A Comprehensive Comparative Performance Analysis of Eigenfaces, Laplacianfaces and Orthogonal Laplacianfaces for Face Recognition

H.Arora[1], K. Tayagi[1], P.Sharma[1], K.Jain[2]

UG student, Dept. of ECE, Amity school of Engineering & Technology, Amity University, Haryana, India [1]

Lecturer, Dept. of ECE, Amity school of Engineering & Technology, Amity University, Haryana, India [2]

ABSTRACT: Facial analysis and recognition is one of the most popular information processing tasks. Face recognition requires some form of dimensionality reduction in the face dataset. There are numerous face representation and recognition techniques which vary in their complexity and efficiency. In this study, we have done a comparative analysis of Eigenfaces based on Principal Component Analysis (PCA), Laplacianfaces based on Locality Preserving Projection (LPP) and Orthogonal Laplacianfaces based on Orthogonal LPP (OLPP), on a standard face database-YALE. PCA is an eigenvector method which aims to preserve the global Euclidean structure of the face space. Its goal is to find a set of mutually orthogonal basis functions that capture the directions of maximum variance in the data. LPP method finds an embedding that preserves local information and obtains a face space that explicitly considers the manifold structure. LPP algorithm aims at finding a linear approximation to the eigen functions of the Laplace Beltrami operator on the face manifold which reflects the intrinsic face manifold structure. OLPP algorithm is based on the LPP algorithm. However, LPP is non-orthogonal and this makes it difficult to reconstruct the data. The OLPP method produces orthogonal basis functions and can have more locality preserving power than LPP. Nearest neighbor classification using OLPP are used to obtain the error rate.

Keywords: Face Recognition, Principal Component Analysis, Locality Preserving Projections, Orthogonal locality preserving projection, Face Manifold, Subspace Learning

I.   INTRODUCTION

Face recognitions an emerging concept in the field of research in Computer science and technology[1]. There are numerous face recognition techniques developed in past few decades. The face recognition techniques can be grouped into: appearance based technique, feature based technique, model based technique [1][6]. One of the most successful and well studied techniques is the appearance based methods[6]. In this method, we consider an image of size m by n pixels as a vector drawn in a mn dimensional space. However, this 'mn' dimensional space is too large to allow efficient and quick computation and also results in a training set that is too big to work on. Hence, some form of dimensionality reduction is required to work with such methods [3][4][5][6]. The classical but popular form of linear technique is method of Principal Component Analysis (PCA) [6][13][17]which is an eigenvector method designed to model linear variations in a high-dimensional data.

PCA approach was developed by Turk and Pentland [6] to describe face images in term of set of basic functions or "Eigenfaces". PCA performs dimensionality reduction by projecting the high dimensional data onto a lower dimension[16] subspace that is formed by taking "principal" eigenvectors of the covariance matrix of data. PCA effectively view the overall global or Euclidean structure of the face images and do not take into account the nonlinear structure of the face manifold hidden in image space. The Laplacianface method [4] is proposed by Xiao fei He, to model the local manifold structure. The Laplacianfaces are the linear approximations to the eigen functions of the Laplace Beltrami operator on the face manifold[16]. A face subspace is obtained by Locality Preserving Projections (LPP) [4][9][3]. Each face image in the image space is mapped to a low dimensional face subspace, which is characterized by a set of feature images, called Laplacianfaces. The face subspace preserves local structure, seems to have more discriminating power than the PCA approach for classification purpose However, the basic functions obtained by the Laplacianface method are non-orthogonal. This makes it difficult to reconstruct the data.

An appearance based approach called orthogonal laplacianfaces proposed by Deng caiet al.[3]. O-Laplacianface is fundamentally based on the Laplacianface method. It builds an adjacency graph which can best reflect the geometry of the face manifold and the class relationship between the sample points. The projections are then obtained by preserving such a graph structure. It shares the same locality preserving character as Laplacianface, but it also requires the basis functions to be orthogonal. Orthogonal basis functions preserve the metric structure of the face space.

In fact, if we use all the vectors obtained by O-Laplacianface, the projection subspace is a rotation map which does not distort the metric structure. As it is given that the locality preserving power is directly related to the discriminating power [11], theO-Laplacianface is expected to have more discriminating power than Laplacianface. In this study, we have done a comparative analysis of Eigenfaces based on Principal Component Analysis (PCA), Laplacianfaces based on Locality Preserving Projection (LPP) and Orthogonal Laplacianfaces based on Orthogonal LPP (OLPP), on a standard face database- YALE. The general flow diagram of above approaches is shown below:
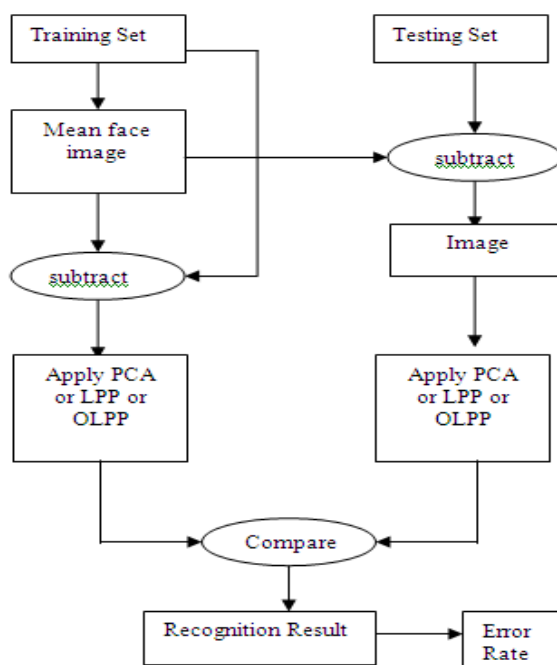

Fig.1. General flow diagram

II.   LITERATURE SURVEY

In LPP, the face images are mapped into a face subspace for analysis[4][8]. Different from Principal Component Analysis (PCA) which effectively see only the Euclidean structure of face space [6][9].LPP finds an embedding that preserve local information, and obtains a face subspace that best detects the essential face manifold structure[4].
It was the first devoted work on face representation and recognition which explicitly considers the manifold structure. The manifold structure is approximated by the adjacency graph computed from the data points. Using the notion of the Laplacian of the graph, a transformation matrix was computed which maps the face images into a face subspace.

OLPP is based on the LPP algorithm, which aims at finding a linear approximation to the eigenfunctions of the Laplace Beltrami operator on the face manifold[3]. However, LPP is non- orthogonal and this makes it difficult to reconstruct the data.The OLPP method produces basis functions and can have more locality preserving power than LPP. Since the

locality preserving power is potentially related to the discriminating power, the OLPP is expected to have more discriminating power too.

III. EIGENFACES APPROACH

Proposed in 1991 by Turk and Pentland [6], this was the first genuinely successful system for automatic recognition of human faces. It was a breakaway from contemporary research trend on face recognition which focused on detecting individual features such as eyes, nose, mouth, and head outline, and defining a face model based on position and size of these features, as well as geometrical relationship between them. Despite being economical representations of a face, these methods are quite sensitive to the feature extraction and measurement process [18]. Lack of existing techniques for effective extraction and measurement of facial features presents a drawback for such methods [19].

The task of facial recognition is discriminating input signals (image data) into several classes (persons). The input signals are highly noisy (e.g. the noise is caused by differing lighting conditions, pose etc.), yet the input images are not completely random and in spite of their differences there are patterns which occur in any input signal. Such patterns, which can be observed in all signals could be - in the domain of facial recognition - the presence of some objects (eyes, nose, mouth) in any face as well as relative distances between these objects. These characteristic features are called eigenfaces in the facial recognition field (or principal components generally). They can be extracted out of original image data by means of a mathematical tool known as PCA.   These components span a subspace within the image space where each different face in the training set has a unique position. The Eigenfaces emphasize significant discriminatory "features" that causes large variations between the faces used for training, which subsequently allows them to be differentiated. It should be noted that these features may not necessarily correspond to facial features mentioned earlier. Recognition is then performed by projecting the test image into the subspace spanned by the Eigenfaces and classified based on the distance of the projection from positions of known faces.

By means of PCA one can transform each original image of the training set into a corresponding eigenface. An important feature of PCA is that one can reconstruct any original image from the training set by combining the eigenfaces. Remember that eigenfaces are nothing less than characteristic features of the faces. Therefore one could say that the original face image can be reconstructed from eigenfaces if one adds up all the eigenfaces (features) in the right proportion.

Principal Component Analysis

Given an s-dimensional vector representation of each face in a training set of M images, PCA [6][22][9] tends to find a t-dimensional subspace whose basis vectors correspond to the maximum variance direction in the original image space. This new subspace is normally lower dimensional (t $\ll$ s). New basis vectors define a subspace of face images called face space. All images of known faces are projected onto the face space to find a set of weights that describes the contribution of each vector. To identify an unknown image, that image is projected onto the face space to obtain its set of weights. By comparing a set of weights for the unknown face to sets of weights of known faces, the face can be identified. If the image elements are considered as random variables, the PCA basis vectors are defined as eigen vectors of the scatter matrix S_T defined as:

$$S_T = \sum_{i=1}^{M} (x_i - \mu) - (x_i - \mu)^T$$

where μ is the mean of all images in the training set (the mean face) and x_i is the i-th image with its columns concatenated in a vector. The projection matrix W_{PCA} is composed of teigenvectors corresponding to t largest eigenvalues, thus creating a t-dimensional face space. We implemented PCA procedure as described in [6].

IV.   LAPLACIANFACES APPROACH

In this section, we describe LPP [8][4], an algorithm for learning a locality preserving subspace. Each face image in the image space is mapped to a low dimensional face subspace which is obtained by LPP, which is characterized by a set of feature images, called Laplacianfaces. It builds a graph incorporating neighborhood information of the data set. Using the notion of the Laplacian of the graph[11], we then compute a transformation matrix which maps the data points to a subspace. This linear transformation optimally preserves local neighborhood information in a certain sense. The representation map generated by the algorithm may be viewed as a linear discrete approximation to a continuous map that naturally arises from the geometry of the manifold [1][2][12]. The LPP algorithm is interesting from a number of perspectives [8]. 1) The maps are designed to minimize a different objective criterion from the classical linear techniques. 2) The locality preserving quality of LPP is likely to be of particular use in information retrieval applications. If one wishes to retrieve audio, video, text documents under a vector space model, then one will ultimately need to do a nearest neighbor search in the low dimensional space. Since LPP is designed for preserving local structure, it is likely that a nearest neighbor search in the low dimensional space will yield similar results to that in the high dimensional space. This makes for an indexing scheme that would allow quick retrieval. 3) LPP is linear. This makes it fast and suitable for practical application. While a number of non linear techniques have properties (1) and (2) above, we know of no other linear projective technique that has such a property. 4) LPP is defined everywhere. Recall that nonlinear dimensionality reduction[7] techniques like ISOMAP[6], LLE[5], Laplacian eigenmaps[2] are defined only on the training data points and it is unclear how to evaluate the map for new test points. In contrast, the Locality Preserving Projection may be simply applied to any new data point to locate it in the reduced representation space.

As a result of all these features, we expect the LPP based techniques[4][3] to be a natural alternative to PCA based techniques in exploratory data analysis, information retrieval, and pattern classification applications. The complete derivation and theoretical justifications of LPP can be traced back to [8]. LPP seeks to preserve the intrinsic geometry of the data and local structure. Given a set of face images $\{x_1, x_2 \ldots\ldots, x_n\} \subset \Re^m$, let X=$\{x_1, x_2 \ldots\ldots, x_n\}$. Let S be a similarity matrix defined on the data points. Laplacianface can be obtained by solving the following minimization problem

$$a_{opt} = arg \min_{a} \sum_{i=1}^{m} \sum_{j=1}^{m} (a^T x_i - a^T x_j)^2 S_{ij}$$

$$= arg \min_{a} a^T XLX^T a$$

with constraint $a^T XDX^T a = 1$ where L=D-S is the graph Laplacian[4] and $D_{ii} = \sum_{j} S_{ij} . D_{ii}$ measures the local density around x_i .

A.   Calculation of Laplacianfaces with LPP

   The algorithmic procedure of LPP is given below:
1)   PCA projection.
     We project the image set into the PCA subspace by throwing away the smallest principal components. x to denote the images in the PCA subspace in the following steps. We denote by W_{PCA} the transformation matrix of PCA.
2)   Constructing the nearest-neighbor graph.
     Let G denote a graph with n nodes. The ith node corresponds to the face image x_i. We put an edge between nodes i and j if xi and x_i are "close," i.e., x_i is among k nearest neighbors of x_i, or xi is among k nearest neighbors of x_j. The constructed nearest neighbor graph is an approximation of the local manifold structure.
3)   Choosing the weights.
     If node i and j are connected, put

$$S_{ij} = e^{\frac{-\|x_i - x_j\|^2}{t}}$$

where t is a suitable constant. Otherwise, put $S_{ij} = 0$. The weight matrix S of graph G models the face manifold structure by preserving local structure.

4) Eigenmap:

Compute the eigenvectors and eigenvalues for the generalized eigenvector problem:

$$XLX^T w = XDX^T w \qquad (1)$$

where D is a diagonal matrix whose entries are column (or row, since S is symmetric) sums of S, $D_{ii} = \sum_j S_{ji}$. $L = D - S$ is the Laplacian matrix. The ith row of matrix X is x_i.

Let w_0, w_1, …,w_{k-1} be the solutions of equation (1), ordered according to their Eigenvalues,$0 \leq \lambda_0 \leq \lambda_1 \ldots \leq \lambda_{k-1}$. These Eigenvalues are equal to or greater than zero because the matrices XLX^T and XDX^T are both symmetric and positive semi definite. Thus, the embedding is as follows:

$$x \rightarrow y = W^T x$$
$$W = W_{PCA} W_{LPP}$$

where,

$W_{LPP} = [w_0; w_1; \ldots ; w_{k-1}]$;
W_{PCA} = Transformation matrix of PCA;
W = Transformation matrix of Laplacianface.

This linear mapping best preserves the manifold's estimated intrinsic geometry in a linear sense. The column vectors of W are the so-called Laplacianfaces. XDX^T is non-singular after some pre-processing steps on X in Laplacianface, thus, the basic functions of Laplacianface can also be regarded as the eigenvectors of the matrix $(XDX^T)^{-1} XLX^T$ associated with the smallest eigenvalues. Since $(XDX^T)^{-1} XLX^T$ is not symmetric in general, the basic functions of Laplacianface are non-orthogonal.

V.   ORTHOGONAL LAPLACIANFACES APPROACH

Orthogonal Laplacianfaces is based on Orthogonal LPP. OLPP is a novel subspace learning algorithm presented by Den cai et al.[3].This algorithm is built on the base of the LPP method[5] and primarily devised to solve the problem of face recognition. Actually, OLPP is an effective dimensionality reduction method falling into category of manifold learning[15].In appearance-based face analysis one is often confronted with the fact that the dimension ofthe face image vector (m) is much larger than the number of face images (n). Thus, the m × m matrix XDX^T is singular. To overcome this problem, we can first apply PCA to project the facesinto a subspace without losing any information and the matrix XDX^T becomes non-singular.

The algorithmic procedure of OLPP is stated below:

1) PCA projection
   Projecting the high dimensionality points x_i into the PCA subspace by throwing away the components corresponding to zero eigenvalue. The transformation matrix of PCA is W_{PCA} .

2) Constructing the nearest-neighbor graph
   Let G denote a graph with n nodes. The ith node corresponds to the face image x_i. We put an edge between nodes i and j if x_i and x_j are "close," i.e., x_j is among k nearest neighbors of x_i, or x_i is among k nearest neighbors of x_j. The constructed nearest neighbor graph is an approximation of the local manifold structure. Note that here we do not use the neighborhood to construct the graph. This is simply because it is often difficult to choose the optimal "in the real-world applications, while k nearest-neighbor graph can be constructed more stably. The disadvantage is that the

k nearest-neighbor search will increase the computational complexity of algorithm. When the computational complexity is a major concern, one can switch to the ε -neighborhood.

3) Choosing the weights
   If node i and j are connected, put

$$S_{ij} = e^{\frac{-\|x_i - x_j\|^2}{t}}$$

Where t is a suitable constant. Otherwise, put $S_{ij} = 0$. The weight matrix S of graph G models the face manifold structure by preserving local structure.

4) Computing the Orthogonal Basis Function
   Let $w_1, w_2, ..., w_k$ be orthogonal basis vectors, defining

$$A^{(k-1)} = [w_1, w_2, ..., w_k]$$
$$B^{(k-1)} = [A^{(k-1)}]^T (XDX^T)^{-1} A^{(k-1)}$$

The orthogonal basis vectors $w_1, w_2, ..., w_k$ can be computed as follow.
   ▪ Compute w_1 as the eigenvector of $(XDX^T)^{-1} XLX^T$ associated with the smallest eigenvalue.
   ▪ Compute w_k as the eigenvector of

$$M^{(k)} = \{I - (XDX^T)^{-1} A^{(k-1)} [B^{(k-1)}]^{-1} [A^{(k-1)}]^T\}.(XDX^T)^{-1} XLX^T$$

associated with the smallest eigenvalue of $M^{(K)}$.

5) OLPP Embedding
   Let $W_{OLPP} = [w_1, w_2, ..., w_l]$, the embedding is as follows.

$$x \rightarrow y = W^T x$$
$$W = W_{PCA} W_{OLPP}$$

Where y is a l -dimensional representation of the face image x, and W is the transformation matrix.

In this paper, utilizing the orthogonality of the base functions, we reduce the dimensionality of face space and then reconstruct the data.

VI.   EXPERIMENTAL RESULTS

A face image can be represented as a point in image space. However, due to the unwanted variations resulting from changes in lighting, facial expression, and pose, the image space might not be an optimal space for visual representation. We can display the eigenvectors as images. The recognition process has three steps. First, we calculate the face subspace from the training samples; then the new face image to be identified is projected into d-dimensional subspace by using our algorithm; finally, the new face image is identified by a nearest neighbor classifier[3][4]All recognition algorithms and all image preprocessing[2] steps were implemented in MATLAB 7.8.0.347

Recognition Analysis

In this sub-section, we compare the three approaches for face representation, i.e., Eigenface, Laplacianface and O-Laplacianface. For each of them, the basis vectors can be thought of as the basis images and any other image is a linear combination of these basis images. The Yale face database was constructed at the Yale Center   for Computational Vision and Control [20]. It contains 165 grayscale images of 15 individuals. The database contains 165 GIF images of 15 subjects There are 11 images per subject, one for each of the following facial expressions or configurations: center-light, w/glasses, happy, left-light, w/no glasses, normal, right-light, sad, sleepy, surprised, and wink. Images are of GIF

format and size is 320x240.By varying images from 1 to 10 per subject, the training set has been created. The rest was taken for testing. The testing samples were then projected into the low-dimensional Representation. Recognition was performed using a nearest-neighborclassifier.

In general, the performance of the Eigenfaces method, the Laplacianfaces method and O-Laplacianfaces method varies with the number of images used for training. We show the best results obtained by Eigenfaces, Laplacianfaces and O-Laplacainfaces. The performance comparison between Laplacianfaces, PCA and O-Laplacianfaces on 2,4,5 training images per subject with different value of subDim given in bracket are shown in Table 1. It is found that the O-Laplacianfaces method significantly outperforms Eigenfaces method and Laplacianfacs method. Note: subDim here indicates number of principal components chosen for an image. Graph has been plotted for the 2,4,5 training images per subject with varying subDim shown in Fig. 2.Table 1 shows the best results obtain by above three method with different training images for particular subDim. Table 2 presents percentage error rate reduction for LPP and OLPP on 2,4,5 training images per subject.
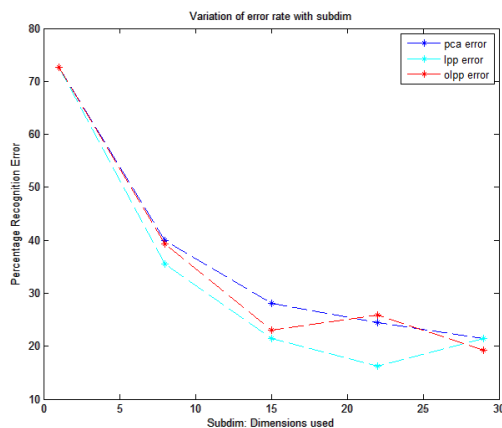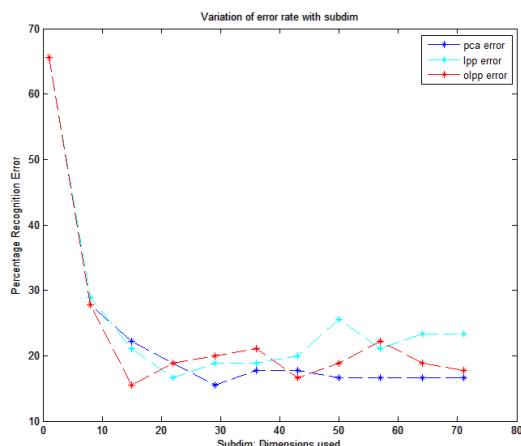

(a) 2 Training images per subject
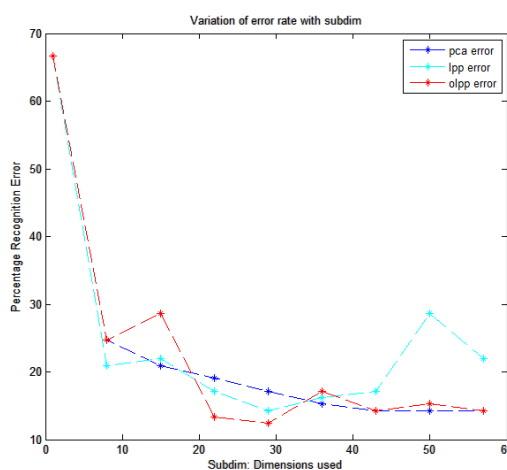
(b) 4 Training images per subject

(c) 5 Training images per subject
Fig. 2: Error rate vs. subdim on Yale database

TABLE I
PERCENTAGE ERROR RATE COMPARISON

Approach	2 Train	4 Train	5 Train
PCA	21.48(29)	17.14(29)	22.22(15)
Laplacianfaces	21.48(29)	14.29(29)	21.11(15)
O-Laplacianfaces	19.26(29)	12.38(29)	15.56(15)

TABLE 2
PERCENTAGE REDUCTION IN ERROR RATE

Approach	2 Train	4 Train	5 Train
Laplacianfaces	21.48(29)	14.29(29)	21.11(15)
O-Laplacianfaces	19.26(29)	12.38(29)	15.56(15)
Reduction in error rate (%)	10.33	13.36	26.29

VII. CONCLUSION

A comparative study of the error rate between PCA approach, Laplacianfaces approach and O-Laplacianfaces has been done. We have found that O-Laplacianfaces consistently perform better than Eigenfaces and Laplacianfaces for 2, 4 and 5 training images per subject. The algorithms applied takes advantage of more training samples, which is important to the real world face recognition systems. Comparing to the Eigenfaces method and Laplacianfaces method, the O-Laplacianfaces method encodes more discriminating information by preserving local structure. By discovering the face manifold structure, Laplacianfaces can identify the person with different expressions, poses and lighting conditions. Experimental results show the effectiveness of the O-Laplacianface method, Laplacianfaces method and Eigenfaces method. Thus we can conclude that local manifold learning techniques prove to be better than the traditional global manifold based techniques, which consider only the Euclidean structure and do not take into account the local features. Also, considering the Fig 2 we can conclude which method out of the three is better on what range of sub dimensions taken.

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