity Ω with the upper free surface open to the ambient air, where surface tension gradients arise due to temperature perturbations. We choose a Cartesian coordinate system of axes with the *x* and *y* axis in the plane of the lower surface and the *z*-axis along the vertically upward direction so that the fluid is confined between the planes at $z = 0$ and $z = d$. A temperature gradient is maintained across the layer by maintaining the lower boundary at a constant temperature T_0 and the upper boundary at T_1 ($< T_0$). It is assumed that surface tension is given by the simple linear law $\tau = \tau_1 - \sigma(T - T_1)$ where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = (\partial\tau/\partial T)_{T=T_1}$ represents the rate of change of surface tension with temperature, evaluated at temperature T_1, and surface tension being a monotonically decreasing function of temperature, σ is positive.

Following Gupta and Shandil [16], we can write simplified linearized perturbation equations under rotation as

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w = -2\Omega\frac{\partial\zeta}{\partial z} \tag{1}$$

$$(1-\alpha_2 T_0)\left(\frac{\partial\theta}{\partial t} - \beta w\right) = \kappa\nabla^2\theta \tag{2}$$

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\zeta = 2\Omega\frac{\partial w}{\partial z} \tag{3}$$

where w, θ and ζ denote respectively the *z*-component of velocity perturbation and temperature perturbation from uniform vertical temperature gradient, and the *z*-component of vorticity. The kinematic viscosity ν, the thermal diffusivity κ and the temperature gradient $\beta = (T_1 - T_0)/d$ are each assumed constant. Further, that coefficient α_2 (due to variation in the temperature) is a constant that ranges from 0 to 10^{-3} and the range of the parameter $\alpha_2 T_0$ covering usual laboratory conditions is $0 \le \alpha_2 T_0 \le 0.5$ for liquids with which we are mostly concerned. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and *t* denotes time.

In seeking solutions of the equations (1), (2) and (3), we must satisfy certain boundary conditions, The boundary conditions at the lower rigid and thermally conducting surface $z = 0$ are given by

$$w = 0 \tag{4}$$

$$\frac{\partial w}{\partial z} = 0 \tag{5}$$

$$\theta = 0 \tag{6}$$

$$\zeta = 0 \tag{7}$$

The boundary conditions at the upper free non-deflecting surface (Pearson [2], Vidal and Acrivos [17]) $z = d$ are

$$w = 0 \tag{8}$$

$$\rho\nu\frac{\partial^2 w}{\partial z^2} = \sigma\nabla_1^2\theta \tag{9}$$

$$-k\frac{\partial\theta}{\partial z} = q\theta \tag{10}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

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$$\frac{\partial \zeta}{\partial z} = 0 \qquad (11)$$

where ρ is the density and $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The boundary condition (9) is usually referred to as the Marangoni boundary condition. k is the thermal conductivity of the fluid and q is the heat transfer coefficient.

We now suppose that the perturbations w, θ and ζ are of the form

$$w(x,y,z,t) = w(z)\exp[i(a_x x + a_y y) + pt]$$
$$\theta(x,y,z,t) = \theta(z)\exp[i(a_x x + a_y y) + pt]$$
$$\zeta(x,y,z,t) = \zeta(z)\exp[i(a_x x + a_y y) + pt]$$

where $a = \sqrt{a_x^2 + a_y^2}$ is the wave number of the disturbance and p is a time constant (which can be complex). We now introduce the non-dimensional quantities using $d, \nu/d, d^2/\nu$ and $\beta d\nu/\kappa$ as the appropriate scales for length, velocity, time and temperature respectively and putting $z_* = \frac{z}{d}$, $a_* = ad$, $Z_* = \frac{\zeta \nu}{d^2}$, $W_* = \frac{wd}{\nu}$, $\Theta_* = \frac{\theta \kappa}{\beta d \nu}$ and $D = d\frac{d}{dz}$. We now let x, y and z stand for co-ordinates in the new units and omitting asterisk for simplicity, equations (1)-(3) and boundary conditions (4)-(11) can be reduced to the following non-dimensional form

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - p\right)W = \sqrt{Ta}DZ \qquad (12)$$

$$\left(D^2 - a^2 - (1-\alpha_2 T_0)pP_r\right)\Theta = -(1-\alpha_2 T_0)W \qquad (13)$$

$$\left(D^2 - a^2 - p\right)Z = -\sqrt{Ta}DW \qquad (14)$$

$$\left.\begin{array}{l} W(0) = 0 \\ DW(0) = 0 \\ \Theta(0) = 0 \\ Z(0) = 0 \end{array}\right\} \qquad (15a, b, c, d)$$

evaluated on the lower boundary $z = 0$, and

$$\left.\begin{array}{l} W(1) = 0 \\ D^2W(1) = -Ma^2\Theta(1) \\ D\Theta(1) = -L\Theta(1) \\ DZ(1) = 0 \end{array}\right\} \qquad (16a, b, c, d)$$

evaluated on the upper boundary $z = 1$. Here $Ta = \frac{4\Omega^2 d^2}{\nu^2}$ is the Taylor number, $M = \frac{\sigma\beta d^2}{\rho\kappa\nu}$ is the Marangoni number and $L = \frac{qd}{k}$ is the Biot number.

We consider the case when neutral state is stationary, in which $p = 0$ and equations (12)-(14) become

$$\left(D^2 - a^2\right)^2 W = \sqrt{Ta}DZ \qquad (17)$$

$$(D^2 - a^2)\Theta = -(1-\alpha_2 T_0)W \qquad (18)$$

$$\left(D^2 - a^2\right)Z = -\sqrt{Ta}DW \qquad (19)$$

Solution to equations (17)-(19) is sought subject to boundary conditions (15a, b, c, d)-(16a, b, c, d). Thus we have an eigenvalue system of eighth order. It is evident that when $\alpha_2 T_0 = 0$, the system reduces to the case which has been

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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analysed by Vidal and Acrivos [17], and when Ta = 0 the system reduces to the case which has been analysed by Gupta and Shandil [16].

IV. SOLUTION OF THE PROBLEM

The Fourier series method as presented by Nield [7] is convenient for the problem under consideration. The constants to be eliminated are denoted by

$$\lambda_1 = D^2W(0),\ \lambda_2 = D^2W(1),\ \lambda_3 = \theta(1),\ \lambda_4 = DZ(0)$$

We let

$$W = \sum_{n=1}^{\infty}[A_n - \frac{2}{n^3\pi^3}\{\lambda_1 - (-1)^n\lambda_2\}]\sin n\pi z \quad (20)$$

$$\theta = \sum_{n=1}^{\infty}[B_n - \frac{2}{n\pi}(-1)^n\lambda_3]\sin n\pi z \quad (21)$$

$$Z = \frac{1}{3}DZ(0) + C_0 + \sum_{n=1}^{\infty}[C_n + \frac{2}{n^2\pi^2}\lambda_4]\cos n\pi z \quad (22)$$

where the boundary conditions (15a), (15c), (16a) and (16d) have already been used while writing equations (20)-(22). We have

$$D^2W = \sum_{n=1}^{\infty}[A_n(-n^2\pi^2) + \frac{2}{n\pi}\{\lambda_1 - (-1)^n\lambda_2\}]\sin n\pi z \quad (23)$$

$$D^4W = \sum_{n=1}^{\infty}A_n(n^4\pi^4)\sin n\pi z \quad (24)$$

$$D^2\theta = \sum_{n=1}^{\infty}B_n(-n^2\pi^2)\sin n\pi z \quad (25)$$

$$DZ = \sum_{n=1}^{\infty}[C_n(-n\pi) + \frac{2}{n\pi}\lambda_4]\sin n\pi z \quad (26)$$

$$D^2Z = -\lambda_4 - \sum_{n=1}^{\infty}C_n(n^2\pi^2)\cos n\pi z \quad (27)$$

The differential equations (17), (18) and (19) are satisfied by substituting the complete Fourier expansions for W, θ, Z and their derivatives using equations (20)-(27) and equating the coefficients of $\sin n\pi z$, we obtain

$$(n^2\pi^2 + a^2)^2A_n + n\pi\sqrt{Ta}C_n = \frac{2}{n^3\pi^3}[\lambda_1 - (-1)^n\lambda_2]\{a^2(2n^2\pi^2 + a^2)\} + \frac{2\sqrt{Ta}}{n\pi}\lambda_4 \quad (28)$$

$$(1-\alpha_2T_0)A_n - (n^2\pi^2 + a^2)B_n = \frac{2(1-\alpha_2T_0)}{n^3\pi^3}[\lambda_1 - (-1)^n\lambda_2] - \frac{2a^2}{n\pi}(-1)^n\lambda_3, \quad (29)$$

$$n\pi\sqrt{Ta}A_n - (n^2\pi^2 + a^2)C_n = \frac{2\sqrt{Ta}}{n^2\pi^2}[\lambda_1 - (-1)^n\lambda_2] - \frac{2a^2}{n^2\pi^2}\lambda_4 \quad (30)$$

and

$$a^2C_0 = -[1 + \frac{a^2}{n^2\pi^2}]\lambda_4 \quad (31)$$

The remaining boundary conditions require that

$$\sum_{n=1}^{\infty}n\pi A_n - \frac{2}{n^2\pi^2}\lambda_1 + \frac{2}{n^2\pi^2}(-1)^n\lambda_2 = 0, \quad (32)$$

$$\lambda_2 + a^2M\lambda_3 = 0, \quad (33)$$

$$\sum_{n=1}^{\infty}(-1)^n n\pi B_n + (1+L)\lambda_3 = 0, \quad (34)$$

DOI: 10.15680/IJIRSET.2014.0308014

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$\sum_{n=1}^{\infty} C_n + C_0 = 0. \quad (35)$$

From equations (28)-(30), A_n, B_n and C_n can be expressed in terms of λ_1, λ_2, λ_3 and λ_4 Substitution in equations (32), (34), (35) and on making use of equation (33), then yield four homogeneous equations in λ_1, λ_2 and λ_4. Elimination of these constants gives

$$\begin{vmatrix} \sum_{n=1}^{n} E_n & \sum_{n=1}^{n} (-1)^n E_n & Ta\sum_{n=1}^{n} F_n \\ \sum_{n=1}^{n} F_n & \sum_{n=1}^{n} (-1)^n F_n & -\frac{1}{2a^2} - \sum_{n=1}^{n} H + Ta\sum_{n=1}^{n} G_n \\ (1-\alpha_2 T_0)\sum_{n=1}^{n}(-1)^n F_n & (1-\alpha_2 T_0)\sum_{n=1}^{n} F_n - \frac{1}{M\,a^2}(a^2\sum_{n=1}^{n} H + \frac{L+1}{2}) & (1-\alpha_2 T_0)Ta\sum_{n=1}^{n}(-1)^n G_n \end{vmatrix} = 0 \quad (36)$$

where

$$E_n = \frac{n^2\pi^2(n^2\pi^2 + a^2)}{R_n},\ F_n = \frac{n^2\pi^2}{R_n},\ G_n = \frac{n^2\pi^2}{(n^2\pi^2 + a^2)R_n},\ H_n = \frac{1}{(n^2\pi^2 + a^2)} \quad (37)$$

with $\quad R_n = (n^2\pi^2 + a^2)^3 + n^2\pi^2 Ta$

From the eigenvalue equation (36), M can be determined as a function of $a, \alpha_2 T_0, Ta$ and L as the ratio of two determinants, that is

$$M(a, \alpha_2 T_0, Ta, L) = \frac{1}{a^2(1-\alpha_2 T_0)} \frac{\begin{vmatrix} \sum_{n=1}^{n} E_n & 0 & Ta\sum_{n=1}^{n} F_n \\ \sum_{n=1}^{n} F_n & 0 & -\frac{1}{2a^2} - \sum_{n=1}^{n} H + Ta\sum_{n=1}^{n} G_n \\ \sum_{n=1}^{n}(-1)^n F_n & a^2\sum_{n=1}^{n} H + \frac{L+1}{2} & Ta\sum_{n=1}^{n}(-1)^n G_n \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^{n} E_n & \sum_{n=1}^{n}(-1)^n E_n & Ta\sum_{n=1}^{n} F_n \\ \sum_{n=1}^{n} F_n & \sum_{n=1}^{n}(-1)^n F_n & -\frac{1}{2a^2} - \sum_{n=1}^{n} H + Ta\sum_{n=1}^{n} G_n \\ \sum_{n=1}^{n}(-1)^n F_n & \sum_{n=1}^{n} F_n & Ta\sum_{n=1}^{n}(-1)^n G_n \end{vmatrix}} \quad (38)$$

V. NUMERICAL RESULTS AND DISCUSSION

The numerical calculations may be carried out in the following fashion. For fixed values of $\alpha_2 T_0$, Ta and L we may seek the lowest value of Marangoni number M as a function of the wave number a to obtain the critical Marangoni number M_c. The value of a at which M attains the minimum is the critical wave number a_c. For given values of $\alpha_2 T_0$ and Ta it may be mentioned here that the system was found to be less stable for an insulating upper free surface ($L = 0$), which is as expected, since an isothermal upper free surface ($L \to \infty$) yields a completely stable system so far as the

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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surface tension driven convection is concerned. Values of critical Marangoni number M_c and the critical wave number a_c computed from equation (38) are listed in Table 1 for assigned values of $\alpha_2 T_0$ and *Ta* when *L* = 0.

Table 1. Numerical values of M_c and a_c for various values of $\alpha_2 T_0$ and *Ta* for *L* = 0.

Ta	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	M_c	a_c	M_c	a_c	M_c	a_c
0	79.607	1.993	113.724	1.993	159.214	1.993
1	79.733	1.995	113.904	1.995	159.466	1.995
10	80.859	2.012	115.513	2.012	161.718	2.012
10^2	91.354	2.166	130.505	2.166	182.708	2.166
10^3	163.405	2.973	233.435	2.973	326.81	2.973
10^4	456.923	5.006	652.747	5.006	913.846	5.006
10^5	1401.76	8.634	2002.52	8.634	2803.52	8.634
10^6	4405.48	15.11	6293.54	15.11	8810.96	15.12

From Table 1, it is evident that for a given value of $\alpha_2 T_0$, both M_c and a_c increase monotonically with *Ta*, and that for a given value of *Ta*, an increase in the value of $\alpha_2 T_0$ leads to an increased value of M_c and the value of critical wave number a_c remains unchanged for various values of $\alpha_2 T_0$. When $\alpha_2 T_0$ = 0, the critical Marangoni number and the corresponding wave number we obtain are identical to the results obtained by Namikawa et al. [18] under the condition when the Rayleigh number *R* = 0. Further, we found that

$$M_c \rightarrow \frac{4.41\sqrt{Ta}}{1-\alpha_2 T_0} \quad \text{and} \quad a_c \rightarrow 0.47(Ta)^{\frac{1}{4}}$$

as $Ta \rightarrow \infty$. These asymptotic expressions are in good agreement with those obtained by Vidal and Acrivos [18].

In Fig. 1 (a) and (b), the neutral stability curves are plotted for $\alpha_2 T_0$ = 0 and $\alpha_2 T_0$ = 0.3 respectively, using relation (38) for various values of *Ta*. The region below each curve represents the stable state. From Fig. 1(a) and (b), we observed that the neutral stability curves move upwards in each case for increasing values of *Ta*, clearly showing the stabilizing effect of *Ta* for a fixed value of $\alpha_2 T_0$.

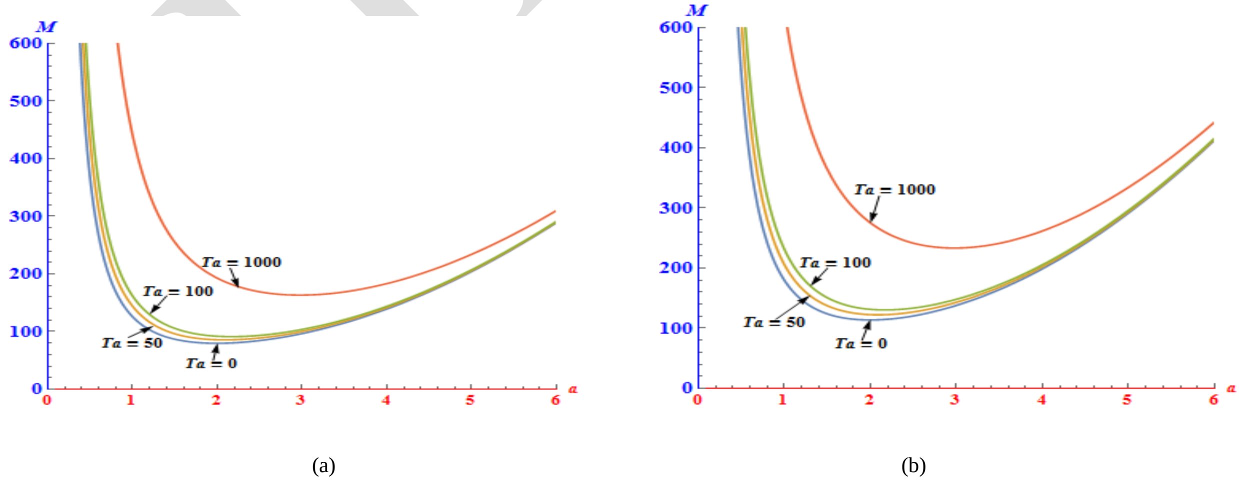

Fig.1: Neutral stability curves for various values of *Ta* when (a) $\alpha_2 T_0$ = 0 and *L* = 0, (b) $\alpha_2 T_0$ = 0.3 and *L* = 0.

We observe that there is smooth increase in the value of M_c with increase in value of Ta, clearly showing the stabilizing effect of Ta, irrespective of whether the liquid layer is relatively hotter ($\alpha_2 T_0 = 0.3$) or cooler ($\alpha_2 T_0 = 0$).

REFERENCES

[1] M.J. Block, "Surface tension as the cause of Bénard cells and surface deformation in a liquid film", Nature, vol.178, pp.650-651, 1956.
[2] J.R.A. Pearson, "On convection cells induced by surface tension", J. Fluid Mech., vol.4, pp.489-500, 1958.
[3] H. Bénard, "Les tourbillons cellulaires dans une napple liquid", Revue général des Sciences pures et appliqués, vol. 11, pp.1261-1271, 1900.
[4] H. Bénard, "Les tourbillons cellulaires dans une napple liquide transportant de la chaleur par convection en régime permanent", Annales de Chimie et de. Physique, vol. 23, pp. 62-144, 1901.
[5] L. Rayleigh, "On convection currents in a horizontal layer of fluid, when the higher temperature is on the underside," Phil. Mag., vol.32(6), pp.529-546, 1916.
[6] S. Chandrasekhar, "Hydrodynamic and Hydromagnetic Stability", London: Oxford University Press, 1961.
[7] D. A. Nield, "Surface tension and buoyancy effects in cellular convection", J. Fluid Mech., vol.19, pp.341-352, 1964.
[8] L.E. Scriven and C.V. Sternling, "On cellular convection driven by surface tension gradients: effects of mean surface tension and surface viscosity", J. Fluid Mech., vol.19, pp.321-340, 1964
[9] K.A. Smith, "On convective instability induced by surface tension gradients", J. Fluid Mech., vol.24(2), pp.401-414, 1966.
[10] S. H. Davis, "Buoyancy-surface tension instability by the method of energy", J. Fluid Mech., vol.39(2), pp.347-359, 1969.
[11] M. Takashima, Surface tension driven instability in a horizontal liquid layer with deformable free surface: I. Stationary convection, J. Phys. Soc. Jpn., vol.50, pp.2745-2750, 1981(a).
[12] M. Takashima, "Surface tension driven instability in a horizontal liquid layer with a deformable free surface. II. Overstability," J. Phys. Soc. Jpn. vol.50, pp.2751–2756, 1981(b).
[13] C. Normand, Y. Pomeau and M. Velarde, "Convective instability: a physicists approach," Rev. Mod. Phys., vol.49, pp.581-624, 1977.
[14] E.L. Koschmieder, "Bénard cells and Taylor vortices," Cambridge: Cambridge University Press, 1993.
[15] M.F. Schatz, S.J. VanHook, W.D. McCormick, J.B. Swift and H.L. Swinney, "Onset of surface tension driven Bénard convection," Phys. Rev. Lett., vol.75, pp.1938-1941, 1995.
[16] A.K. Gupta and R.G. Shandil, "Marangoni convection in a relatively hotter or cooler liquid layer", Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci., vol.28(2) pp.103-106, 2012.
[17] Vidal and A. Acrivos, "The influence of Coriolis force on surface tension-driven convection", J. Fluid. Mech.., vol.26(4), pp.807-818, 1966.
[18] T. Namikawa, M. Takashima and S. Matsushita, "The effect of rotation on convective instability induced by surface tension and buoyancy", J. Phys. Soc. Jpn., vol.28(5), pp.1340-1349, 1970.