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AN INTRODUCTION TO QUANTUM NEURAL COMPUTING

Shaktikanta Nayak*[1], Sitakanta Nayak[2] and Prof. J.P.Singh[3]
[1, 2] PhD student, Department of Management Studies, Indian Institute of Technology, Roorkee
[3] Professors, Department of Management Studies, Indian Institute of Technology, Roorkee, Uttarakhand (India)-247667
[1]shakti302@gmail.com
[2]purieskay@gmail.com
[3]jpsiitr@gmail.com

Abstract: The goal of the artificial neural network is to create powerful artificial problem solving systems. The field of quantum computation applies ideas from quantum mechanics to the study of computation and has made interesting progress. Quantum Neural Network (QNN) is one of the new paradigms built upon the combination of classical neural computation and quantum computation. It is argued that the study of QNN may explain the brain functionality in a better way and create new systems for information processing including solving some classically intractable problems. In this paper we have given an introductory representation of quantum artificial neural network to show how it can be modelled on the basis of double-slit experiment. Also an attempt is made to show the quantum mechanical representation of a classical neuron to implement Hadamard transformation.

INTRODUCTION

The strong demand in neural and quantum computation is driven by the limitations in the hardware implementation of classical computation. Classical computers efficiently process numbers and symbols with relatively short bit registers d < 128.But it has two major limitations to process Patterns, which are wide-band signals having d>100 bits. The first shortcoming is due to the hardware implementation of it i.e. in case of pattern processing, classical computers require enormous number of gates $\propto d^{4.8}$ (According to Rent's Law) to process d-bit registers [9]. On the other hand a typical computer program able to perform universal calculations on patterns requires $\propto 2^d$ operators [10].This fact excludes the possibility to use algorithmic approach for pattern processing. Artificial neural network (ANN) can solve this problem because it uses the novel architecture which is able to process long bit strings and learning by example not by programming.ANN can solve complex problems which have poor knowledge domains.ANN also have some other features like parallel distributed processing and robustness. The main objective of Quantum computation is to minimize the size of the computer elements, which will be governed by the quantum laws.

The research in quantum computing deals with the quantum analog of classical computational architecture which operates with quantum bits and quantum gates. Quantum computers retain many features inherent in classical computers. They cannot operate wide band signals and cannot be simply trained by examples. Their efficiency will depend on the powerful quantum algorithms.

Classical neural networks also face many problems like absence of rule for optimal architectures, time consuming training, limited memory capacity. Quantum computation based on the quantum mechanical nature of physics [3, 4, 5] , which is inherently a parallel distributed processor having exponential memory capacity and easily trainable, but it has severe hardware limitations. Quantum computation is a linear theory but ANN depends upon non linear approach. The field of ANN contains several important ideas, which

include the concept of a processing element (neuron), the transformation performed by this element (in general, input summation and nonlinear mapping of the result in to an output value), the interconnection structure between neurons, the network dynamics and the learning rule which governs the modification of interaction strengths.

Hence it is needed to combine both quantum computation and neural computation (Which is called Quantum Neural Network) to overcome the difficulties of classical computers, quantum computers and neurocomputers [1-2]. Many researchers use their own analogies in establishing a connection between quantum mechanics and neural networks [7]. The power of ANN is due to their massive parallel, distributed processing of information and due to the nonlinearity of the transformation performed by the network nodes (neurons) .On the other hand; quantum mechanics offers the possibility of an even more powerful quantum parallelism which is expressed in the principle of superposition .Thus we can apply quantum computing principles and algorithms in neural network for processing large data sets.

The rest of the paper is organized as: section-2 introduces concepts of quantum computation, section-3 quantum concepts used in computation, section-4 how the quantum computer works, section-5 motivation towards quantum neural computing, section-6 mathematical representation of quantum neuron, section-7 learning rule, section-8 advantage over classical neural network, section-9 implementation obstacles and finally the conclusion and references.

QUANTUM COMPUTATION

In the classical computation information stored in Von Neumann computer is in the form of independent binary bits {0, 1} .These classical bits can be represented in a scalar space .The possible outcome in any classical measurement will be either 0 or 1 .Hence in classical computer the storage, manipulation and measurement of information are done through classical bits .Where as in quantum computation (QC) the information is stored, manipulated and measured in the form of qubits. The qubit is a physical entity described by quantum mechanical principles.

Physically, two spin states of a photon in a magnetic field represent quantum bits or qubits. But the basic difference between the classical bit and qubit depends upon the independentness of the bits/qubits. In QC the qubits are not independent, which leads towards high speed computing capacity i.e. the parallel processing ability .Mathematically a qubit is represented as a vector $|\Psi\rangle$ in a two dimensional complex vector space .The qubit $|\Psi\rangle$ can be written in general form as $|\Psi\rangle = \alpha\,|p\rangle + \beta\,|q\rangle$, where the complex coefficients α and β satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.Hence $|\Psi\rangle$ is the superposition of the basis states $|p\rangle$ and $|q\rangle$ and can be represented in an infinite number of ways simply by varying values of the coefficients α and β subject to the normalization condition. Two types of operations under go in a quantum system: measurement and quantum state transformations. In classical computing set of universal gates are used for computational purpose, where as in QC most algorithms follow a sequence of quantum state transformations followed by measurement. Actual quantum computation processes are very different from that of classical counterpart. In classical computer we give input data through the input devices.

The input signal is stored in computer memory, then fed into the microprocessor and the result is stored in memory before it is displayed in the screen .Thus the information travels around the circuit .In contrast, information in quantum computation is first stored in a register and then external fields , such as oscillating magnetic fields , electric fields or laser beams are applied to produce gate operations on the register. These external fields are designed so that they produce desired gate operation, i.e. unitary matrix acting on a particular set of qubits .Hence information sits in the register and they are updated each time the gate operation acts on the register.

QUANTUM CONCEPTS USED IN COMPUTATION

The main concepts of quantum mechanics are wave function, superposition, coherence and decoherence, operators, measurement, entanglement, and unitary transformations.

Superposition:-

Linear superposition is the physical representation of mathematical principle of linear combination of vectors. The wave function Ψ , which describes the quantum system, exists in a complex Hilbert space .The Hilbert space has a set of states Φ that form a basis .Hence the quantum system can be described mathematically as follows:

$$|\Psi\rangle = \sum_i C_i \; |\Phi_i\rangle$$

Where $|\Psi\rangle$ is the superposition of basis states $|\Phi_i\rangle$ and C_i is the complex coefficients.

Coherence and Decoherence:-

A quantum system is said to be coherent if it is a linear superposition of its basis states. If a quantum system is in a linear super position of states and interacts with its environment, the superposition is destroyed. This loss of coherence is called decoherence.

Operator:-

Operator transforms one wave function in to another .It is represented by capital letter with a hat and is represented by matrix. Mathematically,

$\hat{A}|\Phi_i\rangle = a_i|\Phi_i\rangle$, Where a_i is the eigenvalue and the solution of this equation are called eigen states, which are used to construct the Hilbert space.

ENTANGLEMENT

It is a quantum mechanical phenomenon to establish correlation between two or more quantum states. Entanglement seems to be intuitive from computational point of view but it is little understood from physical stand point of view. The correlation between states comes into picture when states exist as superpositions .When superposition is destroyed the proper correlation is somehow communicated between the qubits and this communication is achieved due to entanglement. The power of quantum computation derives from the exponential state spaces of multiple quantum bits: just as a single qubit can be in a superposition of 0 and 1, a register of n qubits can be in a superposition of all 2^n possible values. The “extra” states that have no classical analog and lead to the exponential size of the quantum state space are the entangled states. Mathematically entanglement is described using density matrix formalism [11-13]

HOW THE QUANTUM COMPUTER WORKS

Consider a classical register has 3 bits, then possible outcome will be $2^3=8$ (it would be possible to use this register to represent any one of the numbers from 0 to 7 at any instance of time).If we will consider a register of 3 qubits, then the register can represent all the numbers from 0 to 7 simultaneously. A single processor having qubit registers will be able to perform calculations using all possible values of the input registers simultaneously. This phenomenon is called quantum parallelism. Unlike classical bits qubits can exist simultaneously as 0 and 1, with probability for each state given by numerical coefficients. A quantum computation [11, 12] is a collection of the following three elements:

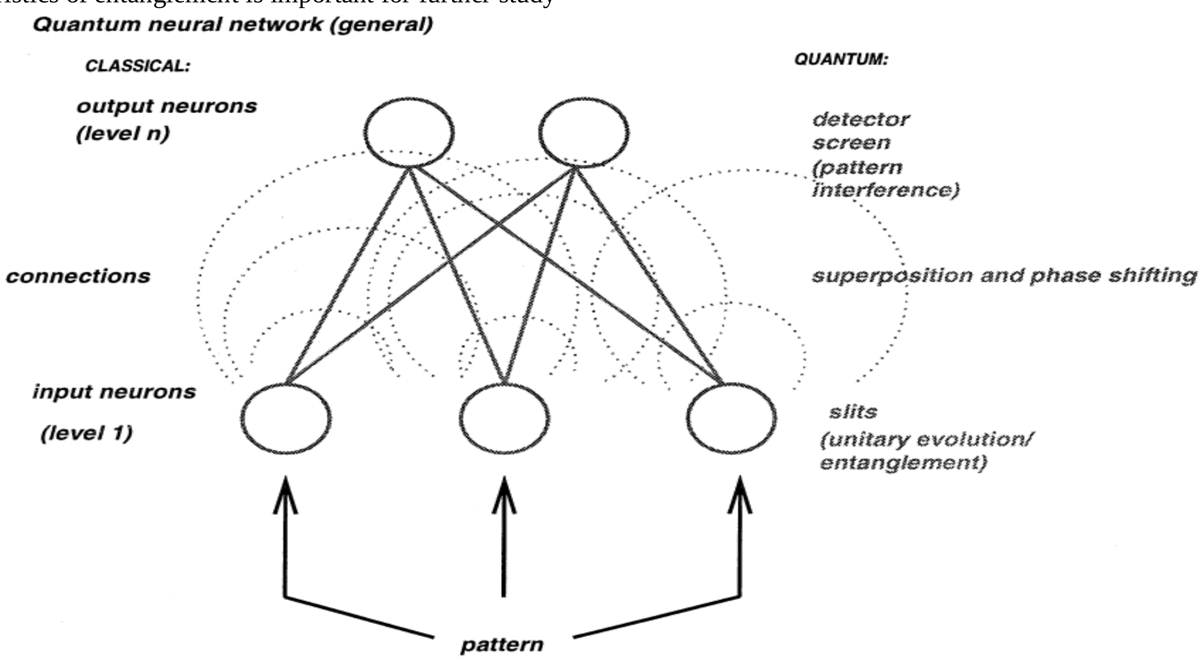
(i) A register or a set of registers.
(ii) A unitary matrix, which is used to execute the quantum algorithms.
(iii) Measurements to extract information.

Hence quantum computation is the set $\{H, U, \{M_m\}\}$, where H is the Hilbert space of n-qubit register .$U \in U(2^n)$ is the quantum algorithm. $\{M_m\}$ is the set of measurement operators .Actual quantum computation processes are very different from that of classical counterpart. In classical computer we give input data through the input devices .The input signal is stored in computer memory, then fed into the microprocessor and the result is stored in memory before it is displayed in the screen .Thus the information travels around the circuit .In contrast information in quantum computation is first stored in a register and then external fields such as oscillating magnetic fields, electric fields or laser beams are applied to produce gate operations on the register. These external fields are designed so that they produce desired gate operation, i.e. unitary matrix acting on a particular set of qubits .Hence information sits in the register and they are updated each time the gate operation acts on the register.

MOTIVATION TOWARDS QUANTUM NEURAL COMPUTING

Quantum computing (computation based on principles of quantum mechanics) is becoming popular due to its massive parallel information processing ability though it is facing problem to implement physical hard ware. The concept of classical computing based on principles of quantum physics is first introduced by Beniof [3], Feynman [4] and formalized by Deutsch [5] in 1985.Quantum computing became popular after shore's factorization algorithm [6]. The motivation for applying quantum computing to the field of artificial neural computing is the natural step to make the generalization of classical neurocomputing to quantum neurocomputing in a hope to enhance the computational capabilities not available using classical neurocomputing. In order to get super classical computational capacity over classical neural computing the quantum mechanical characteristics of entanglement is important for further study for combining quantum computation with neural computation .

The new field which integrates classical neurocomputing with quantum computing is quantum neural computing. Quantum computation is a linear theory but ANN depends upon non linear approach. The field of ANN contains several important ideas, which includes the concept of a processing element (neuron), the transformation performed by this element (in general, input summation and nonlinear mapping of the result in to an output value), the interconnection structure between neurons, the network dynamics and the learning rule which governs the modification of interaction strengths. The main concepts of quantum mechanics are wave function, superposition (coherence), measurement (Decoherence), entanglement, and unitary transformations. In order to establish such correspondence is a major challenge in the development of a model of QNN. Lewestein [14] proposed quantum perceptron, where classical weights in the perceptron are replaced as unitary operator to map input to output. Chrisly proposed a design for feed forward artificial neural network using double-slit experiment. Menneer and Narayanan [15] have extended chrisly proposal for single pattern quantum neural network .Where the concept of multiple universes or superposition principle of quantum theory is applied to neural computing .The architecture of double-slit experiment provides the basis for quantum artificial neural network .Hence a quantum neural network can be modelled on the basis of double-slit experiment, where the photon gun is equivalent to the input pattern, the slit is equivalent to input neurons, the waves between the slits and the screen is equivalent to the connections between the input and output neurons and the screen is equivalent to the output neurons.



ANN	DOUBLESLIT EXPT. SET UP	QNN
Pattern	Photon gun	Quantum Register holds input Pattern
Input Neuron	Slits	Entanglement/unitary evolution of input pattern
Connection	Waves	Superposition of waves created by input pattern
Output neuron	Detector screen	Pattern Interference

Weight	Link	Phase shift

MATHEMETICAL REPRESENTATION OF A QUANTUM NEURON ARTIFICIAL NEURON

A classical artificial neuron is an adjustable unit performing, in general, a non linear mapping of the set of many, N, input (perceptron) values x_1 ...... x_N to a single output value Y. The output value of a classical perceptron is [6]

$$Y= f\left(\sum_{j=1}^{N} w_j x_j\right) \quad \text{.. (1),}$$

where f ( ) is the perceptron activation function and w_j are the weights tuning during learning process.

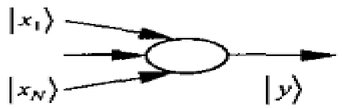
The perceptron learning algorithm works as follows:

1. The weights w_j are initialized to small random numbers.
2. A *pattern* vector ($x_{1........}x_N$) is presented to the perceptron and the output Y generated according to the rule -1.
3. The weights are updated according to the rule $w_j(t+1) = w_j(t) + \eta(d - Y)$........ ..(2), Where *t* is discreet time, *d* is the desired output provided for training and 0<η<1 is the step size.

Quantum Neuron:-

Many researchers use their own analogies in establishing a connection between quantum mechanics and neural networks [7]. The power of ANN is due to their massive parallel, distributed processing of information and due to the nonlinearity of the transformation performed by the network nodes (neurons) .On the other hand; quantum mechanics offers the possibility of an even more powerful quantum parallelism which is expressed in the principle of superposition .This principle provides the advantage in processing large data sets.

Consider a neuron with N inputs$|x_1\rangle$.......$|x_N\rangle$ as shown in the figure.



$|x_j\rangle$ is a quantum bit of the form $|x_1\rangle = a_j|0\rangle + b_j|1\rangle = (a_j, b_j)^T$................................(3) ,where $|a_j|^2 + |b_j|^2 =$

1.The output $|y\rangle$ can be derived by the rule [8]

$$|y\rangle = \hat{F}\sum_{j=1}^{N} \widehat{w_j}\,|x_j\rangle\text{,...(4).}$$

Where $\widehat{w_j}$ is a 2$\times$ 2 matrices acting on the basis {$|0\rangle, |1\rangle$} and $\hat{F}$ is an unknown operator that can be implemented by the network of quantum gates. Let us consider $\hat{F} = \hat{I}$ be the identity operator .The output of the perceptron at the time t will be

$$|y\ (t)\rangle = \hat{F}\sum_{j=1}^{N} \widehat{w_j}(\mathrm{t})\,|x_j\rangle \quad \text{................................. (5)}$$

In analogy with classical case equation (2) we can update the weights as follows

$$\hat{w}_j(\mathrm{t}+1)= \hat{w}_j(\mathrm{t})+\eta(|d\rangle - |y\ (t)\rangle)\langle x_j| \text{…………….(6)}$$

Where $|d\rangle$the desired output .It is is can be shown eq. (6) derives the quantum neuron in to desired state$|d\rangle$. Using rule (6) and taking modulo-square difference of real and desired outputs, we can get

$$\||d\rangle - |y\ (t+1)\rangle\|^2$$
$$= \left\||d\rangle - \sum_{j=1}^{N} \widehat{w_j}(\mathrm{t}+1)\,|x_j\rangle\right\|^2$$
$$= \left\||d\rangle - \sum_{j=1}^{N} \widehat{w_j}(\mathrm{t})\,|x_j\rangle + \eta\,(|d\rangle - |y\ (t)\rangle\,\langle x_j|x_j\rangle)\right\|^2$$
$$= \left\||d\rangle - |y\ (t)\rangle - \sum_{j=1}^{N} \eta(|d\rangle - |y\ (t)\rangle)\right\|^2$$
$$=(1-N\eta)^2\,\||d\rangle - |y\ (t)\rangle\|^2 \text{……………………….. (7)}$$

For small η (0< η< $\frac{1}{N}$) and normalized input states the result of iteration converges to the desired state$|d\rangle$.The whole network can be then composed from the primitives elements using the standard rules of ANN architecture .

LEARNING RULE

The gradient-descent-based algorithm can be used for training the QNN.

Step-1 Choose η >0and lower limit of error E_{min}.

Step-2 Initialize $\hat{w}_j = \hat{w}_j(0)$, as a random matrix, set error E=0 and step counter t=1

Step-3 Calculate output of QNN

$$|y\ (t)\rangle = \hat{F}\sum_{j=1}^{N} \widehat{w_j}(\mathrm{t})\,|x_j\rangle$$

Step-4 Update the weight according to learning rule

$$\hat{w}_j(\mathrm{t}+1)= \hat{w}_j(\mathrm{t})+\eta(|d\rangle - |y\ (t)\rangle)\langle x_j|$$

Step-5 Calculate the error

$$E= \||d\rangle - |y\ (t)\rangle\|^2$$

Step-6 If E< E_{min}. go to step-7 otherwise set t=t+1

Step-7 Save the weight matrix $\hat{w}_j$ and stop.

APPLICATION

In quantum information processing the Hadamard transformation or Hadamard gate is a one qubit rotation mapping the qubit basis states $|0\rangle$and $|1\rangle$to two superposition states with equal weight of the computational basis states $|0\rangle$and $|1\rangle$.Quantum algorithms use Hadamard gate as an initial step to map n-qubits with $|0\rangle$to a superposition of all 2^n orthogonal states in the$|0\rangle$, $|1\rangle$ basis with equal weight. When the Hadamard gate is applied twice in succession, then the final state is always the same as the initial state. Here we will see how the Hadamard transformation can be implemented using quantum artificial neural network. Considering a quantum neuron with input $|x\rangle$ and output Y= $f\left(\sum_{j=1}^{N} w_j x_j\right)$

Taking weight $\hat{w}$ and operator $\hat{F}$ as

$\hat{w} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and

$$\hat{F} = \hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (8)$$

Where $\hat{H}$ is the Walash Hadamard transformation.

When input $|x\rangle = |0\rangle$

$$|y\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

When input $|x\rangle = |1\rangle$

$$|y\rangle = |0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

When $\hat{H}$ is applied to n qubits in the state $|0\rangle$ individually, $\hat{H}$ generates a superposition of all 2^n possible states .i.e. 0 to $2^n - 1$.

$$(\hat{H} \otimes \hat{H} \otimes \ldots \otimes \hat{H}) \, |0000 \ldots 0\rangle$$

$$= \frac{1}{\sqrt{2^n}} \; (|0\rangle + |1\rangle) \;\otimes\; (|0\rangle + |1\rangle) \otimes \ldots \otimes (|0\rangle + |1\rangle) \, |0000 \ldots 0\rangle$$

$$= \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n - 1} |x_j\rangle \quad (9)$$

Comparing equation (9) with equation (4)

We can conclude that Walash-Hardamard transformation can be implemented by a quantum neuron with 2^n inputs and one output choosing $\hat{w} = \frac{1}{\sqrt{2^n}} \hat{I}$ and $\hat{F} = \hat{I}$.

ADVENTAGE OVER ANN

1. N-set of training patterns forms a set of N-homogenous components in QNN where each training pattern is channelled to one component part and the set of weights connected to this component is changed to learn this training pattern .Interference does not take place in the leaning patterns due to the independent components for each training pattern .But in CNN there is most possibility of pattern interference where the network unlearns i.e. a form of catastrophic forgetting.
2. Patterns are separated by using different components in QNN .So there is no necessity of decision plane to separate patterns in to classes which is required in CNN.
3. The components can be trained as a superposition of networks.
4. Also we can train classical networks and combine in to superposition later.
5. Less training time is required as compared to classical neural network.

OBSTACLES FOR IMPLIMENTATION

(i) Quantum Coherence –The system should maintain coherence until the computation is complete. But due to the interaction of the system with the environment, it is not possible to maintain the coherence.

(ii) Connections –The connections are described by entanglement of qubits. The measurement of entanglement is the main obstacle.

CONCLUSION

In this paper we have discussed the basic concepts of quantum neural computing and a mathematical model of quantum neuron to discuss the training algorithm. We have explained the implementation of Hadamard transformation using QNN. A comparison is made between ANN and QNN. Finally the obstacles to implement the QNN are discussed.

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