ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

An approximation method for the solving a class of nonlinear integral equations

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Abstract: A Chebyshev collocation method has been presented to solve nonlinear integral equations in terms of Chebyshev polynomials. This method transforms the integral equation to a matrix equation which corresponds to a system of nonlinear algebraic equations with unknown Chebyshev coefficients. Finally, some examples are presented to illustrate the method and results discussed.

Keywords: Chebyshev collocation method; Fredholm–Volterra integral equations; Nonlinear integral equation

I. INTRODUCTION

As we know the Chebyshev polynomials are one of the best orthogonal polynomials that have important role particularly in numerical analysis. AChebyshev-matrix method for solving nonlinear integral equations have been presented by Sezer and Doğan [15]. In this study, Chebyshev collocation method, which is given by Akyüz and Sezer [3], is developed for nonlinear integral equation.
Fredholm nonlinear integral equation of the second kind and first kind are given by

$$y(s) + p(s)y\big(h(s)\big) + \lambda \int_a^b k(s,t)[y(t)]^3\, dt = g(s)\,, \quad a \le s \le b\,,$$

and

$$p(s)y\big(h(s)\big) + \lambda \int_a^b k(s,t)[y(t)]^3 dt = g(s)\,, \qquad a \le s \le b\,,$$

respectively where λ is a real parameter. Also Volterra nonlinear integral equation of the second and first kind are given by

$$y(s) + p(s)y\big(h(s)\big) + \lambda \int_a^s k(s,t)[y(t)]^3 dt = g(s)\,, \qquad a \le s \le b\,,$$

and

$$p(s)y\big(h(s)\big) + \lambda \int_a^s k(s,t)[y(t)]^3 dt = g(s)\,, \qquad a \le s \le b\,.$$

respectively. Here we assume that all functions are defined in interval [-1,1], otherwise by using suitable change of variable we obtain this interval, which is the domain of the Chebyshev polynomials of the first kind. In this paper, we approximate the solution of nonlinear IE using Chebyshev basis and the method of collocation with Chebyshev points. This method had been used for systems of nonlinear IE [17]. First we consider Fredholm nonlinear integral equations of the second and first kind as

$$y(s) + p(s)y\big(h(s)\big) + \lambda \int_{-1}^1 k(s,t)[y(t)]^3 dt = g(s)\,, \qquad -1 \le s \le 1, \tag{1}$$

and

$$p(s)y\big(h(s)\big) + \lambda \int_{-1}^1 k(s,t)[y(t)]^3 dt = g(s)\,, \quad -1 \le s \le 1. \tag{2}$$

The aim of our method is to get solution as truncated Chebyshev series defined by

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

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$$y(s) \simeq y_N(s) = \sum_{j=0}^{N}{}' a_j T_j(s), \qquad -1 \le s \le 1, \tag{3}$$

where $T_j(s)$ denote the Chebyshev polynomials of the first kind, a_j are unknown Chebyshev coefficients, N is chosen any positive integer and $\sum'$ is a sum whose first term is halved.

II.FUNDAMENTAL RELATIONS

We suppose that kernel functions and solutions of equations (1) and (2) can be expressed as a truncated Chebyshev series. Then (3) can be written in the matrix form

$$y_N(s) = \mathrm{T}(s)\mathrm{A}, \tag{4}$$

where

$$\mathrm{T}(s) = [T_0(s)\ T_1(s)\ \cdots\ T_N(s)],\ \mathrm{A} = [\tfrac{a_0}{2}\, a_1\ \ldots\ a_N]^{\mathrm{T}}.$$

Besides, $[y(s)]^2$ function can be written in the [5]

$$[y(s)]^2 = \overline{\mathrm{T}(s)}\mathrm{B}$$

in which

$$\overline{\mathrm{T}(s)} = [T_0(s)\ T_1(s)\ \cdots\ T_{2N}(s)], \qquad \mathrm{B} = [\tfrac{b_0}{2}\, b_1\ \ldots\ b_{2N}]^{\mathrm{T}}$$

and the elements b_i of the column matrix B consist of a_i and $a_{-i} = a_i$ as follows:

$$b_i = \begin{cases} \dfrac{(a_{\frac{i}{2}})^2}{2} + \displaystyle\sum_{r=1}^{N-\frac{i}{2}} \left(a_{\frac{i}{2}-r}\right)\left(a_{\frac{i}{2}+r}\right) & \text{for even i,} \\ \displaystyle\sum_{r=1}^{N-\frac{i-1}{2}} \left(a_{\frac{i+1}{2}-r}\right)\left(a_{\frac{i+1}{2}+r}\right) & \text{for odd i,} \end{cases}$$

also, $[y(s)]^3$ function can be written

$$[y(s)]^3 = \overline{\overline{\mathrm{T}(s)}}\mathrm{C}, \tag{5}$$

in which

$$\overline{\overline{\mathrm{T}(s)}} = [T_0(s)\ T_1(s)\ \cdots\ T_{3N}(s)], \qquad \mathrm{C} = \tfrac{1}{2}\bar{\mathrm{C}}, \qquad \bar{\mathrm{C}} = [c_0\ c_1\ \ldots\ c_{3N}]^{\mathrm{T}}$$

and the elements c_i of the column matrix $\bar{\mathrm{C}}$ consist of a_i and b_i also $a_{-i} = a_i$ and $b_{-i} = b_i$ as follows:

$$c_i = \begin{cases} \frac{1}{2}a_0 b_0 + \displaystyle\sum_{r=1}^{N} a_r b_r \text{ for } i = 0, \\ a_{\frac{i}{2}} b_{\frac{i}{2}} + \displaystyle\sum_{r=1}^{N-\frac{i}{2}} \left(a_{\frac{i}{2}-r} b_{\frac{i}{2}+r} + a_{\frac{i}{2}+r} b_{\frac{i}{2}-r}\right) + \sum_{r=0}^{i-1} a_{N-r} b_{N-r+i} & \text{for even } i \text{ and } 1 \le i \le N, \\ \displaystyle\sum_{r=1}^{N-\frac{i-1}{2}} \left(a_{\frac{i+1}{2}-r} b_{\frac{i-1}{2}+r} + a_{\frac{i-1}{2}+r} b_{\frac{i+1}{2}-r}\right) + \sum_{r=0}^{i-1} a_{N-r} b_{N-r+i} & \text{for odd } i \text{ and } 1 \le i \le N, \end{cases}$$

and also

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$c_i=\begin{cases} a_{\frac{i}{2}}b_{\frac{i}{2}}+\sum_{r=1}^{N-\frac{i}{2}}\left(a_{\frac{i}{2}-r}b_{\frac{i}{2}+r}+a_{\frac{i}{2}+r}b_{\frac{i}{2}-r}\right)+\sum_{r=0}^{2N-i}a_rb_{r+i}+\sum_{r=1}^{i-N-1}a_rb_{i-r} \\ \quad \text{for even } i \text{ and} N+1\le i\le 2N, \\ \sum_{r=1}^{N-\frac{i-1}{2}}\left(a_{\frac{i+1}{2}-r}b_{\frac{i-1}{2}+r}+a_{\frac{i-1}{2}+r}b_{\frac{i+1}{2}-r}\right)+\sum_{r=0}^{2N-i}a_rb_{r+i}+\sum_{r=1}^{i-N-1}a_rb_{i-r} \\ \quad \text{for odd } i \text{ and} N+1\le i\le 2N, \\ \sum_{r=i-2N}^{N}a_rb_{i-r} \qquad \text{for } 2N+1\le i\le 3N. \end{cases}$$

The method of collocation solves the FIE (1) using the approximation (3) through the equations

$$r_N(s_i)=y_N(s_i)+p(s_i)y_N\big(h(s_i)\big)+\lambda\int_{-1}^{1}k(s_i,t)[y_N(t)]^3dt\;-\;g(s_i)=0, \qquad (6)$$

forChebyshev collocation points

$s_i=\cos(i\pi/N)\in[-1,1], i=1,\cdots,N.$ (7)

Similarly kernel function *k(s,t)*can be expressed asa truncated Chebyshev series for each $s_i$in the form

$$k(s_i,t)\simeq k_N(s_i,t)=\sum_{r=0}^{N}{}'' k_r(s_i)T_r(t), \qquad (8)$$

where double prime denotes that the first and the last terms are halved, the$k_r(s_i)$ are determined by means ofCleanshaw–Kurtis rule, [5] as follows:

$$k_r(s_i)=\frac{2}{\pi}\int_{-1}^{1}\frac{k(s_i,t)T_r(t)}{(1-t^2)^{.5}}dt\simeq\frac{2}{\pi}\times\frac{\pi}{N}\sum_{m=0}^{N}{}''k(s_i,t_m)T_r(t_m)=\frac{2}{N}\sum_{m=0}^{N}{}''k(s_i,t_m)T_r(t_m),$$

where$t_m=\cos(m\pi/N)$for m = 0, 1, ..., N and $k_N(s_i,t)$can be represented in matrix form

$$k_N(s_i,t)=\mathrm{K}(s_i)\mathrm{T}^T(t), \qquad (9)$$

where

$$\mathrm{K}(s_i)=\left[\tfrac{1}{2}k_0(s_i)\;\;k_1(s_i)\;\;\cdots\;\;k_{N-1}(s_i)\tfrac{1}{2}k_N(s_i)\right]. \qquad (10)$$

III. THE METHOD FOR SOLUTION OF NONLINEAR FREDHOLM INTEGRAL EQUATIONS

In this section, we consider Fredholm equation in (1) and approximate to solution by means of finite Chebyshev series defined in (3). The aim is to find Chebyshev coefficients, that is, the matrix A in the matrix form (4). We use (4), (5) and (9) for s_i in (6) to get

$$\mathrm{T}(s_i)\mathrm{A}+p(s_i)\mathrm{T}\big(h(s_i)\big)\mathrm{A}+\lambda\mathrm{K}(s_i)\left[\int_{-1}^{1}\mathrm{T}^T(t)\overline{\overline{\overline{\mathrm{T}(t)}}}dt\right]\mathrm{C}=g(s_i), \qquad (11)$$

for*i=0, 1, ..., N* and using relation

$$\mathrm{Z}=\int_{-1}^{1}\mathrm{T}^T(t)\,\overline{\mathrm{T}(t)}dt=\left[\int_{-1}^{1}\mathrm{T}_i(t)\mathrm{T}_j(t)dt\right]=[\mathrm{Z}_{ij}],$$

whose entries are given in [6] as

$$\mathrm{Z}_{ij}=\begin{cases}\frac{1}{1-(i+j)^2}+\frac{1}{1-(i-j)^2} & \text{for even i + j,}\\ 0 & \text{for odd i + j,}\end{cases}$$

for*i=0, 1, ..., N* and *j=0, 1, ..., 3N.*

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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Then Eqs. (11) can be in write in matrix form

$$(\mathrm{T}_s + \mathrm{P}_s\mathrm{T}_h)\mathrm{A} + \lambda \mathrm{K}_s \mathrm{Z}\mathrm{C} = \mathrm{G}, \qquad (12)$$

where

$$\mathrm{T}_s = \begin{bmatrix} \mathrm{T}(s_0) \\ \mathrm{T}(s_1) \\ \vdots \\ \mathrm{T}(s_N) \end{bmatrix}, \quad \mathrm{T}_h = \begin{bmatrix} \mathrm{T}(h(s_0)) \\ \mathrm{T}(h(s_1)) \\ \vdots \\ \mathrm{T}(h(s_N)) \end{bmatrix}, \quad \mathrm{K}_s = \begin{bmatrix} \mathrm{K}(s_0) \\ \mathrm{K}(s_1) \\ \vdots \\ \mathrm{K}(s_N) \end{bmatrix}, \quad \mathrm{G} = \begin{bmatrix} g(s_0) \\ g(s_1) \\ \vdots \\ g(s_N) \end{bmatrix},$$

$\mathrm{P}_s = diag(p(s_i))_{i=0}^{N}$,

and for Fredholm NIE of the first kind (2), we have

$$\mathrm{P}_s\mathrm{T}_h\mathrm{A} + \lambda \mathrm{K}_s \mathrm{Z}\mathrm{C} = \mathrm{G}. \ (13)$$

The systems (12) and (13) correspond to a system of (N + 1) nonlinear algebraic equations withthe (N + 1) unknown Chebyshev coefficients.

IV. THE METHOD FOR SOLUTION OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS

We now consider the nonlinear Volterra integral equations of the second and first kind as

$$\mathrm{y(s)} + \mathrm{p(s)y(h(s))} + \lambda \int_{-1}^{s} \mathrm{k(s,t)[y(t)]^3} = \mathrm{g(s)}, \qquad -1 \le s \le 1,$$

and

$$\mathrm{p(s)y(h(s))} + \lambda \int_{-1}^{s} \mathrm{k(s,t)[y(t)]^3} = \mathrm{g(s)}, \quad -1 \le s \le 1.$$

In this case the upper bound of integral in (6) should be change to $s_i$and then (11) changes to

$$\mathrm{T}(s_i)\mathrm{A} + p(s_i)\mathrm{T}(h(s_i))\mathrm{A} + \lambda \mathrm{K}(s_i)\left[\int_{-1}^{s_i} \mathrm{T}^T(t)\overline{\overline{\mathrm{T}(t)}}dt\right]\mathrm{C} = g(s_i),$$

for*i=0, 1, ..., N* and then we get

$$(\mathrm{T}_s + \mathrm{P}_s\mathrm{T}_h)\mathrm{A} + \lambda \overline{\mathrm{K}}_s \overline{\mathrm{Z}}\mathrm{C} = \mathrm{G}, \qquad (14)$$

where

$$\overline{\mathrm{K}}_s = \begin{bmatrix} \mathrm{K}(s_0) & 0 & \cdots & 0 \\ 0 & \mathrm{K}(s_0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathrm{K}(s_0) \end{bmatrix}_{(N+1)\times(N+1)^2}, \qquad \overline{\mathrm{Z}} = \begin{bmatrix} \mathrm{Z}(s_0) \\ \mathrm{Z}(s_1) \\ \vdots \\ \mathrm{Z}(s_N) \end{bmatrix}_{(N+1)^2\times(3N+1)}$$

where

$$\mathrm{Z}(s_i) = \int_{-1}^{s_i} \mathrm{T}^T(t)\,\overline{\mathrm{T}(t)}dt = \left[\int_{-1}^{s_i} \mathrm{T}_i(t)\mathrm{T}_j(t)dt\right] = [\mathrm{Z}_{ij}(s_i)],$$

for*i=0, 1, ..., N* and *j=0, 1, ..., 3N*, whose entries are computed in [2] as

$$\mathrm{Z}_{ij} = \frac{1}{4}\begin{cases} 2x^2-2 & \text{for } i+j=1, \\ \frac{\mathrm{T}_{i+j+1}(s)}{i+j+1} - \frac{\mathrm{T}_{i+j-1}(s)}{i+j-1} - \frac{1}{i+j+1} + \frac{1}{i+j-1} + x^2 - 1 & \text{for } |i-j|=1, \\ \frac{\mathrm{T}_{i+j+1}(s)}{i+j+1} + \frac{\mathrm{T}_{1-i-j}(s)}{1-i-j} + \frac{\mathrm{T}_{1+i-j}(s)}{1+i-j} + \frac{\mathrm{T}_{1-i+j}(s)}{1-i+j} + 2\left[\frac{1}{1-(i+j)^2} + \frac{1}{1-(i-j)^2}\right] & \text{for even } i+j, \\ \frac{\mathrm{T}_{i+j+1}(s)}{i+j+1} + \frac{\mathrm{T}_{1-i-j}(s)}{1-i-j} + \frac{\mathrm{T}_{1+i-j}(s)}{1+i-j} + \frac{\mathrm{T}_{1-i+j}(s)}{1-i+j} - 2\left[\frac{1}{1-(i+j)^2} + \frac{1}{1-(i-j)^2}\right] & \text{for odd } i+j, \end{cases}$$

these equations for first kind will be as

$$\mathrm{P}_s\mathrm{T}_h\mathrm{A} + \lambda \overline{\mathrm{K}}_s \overline{\mathrm{Z}}\mathrm{C} = \mathrm{G}.$$

V. ILLUSTRATIONS

In this section, we consider threeexamples. All results were computed using MATLAB.

Example 1. Let us first consider the nonlinear Volterraintegral equation of second kind

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$y(s)+\frac{s}{2}y\left(\frac{s^2}{2}\right)+\int_{-1}^{s}(s+1)[y(t)]^3dt=\tfrac{1}{4}s^5+\tfrac{5}{4}s^4+\tfrac{11}{4}s^3+\tfrac{5}{2}s^2+\tfrac{11}{4}s+\tfrac{5}{4}$$

with the exact solution

$y(s)=s+1,$

and $\lambda=1$.We choose N = 2 and write the solution by truncated Chebyshev series in the form

$$y(s)\simeq y_2(s)=\sum_{j=0}^{2}{}' a_jT_j(s),\qquad -1\le s\le 1. \tag{15}$$

The Chebyshev collocation points are

$s_0=1,\quad s_1=0,\quad s_2=-1.$

Using these points, the matrices in (14) are

$$\lambda=1\ ,\qquad \mathrm{T}_s=\begin{bmatrix}1&1&1\\1&0&-1\\1&-1&1\end{bmatrix},\quad \mathrm{T}_h=\begin{bmatrix}1&\frac{1}{2}&\frac{-1}{2}\\1&0&-1\\1&\frac{1}{2}&\frac{-1}{2}\end{bmatrix},\quad \mathrm{G}=\begin{bmatrix}\frac{43}{4}\\\frac{5}{4}\\\frac{-3}{4}\end{bmatrix},$$

$$\mathrm{P}_s=diag(0.5\,,0,-0.5),\quad \overline{K}_s=\begin{bmatrix}2&0&00&0&00&0&0\\0&0&01&0&00&0&0\\0&0&00&0&00&0&0\end{bmatrix},$$

$$\mathrm{A}=\begin{bmatrix}\frac{a_0}{2}\\a_1\\a_2\end{bmatrix},\mathrm{B}=\begin{bmatrix}\frac{1}{2}b_0\\b_1\\b_2\\b_3\\b_4\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\left(\frac{a_0{}^2}{2}+a_1{}^2+a_2{}^2\right)\\a_0a_1+a_1a_2\\\frac{a_1{}^2}{2}+a_0a_2\\a_1a_2\\\frac{a_2{}^2}{2}\end{bmatrix},$$

$$\mathrm{C}=\tfrac{1}{2}\overline{\mathrm{C}}=\frac{1}{2}\begin{bmatrix}\frac{1}{2}a_0b_0+a_1b_1+a_2b_2\\a_0b_1+a_1b_0+a_1b_2+a_2b_1+a_2b_3\\a_1b_1+a_0b_2+a_2b_0+a_2b_4+a_1b_3\\a_1b_2+a_2b_1+a_0b_3+a_1b_4\\a_2b_2+a_0b_4+a_1b_3\\a_1b_4+a_2b_3\\a_2b_4\end{bmatrix},$$

$$\overline{\mathrm{Z}}=\begin{bmatrix}2&0&-2/3&0&-2/15&0&-2/35\\0&2/3&0&-2/5&0&-2/21&0\\-2/3&0&14/15&0&-38/105&0&-26/315\\1&-1/2&-1/3&1/2&-1/15&-1/6&-1/35\\-1/2&1/3&0&-1/5&1/6&-1/21&0\\-1/3&0&7/15&-1/3&-19/105&1/3&-13/315\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix}.$$

If these matrices are substituted in (14), it is obtained nonlinear algebraic system.

This system yields the solution

$a_0=2,\ \ a_1=1,\qquad a_2=0$

Substituting these values in (15) we have

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$y_2(s) = s + 1,$
which is the exact solution.
Example 2. Let us now consider the following nonlinearFredholm integral equation of the second kind

$$y(s) + e^{-s}y(h(s)) + \int_{-1}^{1} e^{s-t}[y(t)]^3 dt = g(s),$$

with the exact solution

$$y(s) = e^s.$$

In this example we consider four different options for *h(s)* as
(1) $h(s) = 0.8s,$ (2) $h(s) = s^2/2,$
(3) $h(s) = se^{-s},$ (4) $h(s) = \sin(s),$
and with appropriate right hand side. The numerical results are shown in Table 1.
Example 3. Let us now consider the following nonlinearVolterra integral equation of thesecond kind

$$y(s) + e^{-s}y(h(s)) + \int_{-1}^{s} \sin(s - t)[y(t)]^3 dt = g(s),$$

with the exact solution

$$y(s) = e^s.$$

In this example, again we consider four different options for *h(s)* as
(1) $h(s) = 0.8s,$ (2) $h(s) = s^2/2,$
(3) $h(s) = se^{-s},$ (4) $h(s) = \sin(s),$
and with appropriate right hand side. The numerical results are shown in Table 2.

Table 1
Computed error δ_N for Example 2

N	*h(s)*			
	(1)	(2)	(3)	(4)
3	.0027	0.0042	0.0125	0.0020
5	2.3963e-05	5.5536e-05	0.0010	1.6749e-05
10	3.2459e-11	5.1252e-11	8.1415e-09	1.7490e-11
15	2.4535e-15	5.7179e-15	2.2946e-13	1.5200e-15

Table 2
Computed error δ_N for Example 3

N	*h(s)*			
	(1)	(2)	(3)	(4)
3	0.0044	0.0061	0.0113	0.0039
5	2.5582e-05	6.6308e-05	0.0011	1.8978e-05
10	3.0486e-11	3.4939e-11	7.9672e-09	1.7633e-11

In Examples 2 and 3 wecomputed error δ_n by

$$\delta_N = \frac{\|y_{exact}(s) - y_N(s)\|_2}{\|y_{exact}(s)\|_2} = \left\{\sum_{i=0}^{N} e_n^2(s_i)\right\}^{\frac{1}{2}} / \left\{\sum_{i=0}^{N} y_{exact}^2(s_i)\right\}^{\frac{1}{2}}$$

where

$$e_n(s_i) = y_{exact}(s_i) - y_N(s_i).$$

Numerical examples and Tables 1 and 2 show that the proposed method works quite well in practice for differentchoice of *h(s)*.

VI. CONCLUSIONS

Here, we use the Chebyshev collocation method to solve nonlinear integral equation of the first and second kind Fredholm–Volterra integral equations of the second kind by transformingour problems into a system of nonlinear algebraic equations. With using Chebyshev collocations points,the unknown vector is Chebyshev expansion coefficients of the solution. Numerical examples show the accuracyof this method.

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