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Connectedness of a Graph from its Degree Sequence and it is Relevent with Reconstruction Conjecture

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Abstract: *A sequence α of nonnegative integers can represent degrees of a graph G and β for the graph H. there may be many different 1-to-1 or 1-to-many mapping functions by which G can be mapped into H. That is it is feasible to construct isomorphic or regular or connected or disconnected graphs. Finding connectedness of a graph from degree sequence is analogues to Reconstruction Conjecture problem. It is our intention in this paper to infer about the connectedness of the graph only from the degree sequence and no need of any other information. It is evident that there is no unique conclusion about the connectedness of a given graph from the algorithm we project here. However, we can say that whether the sequence represents a connected or disconnected graph.*

Keywords: *Graphic Sequence, Connectedness, Regular graph, Graph Isomorphism, Reconstruction Conjecture*

INTRODUCTION

A sequence $\xi = d_1, d_2, d_3, \ldots, d_n$ of nonnegative integers is called a degree sequence of given graph G if the vertices of G can be labeled $V_1, V_2, V_3, \ldots, V_n$ so that degree $V_i = d_i$; for all i=1,2,3,…..n [2]. For a given graph G, a degree sequence of G can be easily determined [1]. Now the question arise, given a sequence $\xi = d_1, d_2, d_3, \ldots, d_n$ of nonnegative integers, then under what conditions does there exist a graph G. A necessary and sufficient condition for a sequence to be graphical was found by Havel and later rediscovered by Hakimi [1,2]. As a sequel of this another question arises, if the sequence $\xi = d_1, d_2, d_3, \ldots, d_n$, be a graphic sequence then is there any condition for which we can say that ξ represents a connected or disconnected Graph sequence[4,5,6,7,8].

PRELIMINARIES

Definition 1: A sequence $\xi = d_1, d_2, d_3,\ldots, d_n$ of nonnegative integers is said to be *graphic sequence* if there exists a graph G whose vertices have degree d_i and G is called *realization* of ξ [1].

Definition 2: For any graph G, we define

$$\delta(G) = \min\{ \deg(v) \mid v \in V(G)\} \text{ and}$$

$$\Delta(G) = \max\{ \deg(v) \mid v \in V(G)\}.$$

If all the vertices of G have the same degree d then $\delta(G) = \Delta(G) = d$ and in this case the graph G is called the *Regular graph* of degree d [1].

Theorem 1: If $e > {}^{(n-1)}C_2$ the simple graph is a connected graph[5].

Proof: $e = {}^{(n-1)}C_2$ represents total number of edges possible for a connected, simple (complete graph) G with $(n-1)$ vertices. Now if $e > {}^{(n-1)}C_2$ the extra edges must connect to the extra vertex with G to form another simple graph G' with n vertices. Hence the graph is connected. Hence the theorem is proved.

Theorem 2: A graph G with n vertices and $\delta \geq (n-1)/2$ is connected[1].

Proof: Suppose G is not connected. Then G has more than one component. Consider any component $G_1=(V_1,E_1)$ of G.

Let, $v_1 \in V_1$. Since, $\delta \geq (n-1)/2$ there exists at least $(n-1)/2$ vertices in G1 adjacent to v1 and hence V1 contains at least $(n-1)/2 + 1 = (n+1)/2$ vertices.

Thus each component of G contains at least $(n+1)/2$ and G has at least two components. Hence, number of vertices in G $\geq n+1$ which is a contradiction. Hence the theorem is proved. ■

Now, we are producing seven necessary criteria for a given simple graph to be a connected or disconnected graph.

Criteria 1

A graph G with n vertices and e edges if $e < (n-1)$, then the graph is a disconnected graph.

Criteria 2

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices and e number of edges. If $(n-1) \leq e \leq {}^{(n-1)}C_2$ then at least one graph is connected for the sequence ξ.

Criteria 3

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices, e number of edges, $\square_r = \square(n-1)/2\rfloor$ (required $\delta(G)$ for connectedness), $d_1 \neq (n-1)$ and $\delta_r \leq d_n$ implies that the sequence ξ represents connected graph sequence.

Criteria 4

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices, e number of edges, $\delta_r = \lfloor(n-1)/2\rfloor$ (required $\delta(G)$ for connectedness),

$\delta_r > d_n$ and $d_n = d_{n-1} = 1$

or

$\delta_r > d_n$ and $d_n \neq 1$

represents at least one graph can be possible with given sequence ξ which is disconnected graph.

Criteria 5

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices, e number of edges, $\delta_r = \lfloor(n-1)/2\rfloor$ (required $\delta(G)$ for connectedness),

$\delta_r > d_n$ and $d_n = 1 < d_{n-1}$

implies that the sequence ξ must be a connected graph sequence and no disconnected graph can be represented by the sequence ξ.

Criteria 6

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices, e number of edges. If $d_1 = (n-1)$ then ξ represents a connected graph sequence.

Criteria 7

Let $\xi = d_1, d_2, d_3, \ldots, d_n$ be the degree sequence of G with n number of vertices, e number of edges. Number of 1's in ξ (which must be even number) is greater than d_1 and Δ of ξ then the sequence ξ is connected sequence.

PROPOSED THEOREM 1

Let ξ be the degree sequence of given graph G. If $\xi = d_1 \geq d_2 \geq d_3 \geq \ldots, d_n$ then ξ is said to be:

(i) Disconnected graph sequence

if $A' = A \oplus C$ is graphic.

And

$B' = B - C$ is graphic

And

$B' \neq \varnothing$

(ii) Connected graph sequence

if $A' = A \oplus C$ is graphic.

And

$B' = B - C = \varnothing$

Or

$B' = B - C \neq \varnothing$

And

B' is not graphic.

Where, the symbol $\oplus$ means the concatenation symbol here

$\xi = d_1 \geq d_2 \geq d_3 \geq \ldots, d_n$ and

$\xi' = d_2\text{-}1 \geq d_3\text{-}1 \geq \ldots, d_n$

$A = d_2\text{-}1 \geq d_3\text{-}1 \geq \ldots, d_{d1}\text{-}1$

$B = d_{d1+2} \geq \ldots, d_n$ and

$C = \{X_i \in B \mid I = d_{1+2}, \ldots, n\}$

$A' = A \oplus C$ and $B' = B - C$.

Proof: Let, the sequence is $\xi = d_1 \geq d_2 \geq d_3 \geq \ldots, d_n$ a collection of non-negative integers. Deleting d_1 and reducing $d_2, d_3, \ldots, d_{d1+1}$ by unity to produce $\xi' = d_2\text{-}1 \geq d_3\text{-}1 \geq \ldots, d_n$. This is also collection of non-negative integers.

Now, $\xi' = A \oplus B$. where, $A = d_2\text{-}1 \geq d_3\text{-}1 \geq \ldots, d_{d1}\text{-}1$ and $B = d_{d1+2} \geq \ldots, d_n$ also $C \subseteq B = \{ X_i \in B \mid I = d_{1+2}, \ldots, n\}$.

Now,

Case I

A′ and B′ are two components of G. Because, successive reduction[S.L.Hakimi] makes A′ to ∅ and B′ to ∅ individually, without effecting each other.
So, the sequence ξ represents at least one graph sequence which is disconnected graph.

Case II

B′ implies ξ representing G, contains a single component. Hence the sequence ξ represents a connected graph sequence and no disconnected graph is possible for the sequence ξ.

■

ANALYSIS WITH EXAMPLES

Example 1:

Let ξ = (3,3,2,2,2,2,2)
ξ′ = (2,2,2,2,1,1)
A=(2,2,2), B=(2,1,1) and C=(2,1,1)
A′ = (2,2,2,2,1,1)
B′ = ∅

Now, A′ is graphic.

Finally C = ∅, A′ = (2,2,2), B′ = (2,1,1)

A′ and B′ both are graphic.

Then we say that ξ represents disconnected sequence.

Example 2:

Let G_θ = The theta graph of degree sequence of (3,3,2,2,2,2)

Then ξ′ = (2,2,2,1,1)
A=(2,1,1) and B = (2,2)

A is graphic but B is not graphic. Then ξ can not represent any disconnected sequence. This is true for the theta graph.

WHERE THE OWNERS PRESENTLY

The Reconstruction Conjecture states that every finite simple symmetric graph for three or more vertices is resolute, up to isomorphism, by its cluster of induced subgraphs [5]. The conjecture was first invented in 1941 and confers a number of associated problems.

CONCLUSION

The algorithms we proposed actually identifies that the given graphic sequence represents any connected or disconnected Graph sequence or not. Any two isomorphic graphs represent the exactly same sequence. However, the converse is not true [1]. So, the proposed theorem and criterions actually determines that if the given graphic sequence is a connected sequence or disconnected sequence or at least one Graph can be possible for the sequence (which is connected or disconnected).

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