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Degree Equitable Domination Number and Independent Domination Number of a Graph

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Abstract: In this paper, we introduce the degree equitable domination number in graphs. Some interesting relationships are identified between domination and degree equitable domination and independent domination. It is also shown that any positive integers with $\gamma(G) \leq \gamma_N \leq \gamma_i$ are realizable as the domination number, degree equitable domination number and independent domination number of a graph. We also investigate the degree equitable domination in the corona of graphs.

Mathematics Subject Classification: 05C69

Keywords: Degree equitable dominating set, degree equitable domination number,

I. INTRODUCTION

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge[3] in 1958 and 1962 and O.Ore[8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper, G(V, E) a finite, simple, connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G Δ has n vertices and m edges. Degree of a vertex v is denoted by d(v), the *maximum degree* of a graph G is denoted by (G). Let C_n a cycle on n vertices, P_n a path on n vertices and a complete graph on n vertices by K_n. A graph is *connected* if any two vertices are connected by a path. A maximal connected subgraph of a graph G is called a *component* of G .The *number of components* of G is denoted by ω (G). The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A *complete bipartite graph* is a bipartite graph with partitions of order $|V_1|$ = m and $|V_2|$ = n, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a *tree* with one root vertex and n-1 pendant vertices. A bistar, denoted by B(m, n) is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel* graph denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of $C_{n-1.}$ A *Helm* graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n.

The *chromatic number* of a graph G denoted by χ (G) is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours . For any real number x, $\lceil x \rceil$ denotes the largest integer greater than or equal to x. A Nordhaus- G addum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops .The basic definitions and concepts used in this study are adopted from[11].

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Given a graph G = (V(G), E(G)), the cardinality $|V(G)|$ of the vertex set V(G) is the order of G is n. The distance $d_G(u,v)$ between two vertices u and v of G is the length of the *shortest path* joining u and v .If $d_G(u,v) = 1$, u and v are said to be adjacent.

For a given vertex v of a graph G, The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v.

The degree $deg_G(v)$ of v refers to $|N_G(v)|$, and $\Delta(G) = \max\{[deg]_G(v) : v \in V(G)\}$. The closed neighbourhood of v is the set $N_G[v] = N_G(v) \cup \{v\}$ for $S \subseteq V(G)$, $N_G(S) = \bigcup_{v\in S} N_G(v)$ and $N_G[v] = N_G(S) \cup S$. If $N_G[v]$ = V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

A dominating set S in a graph G is an independent dominating set if for every pair of distinct vertices u and v in S, u and v are non adjacent in G . The minimum cardinality $\gamma_i(G)$ of an independent dominating set in G is called the *independent domination number* of G.

II. RELATIONSHIPS BETWEEN DOMINATION AND INDEPENDENT DOMINATION NUMBER

A. Definition:

Given a graph G, choose $v_1 \in V(G)$ and put $S_1 = \{v_1\}$. If $N_G[S_1] \neq V(G)$, choose $v_2 \in V(G)$, $|N_G(v_2)| = |N_G(S_1)|$ and put if $S_2 = \{v_1, v_2\} \neq V(G)$ where possible $k \geq 3$, choose $v_k \in V(G)$; $|N_G(v_k)| = |N_G(S_{k-1})|$ and put $S_k = \{v_1, v_2 \ldots\ldots v_k\}$, there exists a positive integer k such that $N_G[S_k] = V(G)$. A dominating set obtained in this way above is called *a degree equitable dominating set* .The minimum cardinality of a degree equitable dominating set is called the *degree equitable domination number* denoted by γ_N .

B. Example: For all integers n, $\gamma_N(P_n) = \left\lceil \frac{n}{N_G(S)} \right\rceil$. Here $\gamma(G) = 2, \gamma_N(G) = 2, \gamma_i(G) = 3$ Since any independent dominating set is a degree equitable dominating set, so it follows the inequality $\gamma(G) \leq \gamma_N(G) \leq \gamma_i(G)$.

C. Example : For the spider graph, The domination number, degree equitable domination number and independent domination number follows the inequality $\gamma(G) \leq \gamma_N(G) \leq \gamma_i(G)$.

D. Result : For any regular graph G with n vertices is the degree equitable domination number $\gamma_N(G) = \left\lceil \frac{n}{N_G(S)} \right\rceil$

E. Result : For any Complete graph K_n is a degree equitable dominating set with $\gamma_N(K_n) = \left\lceil \frac{n}{N_G(S)} \right\rceil$

F. Result : For any K_n-e is a degree equitable dominating set with $\gamma_N(G) = \left\lceil \frac{n}{N_G(S)} \right\rceil$

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G. Result : For any unicyclic graph with even number of vertices having the pendent vertex is a degree equitable dominating set with γ_N (G) = $\left\lceil \frac{n}{N_G(S)} \right\rceil$

H. Result: For any Complete Bipartite graph $K_{n,m}$ the degree equitable dominating set with γ_N ($K_{n,\ m}$) = $\left\lceil \frac{n+m}{N_G(S)} \right\rceil$ where V(n), V(m) are the vertex set of $K_{n,m}$

I. Result : If G is regular graph the then G-e has a degree equitable dominating set γ_N (G-e) = $\left\lceil \frac{n}{N_G(S)} \right\rceil$

J. Result : For any wheel graph W_n has a degree equitable dominating set with γ_N (W_n) = $\left\lceil \frac{n}{N_G(S)} \right\rceil$

K. Result : If W_n is any wheel graph then W_n –e has a degree equitable dominating set with

γ_N (W_n - e)=$\left\lceil \frac{n}{N_G(S)} \right\rceil$

L. Result : For any Helm graph H_n has a Hamiltonian degree equitable dominating set with γ_N (H_n) = W_n

III. DEGREE EQUITABLE SET OF CORONA

A. Definition:

The Corona G $\circ H$ of a graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^th vertex of G to every vertex in the i^th copy of H . It is customary to denote by H_v that copy of H whose vertices are adjoined with the vertex v of G. In effect G $\circ H$ is composed of the subgraphs H_v+v joined together by the edges of G. Moreover V(G$\circ H) = \cup_{(v \epsilon V(G))} [V(H_{\downarrow}v + v)]$

B. Lemma: Let G and H be any graphs and let S $\subseteq V(G \circ H)$. If S is a dominating set in G $\circ H$ then S $\cap V(H_v + v) \neq \emptyset$, $\forall v \epsilon V(G)$.

Proof: Let $v \epsilon V(G)$ and u$\epsilon V(H_v)$. If S is a dominating set in $G \circ H$, Then $\emptyset \neq N_{\downarrow}(G \circ H)[u] \cap S \subseteq V(H_{\downarrow}v + v) \cap S$ This proves lemma.

In view of lemma $\gamma_N(G \circ H) \geq \gamma_N(G)$ for any graphs G and H.

C. Lemma: Let G and H be any graph and let S $\subseteq V(G \circ H)$. If S is a degree equitable dominating

Set in $G \circ H$, Then S $\cap N(H_v)$ is a degree equitable dominating set in H_v.

Proof: Let v$\epsilon V(G)$ and let S $\subseteq V(G \circ H)$. *By lemma 3.A*, S_v= S $\cap N(H_v) \neq \emptyset$, There exists u ϵ $V(G)$ such that $|N_G(u)| = |N[S_v]|$ then S_v is the degree equitable dominating set in H_v.

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D. Theorem : Let G and H be any two graphs and let S$\subseteq V(G \circ H)$. If S is a degree equitable dominating set in $G \circ H$ then S $\cap N(H_v)$ is degree equitable dominating set iff V(G) is regular.

Proof: Assume G is regular, To prove that H_v is a degree equitable, There exists v$\epsilon V(G)$ and S$\subseteq V(G \circ H)$. *By lemma 3.B*, S_v = S $\cap N(H_v) \neq \emptyset$ there exists u $\epsilon\ V(G)$ such that $|N_G(u)| = |N[S_v]|$ then S_v is the degree equitable dominating set in H_v.

Coversly, Assume that S_v is degree equitable dominating set in H_v, To prove that V(G) is regular. Suppose, V(G) is not regular there exists v$\epsilon V(G)$ and let S $\subseteq V(G \circ H)$. *By lemma 3.B,* S_v= S $\cap N(H_v) \neq \emptyset$ and there exists u$\epsilon V(G)$ such that $|N_G(u)| \neq |N[S_v]|$ which is a contradiction to our assumption that, By the definition of degree equitable dominating set S_v is not a degree equitable dominating set, Hence V(G) must be regular.

E. Theorem : Let G and H be any two graphs and let S $\subseteq V(G \circ H)$. If S is a degree equitable dominating set in $G \circ H$, Then S $\cup N(H_v)$ is a degree equitable dominating set in H_v iff one is contained in other.

Proof: Let G and H be any two graphs, Assume S_v = S $\cup N(H_v)$ is a degree equitable dominating set in H_v. *To prove* that One is contained in other, Suppose S is a dominating set in G and N(H_v) is a set which contains every neighborhood of elements of H_v. Let v$\epsilon V(G)$and S$\subseteq V(G \circ H)$. *By lemma 3.A*, S_v= S$\cap N(H_v) \neq \emptyset$,By definition of H_v we have N(H_v) is a subset of V(G) and S $\subseteq V(G)$, Therefore N(H_v) = S.

Conversely, Let S = N(H_v),To prove that S_v = S $\cup N(H_v)$ is a degree equitable dominating set H_v. There exists v $\epsilon\ V(G)$ and let S $\subseteq V(G \circ H)$. By lemma 3.B, S_v= S $\cap N(H_v) \neq \emptyset$, For every v$\epsilon V(G)$ and assumed that S = N(H_v)such that $|N(H_v)| = |N_G(S)|$, Hence S $\cup N(H_v)$ is a degree equitable dominating set in H_v.

F. Lemma: Let G be a regular graph with n≥ 3 and H be any subgraph of G and let S$\subseteq V(G \circ H)$. If S is a dominating set in $G \circ H$ then < S > is the induced subgraph of $G \circ H$ induced by S is the Hamiltonian cycle which is a degree equitable dominating set.

Proof: Given G be a regular graph and H be any subgraph and let S $\subseteq V(G \circ H)$. *By lemma 3.B* .If S is a dominating set in $G \circ H$ then S $\cap V(H_v + v) \neq \emptyset$,$\forall v \epsilon V(G)$, By choosing v_1 $\epsilon\ V(G \circ H)$ and put S_1 ={v_1}, if $N_G[S_1]$ $\neq V(G \circ H)$ and $N_G(S_1)$ = H_{v_1} ,choose, $v_2\epsilon V(G)$ and S_2 = {v_1,v_2} if $N_G[S_2]$ $\neq V(G \circ H)$and $N_G(S_2)$ ={v_1, v_2} where k≥ 3 , Choose $v_k \epsilon\ V(G \circ H)$ and $|N_G(v_k)| = |N_G(S_{k-1})|$ and put S_k ={v_1,v_2,……v_k}, There exists a positive integer k such that $N_G[S_k]$ = V(G) where $N_G(S_k)$ = {v_1,v_2,…….v_k}, since G is regular and each N_G(S) has a v $\epsilon\ G$, Hence every vertex in S is adjacent to atleast two vertices in S as well as another vertex in H_v, Since S is adjacent to atleast two vertices in S having a spanning cycle among the set S is a Hamiltonian cycle is of equal number of degree . Hence S has a Hamiltonian cycle of degree equitable dominating set.

G. Theorem: Let G be a regular graph with n ≥ 3 and H be any subgraph of G and let S $\subseteq V((G - e) \circ H)$ if S is a dominating set in [$(G - e) \circ H$] then < S > is the induced subgraph of $[(G - e) \circ H]$ induced by S is the degree equitable independent dominating set H_v.

Proof: Given G be a regular graph and H be any subgraph and let S$\subseteq V(G \circ H)$. *By lemma 3.B*, If S is a dominating set in (G - e) $\circ H$ then S $\cap V(H_v + v) \neq \emptyset$,$\forall v \epsilon V(G)$.By choosing $v_1\epsilon V((G - e) \circ H)$ and put S_1 = {H_{v_1}}, if $N_G[S_1]$ $\neq V((G - e) \circ H)$ and $N(S_1)$=H_{v_1}, choose $v_2\epsilon V((G - e) \circ H)$ and put S_2 = {H_{v_1}, H_{v_2}}, If $N_G[S_2]$ $\neq V((G - e) \circ H)$ and $N_G(S_2)$ =$\{H_{v_1}, H_{v_2}\}$ where k≥ 3 , choose $v_k\epsilon V((G - e) \circ H)$ and $|N_G(v_k)| = |N_G(S_{k-1})|$

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and put $S_k = \{H_{v_1}, H_{v_2}, \ldots H_{v_k}\}$, since G is not a regular graph each $N_G(S)$ has a H_v . Therefore every vertex in S is adjacent to only vertex in $v \in$ G but in H_v, Hence every vertex in S is independent and degree equitable.

H. Corrolary :

Let G be any graph with n ≥ 3, Let H be any subgraph of G and let $S \subseteq V(G \circ H)$. If S is a dominating set in $G \circ H$, Then < S > is a induced subgraph $G \circ H$ is the degree equitable independent dominating set in H_v.

Proof: It is obvious as the previous theorem.

REFERENCES

[1] R.B. Allan and R. Laskar, *On domination and independent domination numbers of a graph*, Discrete Mathematics, Vol. 23, No. 2, 73-76, 1978.

[2] I.S. Aniversario, F.P. Jamil and S.R. Canoy Jr., *The closed geodetic numbers of graphs*, Utilitas Mathematica, Vol. 74, pp. 3-18, 2007.

[3] C. Berge, *theory of Graphs and its Applications*, Methuen, London, 1962.

[4] F. Buckley, F. Harary. *Distance in graphs*. Redwood City. C. A: Addition-Wesley. 1990.

[5] W. Duckworth and N. C. Wormald, *On the independent domination number of random regular graphs*, Combinatorics, Probabilty and Computing, Vol. 15, 4, 2006.

[6] T. Haynes, S. Hedetniemi and M. Henning, *Domination in graphs applied to electrical power networks*, J. Discrete Math. 15(4), 519-529, 2000.

[7] T.W. Hanes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*. Marcel Dekker, Inc. New York (1998).

[8] O. Ore, *Theory of graphs*, Amer. Math. Soc. Colloq. Publ., Vol.38, Providence, 1962.

[9] L.Sun and J. Wang, *An upper bound for the independent domination number*, Journal of Combinatorial Theory, Vol.76, 2. 240-246, 1999.

[10] H. Walikar, B. Acharya and E. Sampathkumar, *Recent developments in the theory of domination in graphs*, Allahabad, 1, 1979.

[11] Teresa L. Tacabobo and Ferdinand P.Jamil, *Closed Domination in Graphs*, International Mathematical Forum, Vol. 7, 2012, No. 51, 2509-2518.

[12] T.R.Nirmala Vasantha, *A study on Restrained Domination number of a graph*, Thesis 2007, M.S.U, Tirunelveli.

[13] B.D.Acharya, H.B.Walikar, and E.Sampathkumar, *Recent developments in the theory of domination in graphs*. In MRI Lecture Notes in Math. Mehta Research Instit, Allahabad No.1, (1979).