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Common Fixed Point Theorems for Six Self Mappings under Integral Type Contractive Condition in Fuzzy Metric Spaces

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ABSTRACT: In this paperwe prove a fixed point theorem for six self mappings by using compatibility of type(β) under the contractive conditions of integral type in Fuzzy Metric Spaces and extends the results on metric and fuzzy metric spaces.

KEYWORDS: Compatible maps, R-weakly commuting maps, Reciprocal Continuity, Compatible of type (β) mappings, Integral Type.

I. INTRODUCTION

In 1965, the concept of fuzzy sets was introduced by Zadeh[1]. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by kramosil and Michalek[2] and George and Veeramani[3] modified the notion of fuzzy metric spaces with the help of continuos t-norms. Recently many authors have proved fixed point theorems involving fuzzy sets. Vasuki[4] investigated some fixed point theorems in fuzzy metric spaces for R-weakly commuting maps and Pant[5] introduced notion of reciprocal continuity of mappings in metric spaces. Pant and Jha[6] proved on analogue of results given by Balasubramaniam et.al.[7]. S. Kutukcu .et.al [8] extended the result of Pant and Jha[6].

Branciari [9] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. P. Vijayaraju et al. [10] discussed a pair of mappings in complete metric spaces satisfying contractive condition of integral type. A.Aliouche et al. [11] discussed the existence of fixed point for the pair of weakly compatible mappings satisfying contractive condition of integral type with (E.A) property in symmetric spaces or metric spaces. Xiaofeng Shao et al.[12] discussed the existence of fixed point for weakly compatible mappings under the contractive conditions of integral type in Fuzzy metric spaces.
The purpose of this paper is to prove a common fixed point theorem in fuzzy metric spaces by compatible mappings of type (β) for six self maps under integral Type Contractive condition which generalize the result of Xiaofeng Shao et al.[12], S. Kutukcu et.al.[8] and others.

II. PRELIMINARIES

Definition 1.1[1]: A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if * is satisfying the following conditions

a) * is commutative and associative,
b) * is continuous,
c) $a*1 = a$ for all $a \in [0,1]$,
d) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1]$.

Definition 1.2 [3]: A 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set. * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$, $s, t > 0$
(fm1) $M(x, y, t) > 0$,
(fm2) $M(x, y, t) = 1$ if and only if $x=y$,
(fm3) $M(x, y, t) = M(y, x, t)$,

(fm4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t+s),
(fm5) M(x, y, •) : [0, ∞) →[0, 1] is left continuous.
Then M is called a fuzzy metric on X.The function M(x, y, t) denote the degree of nearness between x and y with respect to 't'. We identify x=y with M(x, y, t)=1 for all t>0 and M(x, y, t)=0 with ∞, and we can find some topological properties and examples of fuzzy metric spaces in paper of George and Veeramani[3].

Example 1.3 (Induced fuzzy metric[3]) : Let (x, d) be a metric space. Define a*b= ab for all a, b∈ [0, 1] and let M_d be fuzzy sets on $X^2 \times (o, \infty)$ defined as follows
$M_d(x, y, t)=\frac{t}{t+d(x,y)}$ then (X, M_d, *) is a fuzzy metric space. We call this fuzzy metric induced by the metric d, the standard fuzzy metric. On the other hand note that there exists no metric on X satisfying the above $M_d(x, y, t)$.
Definition 1.4 ([14]): let (X, M, *) be fuzzy metric space then

a) A sequence $\{x_n\}$ in X is said to be convergent to a point x in X if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x, t) > 1-\varepsilon$ for all $n \ge n_0$ i.e., $M(x_n, x, t) \to 1$ as $n \to \infty$,for all $t > 0$.

b) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if for each $\varepsilon > 0$ and $t > 0$ there exists $n_0 \in N$ such that $M(x_m, x_n, t) > 1-\varepsilon$ for all $m, n \ge n_0$ i.e., $M(x_m, x_n, t) \to 1$ as $n \to \infty$ for all $t > 0$.

c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 1.5: Since * is continuous, it follows from (fm4) that the limit of the sequence in fuzzy metric space is uniquely determined.
Let (X, M, *) is a fuzzy metric space with the following condition (fm6) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y∈X.

Lemma 1.6[14]: for all x, y ∈ X, M(x, y, •) is non decreasing.
Proposition 1.7[6]: In a fuzzy metric space (X, M, *) if a* a ≥ a for all a∈ [0, 1] then a*b= min{a, b} for all a, b ∈ [0, 1].
Definition 1.8 ([13]): Two self maps A and S of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.
Definition 1.9 ([4]) : Two self maps A and S of a fuzzy metric space (X, M, *) are called weakly commuting if $M(ASx, SAx, t) \ge M(Ax, Sx, t)$ for all x ion X and t>0.
Definition 1.10 ([4]): Two self maps A and S of a fuzzy metric space (X, M, *) are called R-weakly commuting if there exists R> 0 such that $M(ASx, SAx, t) \ge M\left(Ax, Sx, \frac{t}{R}\right)$ for all x in X and t>0.

Remark 1.11 : Clearly, point wise R-weakly commuting implies weak commuting only when R≤1.
Remark 1.12 : Compatible mappings are point wise R-weakly commuting but not conversely.
Definition 1.13[7]: Two self maps A and S of a fuzzy metric space (X, M, *) are called reciprocally continuous on X if $\lim_{n\to\infty} ASx_n = Ax$ and $\lim_{n\to\infty} SAx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.
Remark 1.14: If A and S are both continuous then they are obviously reciprocally continuous but converse is not true. More over in the setting of common fixed point theorems for point wise R-weakly commuting mappings satisfying contractive conditions, continuity of one of the mappings A or S implies their reciprocally continuity but not conversely.
Definition 1.15[13]: Two self mappings A and S of a fuzzy metric space (X, M, *) are called

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compatible of type(β) if $\lim_{n\to\infty} M(AAx_n, SSx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x∈X.

Definition 1.16[12]: $\phi_1 : [0,\infty) \to [0,\infty)$ is said to satisfy the condition (Φ_1) if $\phi_1(t)$ is strictly increasing (Φ_1) $: \phi_1(0) = 0, \lim_{n\to\infty} \phi_1^n(t) = \infty$, $\forall t > 0$ where $\phi_1^n(t) = \phi_1(\phi_1^{n-1}(t))$.

It is easy to prove that if (ϕ_1) satisfies that condition we have $\phi_1(t) > t$ and $\phi_1^n(t) > \phi_1^{n-1}(t), \ \forall t > 0, n = 1, 2, \ldots$

Definition 1.17[12]: $\phi_2 : [0,\infty) \to [0,\infty)$ is said to satisfy the condition (Φ_2) if $\phi_2(t)$ is strictly increasing,

$(\Phi_2) : \phi_2(0) = 0$, $\phi_2\left(\int_0^1 \phi(s)ds\right) = \int_0^1 \phi(s)ds$, $\phi_2(t) > t$, for $t \in \left(0, \int_0^1 \phi(s)ds\right) \cup \left(\int_0^1 \phi(s)ds, \infty\right)$

Lemma 1.18[12]: Let (X, M, *) be a fuzzy metric space with continuoua t-norm *, if

$$\int_0^{M(x,y,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(x,y,t)} \phi(s)ds\right), \ \forall t > 0, \text{ then } x = y.$$

Theorem 1.19[12]: Let (X, M, *) be a fuzzy metric space with continuoua t-norm *,A, B, S, T are self mappings on X, (A, S) and (B, T) are two pairs of weakly compatible mappings.

$A(X) \subseteq T(X), B(X) \subseteq S(X)$, one of A(X), B(X), S(X), T(X) is complete subspace of X, if

$$\int_0^{M(Ax,By,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H} \phi(s)ds\right), \ \forall x, y \in X, t > 0 \text{ where}$$

$H = \min\{M(Sx, Ty, \phi_1(t)), M(Sx, Ax, \phi_1(t)), M(Bx, Ty, \phi_1(t))\}$, then A, B, S, T exist a unique common fixed point.

III. MAIN RESULTS

In this section, we extend the **Theorem 1.19**using compatibility of type(β) for six self maps.

Theorem 2.1: Let (X, M, *) be a complete fuzzy metric space with a*a≥a for all a∈ [0, 1] and the condition (fm6).Let A,B, S, T, P and Q be mappings from X into itself such that

(2.1.1) P(X) ⊂ AB(X), Q(X) ⊂ ST(X)

(2.1.2) AB=BA, ST=TS, PS=SP, QA=AQ, PT=TP, QB=BQ

(2.1.3) The pairs (P, ST) and (Q, AB) are compatible of type (β) and one of P, AB, Q, ST is continuos

(2.1.4) $$\int_0^{M(Px,Qy,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H} \phi(s)ds\right), \ \forall x, y \in X, t > 0 \text{ where}$$

$H = \min\{M(STx, Px, \phi_1(t)), M(ABy, Qy, \phi_1(t)), M(STx, ABy, \phi_1(t)),$
$M(ABy, Px, \alpha\phi_1(t)), M(STx, Qy, (2-\alpha)\phi_1(t))\}$,

for all x, y ∈ X, α ∈ (0, 2) and t>0 then A, S, P, T, Q and B have a unique common fixed point in X .

Proof: By (2.1.1), Since P(X) ⊂ AB(X), for any x_0∈ X, there exists a point x_1∈X such that $Px_0 = ABx_1$,

since $Q(X) \subset ST(X)$ for this point x_1 , we can choose a point $x_2 \in X$ such that $Qx_1=STx_2$.
Inductively we can find a sequence $\{y_n\}$ in X as follows

(2.1.5) $y_{2n}= Px_{2n}=ABx_{2n+1}$ and $y_{2n+1}=Qx_{2n+1}=STx_{2n+2}$ for n=0, 1, 2, 3 ……..

By (2.1.4) for all t>0 and α=1-q with q ∈ (0, 1), we have

$$\int_0^{M(y_{2n},y_{2n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{H_1} \phi(s)ds\right)$$

$$H_1 = \min\{M(STx_{2n}, P x_{2n}, \phi_1(t)), M(AB x_{2n+1}, Q x_{2n+1}, \phi_1(t)), M(ST x_{2n}, AB x_{2n+1}, \phi_1(t)),$$
$$M(AB x_{2n+1}, P x_{2n}, (1-q)\phi_1(t)), M(ST x_{2n}, Q x_{2n}, (1+q)\phi_1(t))\},$$

$$\int_0^{M(y_{2n},y_{2n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{H_2} \phi(s)ds\right)$$

$$H_2 = \min\{M(y_{2n-1}, y_{2n}, \phi_1(t)), M(y_{2n}, y_{2n+1}, \phi_1(t)), M(y_{2n-1}, y_{2n}, \phi_1(t)),$$
$$M(y_{2n+1}, y_{2n}, (1-q)\phi_1(t)), M(y_{2n-1}, y_{2n+1}, (1+q)\phi_1(t))\},$$

$$\ge \phi_2\left(\int_0^{H_3} \phi(s)ds\right)$$

$$H_3 = \min\{M(y_{2n-1}, y_{2n}, \phi_1(t)), M(y_{2n}, y_{2n+1}, \phi_1(t)), M(y_{2n-1}, y_{2n}, \phi_1(t)), M(y_{2n}, y_{2n+1}, q\,\phi_1(t))\}$$

Since t-norm * is continuous, letting q→1, we have

$$\ge \phi_2\left(\int_0^{H_4} \phi(s)ds\right)$$

$$H_4 = \min\{M(y_{2n-1}, y_{2n}, \phi_1(t)), M(y_{2n}, y_{2n+1}, \phi_1(t)), M(y_{2n}, y_{2n+1}, \phi_1(t))\}$$

$$\ge \phi_2\left(\int_0^{\min\{M(y_{2n-1}, y_{2n}, \phi_1(t)), M(y_{2n}, y_{2n+1}, \phi_1(t))\}} \phi(s)ds\right)$$

If $M(y_{2n}, y_{2n+1}, \phi_1(t)) \le M(y_{2n-1}, y_{2n}, \phi_1(t))$

Then $\int_0^{M(y_{2n},y_{2n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{M(y_{2n}, y_{2n+1}, \phi_1(t))} \phi(s)ds\right)$

From **lemma1.18** we get $y_{2n} = y_{2n+1}$ and $M(y_{2n}, y_{2n+1}, \phi_1(t)) = 1$

So we get $M(y_{2n-1}, y_{2n}, \phi_1(t)) > 1$, which is a contradiction. So we have

$\int_0^{M(y_{2n},y_{2n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{M(y_{2n-1}, y_{2n}, \phi_1(t))} \phi(s)ds\right)$ so we get

$\int_0^{M(y_{2n},y_{2n-1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{M(y_{2n-1}, y_{2n-2}, \phi_1(t))} \phi(s)ds\right)$ It follows that

$$\int_0^{M(y_n,y_{n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{M(y_n, y_{n-1}, \phi_1(t))} \phi(s)ds\right)$$

Moreover we have

$$\int_0^{M(y_n,y_{n+1},t)} \phi(s)ds \ge \phi_2\left(\int_0^{M(y_n, y_{n-1}, \phi_1(t))} \phi(s)ds\right)$$

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$$\geq \int_0^{M(y_n,\, y_{n-1},\, \phi_1(t))} \phi(s)ds$$

$$\geq \phi_2\left(\int_0^{M\left(y_{n-1},\, y_{n-2},\, \phi_1^2(t)\right)} \phi(s)ds\right)$$

$$\geq \int_0^{M\left(y_{n-1},\, y_{n-2},\, \phi_1^2(t)\right)} \phi(s)ds$$

$$\geq \ldots\ldots \geq \int_0^{M\left(y_0,\, y_1,\, \phi_1^n(t)\right)} \phi(s)ds$$

Which means $\int_0^{M(y_n, y_{n+1}, t)} \phi(s)ds \geq \int_0^{M\left(y_0,\, y_1,\, \phi_1^n(t)\right)} \phi(s)ds$

Thus we have $M\left(y_n,\, y_{n+1},\, t\right) \leq M\left(y_0,\, y_1,\, \phi_1^n(t)\right)$

For all $k \in N$ we have

$M\left(y_n,\, y_{n+p},\, t\right) \geq M\left(y_n,\, y_{n+1},\, \mathrm{p}^{-1}t\right) * M\left(y_{n+1},\, y_{n+2},\, \mathrm{p}^{-1}t\right)$

$*\ldots\ldots* M\left(y_{n+p-1},\, y_{n+p},\, \mathrm{p}^{-1}t\right)$

$\geq M\left(y_0,\, y_1,\, \phi_1^n(\mathrm{p}^{-1}t)\right) * M\left(y_0,\, y_1,\, \phi_1^{n+1}(\mathrm{p}^{-1}t)\right) *$

$*\ldots\ldots* M\left(y_0,\, y_1,\, \phi_1^{n+p-1}(\mathrm{p}^{-1}t)\right)$

Taking limit on both sides we get

$\lim_{n\to\infty} M\left(y_n,\, y_{n+p},\, t\right) \geq \lim_{n\to\infty} M\left(y_0,\, y_1,\, \phi_1^n(\mathrm{p}^{-1}t)\right) * \lim_{n\to\infty} M\left(y_0,\, y_1,\, \phi_1^{n+1}(\mathrm{p}^{-1}t)\right)$

$*\ldots\ldots* \lim_{n\to\infty} M\left(y_0,\, y_1,\, \phi_1^{n+p-1}(\mathrm{p}^{-1}t)\right)$

It follows that $\lim_{n\to\infty} M\left(y_n,\, y_{n+p},\, t\right) = 1$

Therefore $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, $\{y_n\}$ converges to a point z∈X. Since $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$, $\{ABx_{2n+1}\}$ and $\{STx_{2n+2}\}$ are subsequences of $\{y_n\}$, they also converge to the point z that is

(2.1.6) Px_{2n}, Qx_{2n+1}, ABx_{2n+1}, $STx_{2n+1} \to z$ as $n\to\infty$

Assume P is continuous

(2.1.7) $PPx_{2n} = PABx_{2n+1} \to Pz$ and $PQx_{2n+1} = PSTx_{2n+2} \to Pz$ as $n\to\infty$

Since (P, ST) is compatible mappings of type (β),

We have $\lim_{n\to\infty} M(PPx_{2n}, (ST)(ST)x_{2n}, t) = 1$ and $\lim_{n\to\infty} M(Pz, (ST)(ST)x_{2n}, t) = 1$

which shows that $\lim_{n\to\infty} (ST)(ST)x_{2n} = Pz$ then

(2.1.8) $(ST)(ST)x_{2n} \to Pz$ as $n \to \infty$

Now we have to show that z is the fixed point of P.

taking $x = STx_{2n}, y = x_{2n}$ in (2.1.4)we have

$$\int_0^{M(\mathrm{PST}x_{2n}, \mathrm{Q}x_{2n}, t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_5} \phi(s)ds\right), \text{ where}$$

$H_5 = \min\{M(STSTx_{2n}, \mathrm{PST}\,x_{2n}, \phi_1(\mathrm{t})), M(\mathrm{AB}\,x_{2n+1}, \mathrm{Q}\,x_{2n+1}, \phi_1(\mathrm{t})), M(\mathrm{ST}\,STx_{2n}, \mathrm{AB}\,x_{2n+1}, \phi_1(\mathrm{t})),$
$M(\mathrm{AB}\,x_{2n+1}, \mathrm{PST}\,x_{2n}, \alpha\phi_1(\mathrm{t})), M(\mathrm{STST}\,x_{2n}, \mathrm{Q}\,x_{2n+1}, (2-\alpha)\phi_1(\mathrm{t}))\}$,

Let $n \to \infty$ and α=1 and from (2.1.6), (2.1.7), (2.1.8) we have

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$$\int_0^{M(Pz,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_6} \phi(s)ds\right)$$

$H_6 = \min\{M(\text{Pz},\text{Pz},\phi_1(\text{t})), M(\text{z},\text{z},\phi_1(\text{t})), M(\text{Pz},\text{z},\phi_1(\text{t})),$
$M(\text{z},\text{Pz},\phi_1(\text{t})), M(\text{Pz},\text{z},\phi_1(\text{t}))\}$,

$$\int_0^{M(Pz,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(Pz,z,\phi_1(t))} \phi(s)ds\right)$$ which gives

(2.1.9) Thus Pz=z.

Since P(X) ⊂ AB(X), there exists a point x in X such that z=ABu=Pz
nowwe prove z=Qu.

taking $x = x_{2n}, y = u$ in (2.1.4)we have

$$\int_0^{M(P x_{2n},Qu,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_7} \phi(s)ds\right),$$ where

$H_7 = \min\{M(STx_{2n}, \text{P}\,x_{2n}, \phi_1(\text{t})), M(\text{ABu},\text{Qu},\phi_1(\text{t})), M(\text{ST}\,x_{2n},\text{ABu},\phi_1(\text{t})),$
$M(\text{ABu},\text{P}\,x_{2n},\alpha\phi_1(\text{t})), M(\text{ST}\,x_{2n},\text{Qu},(2-\alpha)\phi_1(\text{t}))\}$,

Let $n \to \infty$ and α=1 and from (2.1.6) we have

$$\int_0^{M(z,Qu,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_6} \phi(s)ds\right)$$ where

$H_8 = \min\{M(\text{z},\text{z},\phi_1(\text{t})), M(\text{z},\text{Qu},\phi_1(\text{t})), M(\text{z},\text{z},\phi_1(\text{t})),$
$M(\text{z},\text{z},\phi_1(\text{t})), M(\text{z},\text{Qu},\phi_1(\text{t}))\}$,

$$\int_0^{M(z,Qu,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(z,Qu,\phi_1(t))} \phi(s)ds\right)$$

which shows z=Qu.

Thus z=Pz=ABu=Qu.

since (Q,AB) is compatible mapping of type(β), we have
M(QQu, (AB) (AB)u, t)=1 and M(Qz, ABz, t)=1
thereforeQz=ABz.
Now, we show that z is the fixed point of Q.
from (2.1.4), (2.1.6) we have

taking $x = x_{2n}, y = z$ in (2.1.4)we have

$$\int_0^{M(P x_{2n},Qz,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_8} \phi(s)ds\right),$$ where

$H_9 = \min\{M(STx_{2n}, \text{P}\,x_{2n}, \phi_1(\text{t})), M(\text{ABz},\text{Qz},\phi_1(\text{t})), M(\text{ST}\,x_{2n},\text{ABz},\phi_1(\text{t})),$
$M(\text{ABz},\text{P}\,x_{2n},\alpha\phi_1(\text{t})), M(\text{ST}\,x_{2n},\text{Qz},(2-\alpha)\phi_1(\text{t}))\}$,

Let $n \to \infty$ and α=1

$$\int_0^{M(z,Qz,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_9} \phi(s)ds\right)$$

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where

$$H_{10} = \min\{M(z,z,\phi_1(t)), M(Qz,Qz,\phi_1(t)), M(z,Qz,\phi_1(t)),$$
$$M(z,z,\phi_1(t)), M(z,Qz,\phi_1(t))\},$$

$$\int_0^{M(z,Qz,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(z,Qz,\phi_1(t))} \phi(s)ds\right)$$

(2.1.10) which shows z=Qz

thus z=Pz=Qz=ABz

Since Q(X) ⊂ ST(X) there exists a ν in X such that z=Qz=STν

Now we prove z=Pν

taking $x = \nu, y = z$ in (2.1.4) and from (2.1.6))we have

$$\int_0^{M(P\nu,Qz,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{10}} \phi(s)ds\right), \text{ where}$$

$$H_{11} = \min\{M(ST\nu, P\nu, \phi_1(t)), M(ABz, Qz, \phi_1(t)), M(ST\nu, ABz, \phi_1(t)),$$
$$M(ABz, P\nu, \alpha\phi_1(t)), M(ST\nu, Qz, (2-\alpha)\phi_1(t))\},$$

Let $n \to \infty$ and α=1

$$\int_0^{M(P\nu,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{11}} \phi(s)ds\right)$$

where

$$H_{12} = \min\{M(z,p\nu,\phi_1(t)), M(z,z,\phi_1(t)), M(z,z,\phi_1(t)),$$
$$M(z,P\nu,\phi_1(t)), M(z,z,\phi_1(t))\},$$

$$\int_0^{M(P\nu,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(P\nu,z,\phi_1(t))} \phi(s)ds\right)$$

which shows z=Pν

Since (P, ST) are compatible mappings of type(β)

We have M(PPν, ST(STν), T)=1 and M(Pz, STz, t)=1

which shows Pz=STz

(2.1.11) thus z=Pz=ABz=Qz=STz

Now we show that z=Tz

taking $x = Tz, y = x_{2n+1}$ in (2.1.4)we have

$$\int_0^{M(PTz,Qx_{2n+1},t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{12}} \phi(s)ds\right), \text{ where}$$

$$H_{13} = \min\{M(STTz, PTz, \phi_1(t)), M(ABx_{2n+1}, Qx_{2n+1}, \phi_1(t)), M(STTz, ABx_{2n+1}, \phi_1(t)),$$
$$M(ABx_{2n+1}, PTz, \alpha\phi_1(t)), M(STTz, Qx_{2n+1}, (2-\alpha)\phi_1(t))\},$$

taking $n \to \infty$ and α=1 and from (2.1.6)

$$\int_0^{M(Tz,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{13}} \phi(s)ds\right)$$

where

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$$H_{14} = \min\{M(\mathrm{Tz},\mathrm{T}z,\phi_1(\mathrm{t})), M(\mathrm{z},\mathrm{z},\phi_1(\mathrm{t})), M(\mathrm{Tz},\mathrm{z},\phi_1(\mathrm{t})),$$
$$M(\mathrm{z},\mathrm{T}z,\phi_1(\mathrm{t})), M(\mathrm{Tz},\mathrm{z},\phi_1(\mathrm{t}))\},$$

$$\int_0^{M(\mathrm{T}z,z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(\mathrm{T}z,z,\phi_1(t))} \phi(s)ds\right)$$

(2.1.12) which showsTz=z
Since STz=z therefore
(2.1.13) Sz=z,
Finally we have to show that Bz=z.
taking $x = z, y = Bz$ in (2.1.4)we have

$$\int_0^{M(\mathrm{Pz},\mathrm{QB}z,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{14}} \phi(s)ds\right),$$ where

$$H_{15} = \min\{M(STz,\mathrm{P}z,\phi_1(\mathrm{t})), M(\mathrm{ABB}z,\mathrm{QB}z,\phi_1(\mathrm{t})), M(\mathrm{ST}z,\mathrm{ABB}z,\phi_1(\mathrm{t})),$$
$$M(\mathrm{ABB}z,\mathrm{P}z,\alpha\phi_1(\mathrm{t})), M(\mathrm{ST}z,\mathrm{QB}z,(2-\alpha)\phi_1(\mathrm{t}))\},$$

taking $n \to \infty$ and α=1 and from (2.1.6)

$$\int_0^{M(z,\mathrm{Bz},t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{15}} \phi(s)ds\right)$$

where

$$H_{16} = \min\{M(\mathrm{z},z,\phi_1(\mathrm{t})), M(\mathrm{z},\mathrm{Bz},\phi_1(\mathrm{t})), M(\mathrm{Tz},\mathrm{z},\phi_1(\mathrm{t})),$$
$$M(\mathrm{z},z,\phi_1(\mathrm{t})), M(\mathrm{z},\mathrm{z},\phi_1(\mathrm{t}))\},$$

$$\int_0^{M(z,\mathrm{Bz},t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(z,\mathrm{Bz},\phi_1(t))} \phi(s)ds\right)$$

(2.1.14) therefore z=Bz.
Since ABz=z therefore
(2.1.15) Az=z.
By combining the above results, we have Az=Bz=Sz=Tz=Pz=Qz=z.
That is z is the common fixed point of A, B, S, T, P and Q.
For uniqueness, Let w(w≠z) be another common fixed point of A,B,S,T,P and Q.
then by taking $x = z, y = w$ in (2.1.4)we have

$$\int_0^{M(\mathrm{Pz},\mathrm{Q}w,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{16}} \phi(s)ds\right),$$ where

$$H_{17} = \min\{M(STz,\mathrm{P}z,\phi_1(\mathrm{t})), M(\mathrm{AB}w,\mathrm{Q}w,\phi_1(\mathrm{t})), M(\mathrm{ST}z,\mathrm{AB}w,\phi_1(\mathrm{t})),$$
$$M(\mathrm{AB}w,\mathrm{P}z,\alpha\phi_1(\mathrm{t})), M(\mathrm{ST}z,\mathrm{Q}w,(2-\alpha)\phi_1(\mathrm{t}))\},$$

taking $n \to \infty$ and α=1 and (2.1.6)

$$\int_0^{M(z,w,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H_{17}} \phi(s)ds\right)$$

where

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$H_{18} = \min\{M(z, z, \phi_1(t)), M(w, w, \phi_1(t)), M(z, w, \phi_1(t)),$
$M(w, z, \phi_1(t)), M(z, w, \phi_1(t))\}$,

$$\int_0^{M(z,w,t)} \phi(s)ds \geq \phi_2\left(\int_0^{M(z,w,\phi_1(t))} \phi(s)ds\right)$$

Which shows z=w.

If we put B=T=I_x (the identity map on X) in **Theorem 2.1**We have the following.

Corollary 2.2 :let (X, M, *) be a complete fuzzy metric space with a*a≥ a for all a∈ [0, 1] and the condition (fm6), let A, S, T be mapping from X into itself such that
(2.2.1) P(X) ⊂ A(X), Q(X) ⊂ S(X)
(2.2.2) The pairs (P, S) and (Q, A) are compatible of type (β) and one of P, A, Q and S is continuous.
(2.2.3) There exists a number k ∈ (0, 1) such that

$$\int_0^{M(Px,Qy,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H} \phi(s)ds\right), \forall x, y \in X, t > 0 \text{ where}$$

$H = \min\{M(Sx, Px, \phi_1(t)), M(Ay, Qy, \phi_1(t)), M(Sx, Ay, \phi_1(t)),$
$M(Ay, Px, \alpha\phi_1(t)), M(Sx, Qy, (2-\alpha)\phi_1(t))\}$,

for all x, y ЄX, αЄ(0, 2) and t>0, then A, B, P and Q have a unique common fixed point in X.

If we put P=Q, B=T=I_x in the **Theorem 2.1**we have the following.

Corollary 2.3: Let (X, M, *) be a complete fuzzy metric space with a*a≥ a for all a∈ [0, 1] and the condition (fm6), let A, S, T be mapping from X into itself such that
(2.3.1) P(X) ⊂ A(X), P(X) ⊂ S(X)
(2.3.2) The pairs (P, S) and (P, A) are compatible of type (β) and one of P, A, S is continuous.
(2.3.3) There exists a number k ∈ (0, 1) such that

$$\int_0^{M(Px,Qy,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H} \phi(s)ds\right), \forall x, y \in X, t > 0 \text{ where}$$

$H = \min\{M(Sx, Px, \phi_1(t)), M(Ay, Py, \phi_1(t)), M(Sx, Ay, \phi_1(t)),$
$M(Ay, Px, \alpha\phi_1(t)), M(Sx, Py, (2-\alpha)\phi_1(t))\}$,

for all x, y ∈X, α∈ (0, 2) and t>0 then P, S, A have a unique common fixed point in X.

If we put B=T=I_x , P=Q and A=S in **Theorem 2.1**we have the following.

Corollary 2.4: Let (X, M, *) be a complete fuzzy metric space with a*a≥ a for all a∈ [0, 1] and the condition (fm6). Let P, S be compatible mappings of type (β) on X into itself such that
P(X) ⊂ S(X) and there exists a constant k ∈ (0, 1) such that

$$\int_0^{M(Px,Qy,t)} \phi(s)ds \geq \phi_2\left(\int_0^{H} \phi(s)ds\right), \forall x, y \in X, t > 0 \text{ where}$$

$H = \min\{M(Sx, Px, \phi_1(t)), M(Sy, Py, \phi_1(t)), M(Sx, Sy, \phi_1(t)),$
$M(Sy, Px, \alpha\phi_1(t)), M(Sx, Py, (2-\alpha)\phi_1(t))\}$,

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for all x, y∈X, α∈ (0, 2) and t> 0 then P and S have a unique common fixed point in X.

If we put A=S and B=T=I_x in the **Theorem 2.1** we have the following.

Corollary 2.5: Let (X, M, *) be complete fuzzy metric space with a*a≥a for all a∈ [0, 1] and the condition (fm6).Let P, Q and S be mappings from X to itself such that

(2.5.1) P(X) ⊂ S(X), Q(X) ⊂ S(X)

(2.5.2) The pairs (P, S) or (Q, S) is compatibility of type (β) and one of P, S, Q is continuous

(2.5.3) $\int_0^{M(Px,Qy,t)} \phi(s)ds \geq \phi_2\left(\int_0^H \phi(s)ds\right)$, $\forall x, y \in X, t > 0$ where

$H = \min\{M(Sx, Px, \phi_1(t)), M(Sy, Qy, \phi_1(t)), M(Sx, Sy, \phi_1(t)),$

$M(Sy, Px, \alpha\phi_1(t)), M(Sx, Qy, (2-\alpha)\phi_1(t))\}$,

for all x, y ∈ X, α ∈ (0, 2) and t>0 then P, Q, S have a unique common fixed point in X.

IV.CONCLUSION

This article investigates common fixed point theorems for six self mappings. The concept of compatible mappings of type-beta in Fuzzy metric spaces using integral type contractive conditions has also been used. Several Fixed point theorems in Fuzzy metric spaces such as fixed point theorems for four, three and two self mappings have been derived in the present study as particular cases.

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